

# Mathematica 11.3 Integration Test Results

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Test results for the 3189 problems in "1.1.1.3 (a+b x)^m (c+d x)^n (e+f x)^p.m"

Problem 6: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x) (a c - b c x)^3}{x^3} dx$$

Optimal (type 1, 18 leaves, 1 step):

$$-\frac{c^3 (a - b x)^4}{2 x^2}$$

Result (type 1, 41 leaves):

$$c^3 \left( -\frac{a^4}{2 x^2} + \frac{2 a^3 b}{x} + 2 a b^3 x - \frac{b^4 x^2}{2} \right)$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x) (a c - b c x)^4}{x^7} dx$$

Optimal (type 1, 41 leaves, 2 steps):

$$-\frac{c^4 (a - b x)^5}{6 x^6} - \frac{7 b c^4 (a - b x)^5}{30 a x^5}$$

Result (type 1, 85 leaves):

$$-\frac{a^5 c^4}{6 x^6} + \frac{3 a^4 b c^4}{5 x^5} - \frac{a^3 b^2 c^4}{2 x^4} - \frac{2 a^2 b^3 c^4}{3 x^3} + \frac{3 a b^4 c^4}{2 x^2} - \frac{b^5 c^4}{x}$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x) (a c - b c x)^5}{x^4} dx$$

Optimal (type 1, 18 leaves, 1 step):

$$-\frac{c^5 (a - b x)^6}{3 x^3}$$

Result (type 1, 63 leaves):

$$c^5 \left( -\frac{a^6}{3x^3} + \frac{2a^5b}{x^2} - \frac{5a^4b^2}{x} - 5a^2b^4x + 2ab^5x^2 - \frac{b^6x^3}{3} \right)$$

**Problem 44: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)(ac-bcx)^6}{x^9} dx$$

Optimal (type 1, 41 leaves, 2 steps):

$$-\frac{c^6(a-bx)^7}{8x^8} - \frac{9bc^6(a-bx)^7}{56ax^7}$$

Result (type 1, 112 leaves):

$$-\frac{a^7c^6}{8x^8} + \frac{5a^6bc^6}{7x^7} - \frac{3a^5b^2c^6}{2x^6} + \frac{a^4b^3c^6}{x^5} + \frac{5a^3b^4c^6}{4x^4} - \frac{3a^2b^5c^6}{x^3} + \frac{5ab^6c^6}{2x^2} - \frac{b^7c^6}{x}$$

**Problem 93: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^5 (A+Bx) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(Ab-aB)(a+bx)^6}{6b^2} + \frac{B(a+bx)^7}{7b^2}$$

Result (type 1, 109 leaves):

$$a^5Ax + \frac{1}{2}a^4(5Ab+aB)x^2 + \frac{5}{3}a^3b(2Ab+aB)x^3 + \frac{5}{2}a^2b^2(Ab+aB)x^4 + ab^3(Ab+2aB)x^5 + \frac{1}{6}b^4(Ab+5aB)x^6 + \frac{1}{7}b^5Bx^7$$

**Problem 101: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^5 (A+Bx)}{x^8} dx$$

Optimal (type 1, 44 leaves, 2 steps):

$$-\frac{A(a+bx)^6}{7ax^7} + \frac{(Ab-7aB)(a+bx)^6}{42a^2x^6}$$

Result (type 1, 104 leaves):

$$-\frac{1}{42x^7} (21b^5x^5(A+2Bx) + 35a^4x^4(2A+3Bx) + 35a^2b^3x^3(3A+4Bx) + 21a^3b^2x^2(4A+5Bx) + 7a^4bx(5A+6Bx) + a^5(6A+7Bx))$$

### Problem 113: Result more than twice size of optimal antiderivative.

$$\int x^3 (a+bx)^{10} (A+Bx) dx$$

Optimal (type 1, 112 leaves, 2 steps):

$$\begin{aligned} & -\frac{a^3 (Ab - aB) (a+bx)^{11}}{11b^5} + \frac{a^2 (3Ab - 4aB) (a+bx)^{12}}{12b^5} - \\ & \frac{3a (Ab - 2aB) (a+bx)^{13}}{13b^5} + \frac{(Ab - 4aB) (a+bx)^{14}}{14b^5} + \frac{B (a+bx)^{15}}{15b^5} \end{aligned}$$

Result (type 1, 231 leaves):

$$\begin{aligned} & \frac{1}{4} a^{10} A x^4 + \frac{1}{5} a^9 (10Ab + aB) x^5 + \frac{5}{6} a^8 b (9Ab + 2aB) x^6 + \\ & \frac{15}{7} a^7 b^2 (8Ab + 3aB) x^7 + \frac{15}{4} a^6 b^3 (7Ab + 4aB) x^8 + \frac{14}{3} a^5 b^4 (6Ab + 5aB) x^9 + \\ & \frac{21}{5} a^4 b^5 (5Ab + 6aB) x^{10} + \frac{30}{11} a^3 b^6 (4Ab + 7aB) x^{11} + \frac{5}{4} a^2 b^7 (3Ab + 8aB) x^{12} + \\ & \frac{5}{13} a b^8 (2Ab + 9aB) x^{13} + \frac{1}{14} b^9 (Ab + 10aB) x^{14} + \frac{1}{15} b^{10} B x^{15} \end{aligned}$$

### Problem 114: Result more than twice size of optimal antiderivative.

$$\int x^2 (a+bx)^{10} (A+Bx) dx$$

Optimal (type 1, 87 leaves, 2 steps):

$$\frac{a^2 (Ab - aB) (a+bx)^{11}}{11b^4} - \frac{a (2Ab - 3aB) (a+bx)^{12}}{12b^4} + \frac{(Ab - 3aB) (a+bx)^{13}}{13b^4} + \frac{B (a+bx)^{14}}{14b^4}$$

Result (type 1, 226 leaves):

$$\begin{aligned} & \frac{1}{3} a^{10} A x^3 + \frac{1}{4} a^9 (10Ab + aB) x^4 + a^8 b (9Ab + 2aB) x^5 + \\ & \frac{5}{2} a^7 b^2 (8Ab + 3aB) x^6 + \frac{30}{7} a^6 b^3 (7Ab + 4aB) x^7 + \frac{21}{4} a^5 b^4 (6Ab + 5aB) x^8 + \\ & \frac{14}{3} a^4 b^5 (5Ab + 6aB) x^9 + 3a^3 b^6 (4Ab + 7aB) x^{10} + \frac{15}{11} a^2 b^7 (3Ab + 8aB) x^{11} + \\ & \frac{5}{12} a b^8 (2Ab + 9aB) x^{12} + \frac{1}{13} b^9 (Ab + 10aB) x^{13} + \frac{1}{14} b^{10} B x^{14} \end{aligned}$$

### Problem 115: Result more than twice size of optimal antiderivative.

$$\int x (a+bx)^{10} (A+Bx) dx$$

Optimal (type 1, 61 leaves, 2 steps):

$$-\frac{a(Ab - aB)(a+bx)^{11}}{11b^3} + \frac{(Ab - 2aB)(a+bx)^{12}}{12b^3} + \frac{B(a+bx)^{13}}{13b^3}$$

Result (type 1, 218 leaves):

$$\begin{aligned} & \frac{1}{6} a^{10} x^2 (3A + 2Bx) + \frac{5}{6} a^9 b x^3 (4A + 3Bx) + \frac{9}{4} a^8 b^2 x^4 (5A + 4Bx) + 4 a^7 b^3 x^5 (6A + 5Bx) + \\ & 5 a^6 b^4 x^6 (7A + 6Bx) + \frac{9}{2} a^5 b^5 x^7 (8A + 7Bx) + \frac{35}{12} a^4 b^6 x^8 (9A + 8Bx) + \frac{4}{3} a^3 b^7 x^9 (10A + 9Bx) + \\ & \frac{9}{22} a^2 b^8 x^{10} (11A + 10Bx) + \frac{5}{66} a b^9 x^{11} (12A + 11Bx) + \frac{1}{156} b^{10} x^{12} (13A + 12Bx) \end{aligned}$$

**Problem 116: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^{10} (A+Bx) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(Ab - aB)(a+bx)^{11}}{11b^2} + \frac{B(a+bx)^{12}}{12b^2}$$

Result (type 1, 198 leaves):

$$\begin{aligned} & \frac{1}{132} x (66 a^{10} (2A + Bx) + 220 a^9 b x (3A + 2Bx) + 495 a^8 b^2 x^2 (4A + 3Bx) + 792 a^7 b^3 x^3 (5A + 4Bx) + \\ & 924 a^6 b^4 x^4 (6A + 5Bx) + 792 a^5 b^5 x^5 (7A + 6Bx) + 495 a^4 b^6 x^6 (8A + 7Bx) + \\ & 220 a^3 b^7 x^7 (9A + 8Bx) + 66 a^2 b^8 x^8 (10A + 9Bx) + 12 a b^9 x^9 (11A + 10Bx) + b^{10} x^{10} (12A + 11Bx)) \end{aligned}$$

**Problem 129: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^{10} (A+Bx)}{x^{13}} dx$$

Optimal (type 1, 44 leaves, 2 steps):

$$-\frac{A(a+bx)^{11}}{12ax^{12}} + \frac{(Ab - 12aB)(a+bx)^{11}}{132a^2x^{11}}$$

Result (type 1, 199 leaves):

$$\begin{aligned} & -\frac{1}{132x^{12}} \\ & (66 b^{10} x^{10} (A + 2Bx) + 220 a b^9 x^9 (2A + 3Bx) + 495 a^2 b^8 x^8 (3A + 4Bx) + 792 a^3 b^7 x^7 (4A + 5Bx) + \\ & 924 a^4 b^6 x^6 (5A + 6Bx) + 792 a^5 b^5 x^5 (6A + 7Bx) + 495 a^6 b^4 x^4 (7A + 8Bx) + \\ & 220 a^7 b^3 x^3 (8A + 9Bx) + 66 a^8 b^2 x^2 (9A + 10Bx) + 12 a^9 b x (10A + 11Bx) + a^{10} (11A + 12Bx)) \end{aligned}$$

**Problem 130: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^{10} (A+Bx)}{x^{14}} dx$$

Optimal (type 1, 72 leaves, 3 steps):

$$-\frac{A(a+bx)^{11}}{13ax^{13}} + \frac{(2Ab-13aB)(a+bx)^{11}}{156a^2x^{12}} - \frac{b(2Ab-13aB)(a+bx)^{11}}{1716a^3x^{11}}$$

Result (type 1, 202 leaves):

$$-\frac{1}{1716x^{13}} (286b^{10}x^{10}(2A+3Bx) + 1430ab^9x^9(3A+4Bx) + 3861a^2b^8x^8(4A+5Bx) + 6864a^3b^7x^7(5A+6Bx) + 8580a^4b^6x^6(6A+7Bx) + 7722a^5b^5x^5(7A+8Bx) + 5005a^6b^4x^4(8A+9Bx) + 2288a^7b^3x^3(9A+10Bx) + 702a^8b^2x^2(10A+11Bx) + 130a^9bx(11A+12Bx) + 11a^{10}(12A+13Bx))$$

**Problem 138: Result more than twice size of optimal antiderivative.**

$$\int x^3 (a+bx)(c+dx)^{16} dx$$

Optimal (type 1, 114 leaves, 2 steps):

$$\frac{c^3(bc-ad)(c+dx)^{17}}{17d^5} - \frac{c^2(4bc-3ad)(c+dx)^{18}}{18d^5} + \frac{3c(2bc-ad)(c+dx)^{19}}{19d^5} - \frac{(4bc-ad)(c+dx)^{20}}{20d^5} + \frac{b(c+dx)^{21}}{21d^5}$$

Result (type 1, 359 leaves):

$$\frac{1}{4}ac^{16}x^4 + \frac{1}{5}c^{15}(bc+16ad)x^5 + \frac{4}{3}c^{14}d(2bc+15ad)x^6 + \frac{40}{7}c^{13}d^2(3bc+14ad)x^7 + \frac{35}{2}c^{12}d^3(4bc+13ad)x^8 + \frac{364}{9}c^{11}d^4(5bc+12ad)x^9 + \frac{364}{5}c^{10}d^5(6bc+11ad)x^{10} + 104c^9d^6(7bc+10ad)x^{11} + \frac{715}{6}c^8d^7(8bc+9ad)x^{12} + 110c^7d^8(9bc+8ad)x^{13} + \frac{572}{7}c^6d^9(10bc+7ad)x^{14} + \frac{728}{15}c^5d^{10}(11bc+6ad)x^{15} + \frac{91}{4}c^4d^{11}(12bc+5ad)x^{16} + \frac{140}{17}c^3d^{12}(13bc+4ad)x^{17} + \frac{20}{9}c^2d^{13}(14bc+3ad)x^{18} + \frac{8}{19}cd^{14}(15bc+2ad)x^{19} + \frac{1}{20}d^{15}(16bc+ad)x^{20} + \frac{1}{21}bd^{16}x^{21}$$

**Problem 139: Result more than twice size of optimal antiderivative.**

$$\int x^2 (a+bx)(c+dx)^{16} dx$$

Optimal (type 1, 88 leaves, 2 steps):

$$-\frac{c^2(bc-ad)(c+dx)^{17}}{17d^4} + \frac{c(3bc-2ad)(c+dx)^{18}}{18d^4} - \frac{(3bc-ad)(c+dx)^{19}}{19d^4} + \frac{b(c+dx)^{20}}{20d^4}$$

Result (type 1, 355 leaves):

$$\begin{aligned} & \frac{1}{3} a c^{16} x^3 + \frac{1}{4} c^{15} (b c + 16 a d) x^4 + \frac{8}{5} c^{14} d (2 b c + 15 a d) x^5 + \\ & \frac{20}{3} c^{13} d^2 (3 b c + 14 a d) x^6 + 20 c^{12} d^3 (4 b c + 13 a d) x^7 + \frac{91}{2} c^{11} d^4 (5 b c + 12 a d) x^8 + \\ & \frac{728}{9} c^{10} d^5 (6 b c + 11 a d) x^9 + \frac{572}{5} c^9 d^6 (7 b c + 10 a d) x^{10} + 130 c^8 d^7 (8 b c + 9 a d) x^{11} + \\ & \frac{715}{6} c^7 d^8 (9 b c + 8 a d) x^{12} + 88 c^6 d^9 (10 b c + 7 a d) x^{13} + 52 c^5 d^{10} (11 b c + 6 a d) x^{14} + \\ & \frac{364}{15} c^4 d^{11} (12 b c + 5 a d) x^{15} + \frac{35}{4} c^3 d^{12} (13 b c + 4 a d) x^{16} + \frac{40}{17} c^2 d^{13} (14 b c + 3 a d) x^{17} + \\ & \frac{4}{9} c d^{14} (15 b c + 2 a d) x^{18} + \frac{1}{19} d^{15} (16 b c + a d) x^{19} + \frac{1}{20} b d^{16} x^{20} \end{aligned}$$

**Problem 140: Result more than twice size of optimal antiderivative.**

$$\int x (a + b x) (c + d x)^{16} dx$$

Optimal (type 1, 62 leaves, 2 steps):

$$\frac{c (b c - a d) (c + d x)^{17}}{17 d^3} - \frac{(2 b c - a d) (c + d x)^{18}}{18 d^3} + \frac{b (c + d x)^{19}}{19 d^3}$$

Result (type 1, 347 leaves):

$$\begin{aligned} & \frac{1}{2} a c^{16} x^2 + \frac{1}{3} c^{15} (b c + 16 a d) x^3 + 2 c^{14} d (2 b c + 15 a d) x^4 + \\ & 8 c^{13} d^2 (3 b c + 14 a d) x^5 + \frac{70}{3} c^{12} d^3 (4 b c + 13 a d) x^6 + 52 c^{11} d^4 (5 b c + 12 a d) x^7 + \\ & 91 c^{10} d^5 (6 b c + 11 a d) x^8 + \frac{1144}{9} c^9 d^6 (7 b c + 10 a d) x^9 + 143 c^8 d^7 (8 b c + 9 a d) x^{10} + \\ & 130 c^7 d^8 (9 b c + 8 a d) x^{11} + \frac{286}{3} c^6 d^9 (10 b c + 7 a d) x^{12} + 56 c^5 d^{10} (11 b c + 6 a d) x^{13} + \\ & 26 c^4 d^{11} (12 b c + 5 a d) x^{14} + \frac{28}{3} c^3 d^{12} (13 b c + 4 a d) x^{15} + \frac{5}{2} c^2 d^{13} (14 b c + 3 a d) x^{16} + \\ & \frac{8}{17} c d^{14} (15 b c + 2 a d) x^{17} + \frac{1}{18} d^{15} (16 b c + a d) x^{18} + \frac{1}{19} b d^{16} x^{19} \end{aligned}$$

**Problem 141: Result more than twice size of optimal antiderivative.**

$$\int (a + b x) (c + d x)^{16} dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$-\frac{(b c - a d) (c + d x)^{17}}{17 d^2} + \frac{b (c + d x)^{18}}{18 d^2}$$

Result (type 1, 342 leaves):

$$\begin{aligned}
 & a c^{16} x + \frac{1}{2} c^{15} (b c + 16 a d) x^2 + \frac{8}{3} c^{14} d (2 b c + 15 a d) x^3 + \\
 & 10 c^{13} d^2 (3 b c + 14 a d) x^4 + 28 c^{12} d^3 (4 b c + 13 a d) x^5 + \frac{182}{3} c^{11} d^4 (5 b c + 12 a d) x^6 + \\
 & 104 c^{10} d^5 (6 b c + 11 a d) x^7 + 143 c^9 d^6 (7 b c + 10 a d) x^8 + \frac{1430}{9} c^8 d^7 (8 b c + 9 a d) x^9 + \\
 & 143 c^7 d^8 (9 b c + 8 a d) x^{10} + 104 c^6 d^9 (10 b c + 7 a d) x^{11} + \frac{182}{3} c^5 d^{10} (11 b c + 6 a d) x^{12} + \\
 & 28 c^4 d^{11} (12 b c + 5 a d) x^{13} + 10 c^3 d^{12} (13 b c + 4 a d) x^{14} + \frac{8}{3} c^2 d^{13} (14 b c + 3 a d) x^{15} + \\
 & \frac{1}{2} c d^{14} (15 b c + 2 a d) x^{16} + \frac{1}{17} d^{15} (16 b c + a d) x^{17} + \frac{1}{18} b d^{16} x^{18}
 \end{aligned}$$

**Problem 142: Result more than twice size of optimal antiderivative.**

$$\int x^2 (2+x)^5 (2+3x) dx$$

Optimal (type 1, 12 leaves, 1 step):

$$\frac{1}{3} x^3 (2+x)^6$$

Result (type 1, 42 leaves):

$$\frac{64 x^3}{3} + 64 x^4 + 80 x^5 + \frac{160 x^6}{3} + 20 x^7 + 4 x^8 + \frac{x^9}{3}$$

**Problem 171: Result more than twice size of optimal antiderivative.**

$$\int x^3 (a+bx)^2 (c+dx)^{16} dx$$

Optimal (type 1, 177 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{c^3 (b c - a d)^2 (c + d x)^{17}}{17 d^6} + \frac{c^2 (5 b c - 3 a d) (b c - a d) (c + d x)^{18}}{18 d^6} - \\
 & \frac{c (10 b^2 c^2 - 12 a b c d + 3 a^2 d^2) (c + d x)^{19}}{19 d^6} + \\
 & \frac{(10 b^2 c^2 - 8 a b c d + a^2 d^2) (c + d x)^{20}}{20 d^6} - \frac{b (5 b c - 2 a d) (c + d x)^{21}}{21 d^6} + \frac{b^2 (c + d x)^{22}}{22 d^6}
 \end{aligned}$$

Result (type 1, 589 leaves):

$$\begin{aligned}
 & \frac{1}{4} a^2 c^{16} x^4 + \frac{2}{5} a c^{15} (b c + 8 a d) x^5 + \\
 & \frac{1}{6} c^{14} (b^2 c^2 + 32 a b c d + 120 a^2 d^2) x^6 + \frac{16}{7} c^{13} d (b^2 c^2 + 15 a b c d + 35 a^2 d^2) x^7 + \\
 & \frac{5}{2} c^{12} d^2 (6 b^2 c^2 + 56 a b c d + 91 a^2 d^2) x^8 + \frac{56}{9} c^{11} d^3 (10 b^2 c^2 + 65 a b c d + 78 a^2 d^2) x^9 + \\
 & \frac{182}{5} c^{10} d^4 (5 b^2 c^2 + 24 a b c d + 22 a^2 d^2) x^{10} + \frac{208}{11} c^9 d^5 (21 b^2 c^2 + 77 a b c d + 55 a^2 d^2) x^{11} + \\
 & \frac{143}{6} c^8 d^6 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^{12} + 220 c^7 d^7 (4 b^2 c^2 + 9 a b c d + 4 a^2 d^2) x^{13} + \\
 & \frac{143}{7} c^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{14} + \frac{208}{15} c^5 d^9 (55 b^2 c^2 + 77 a b c d + 21 a^2 d^2) x^{15} + \\
 & \frac{91}{4} c^4 d^{10} (22 b^2 c^2 + 24 a b c d + 5 a^2 d^2) x^{16} + \frac{56}{17} c^3 d^{11} (78 b^2 c^2 + 65 a b c d + 10 a^2 d^2) x^{17} + \\
 & \frac{10}{9} c^2 d^{12} (91 b^2 c^2 + 56 a b c d + 6 a^2 d^2) x^{18} + \frac{16}{19} c d^{13} (35 b^2 c^2 + 15 a b c d + a^2 d^2) x^{19} + \\
 & \frac{1}{20} d^{14} (120 b^2 c^2 + 32 a b c d + a^2 d^2) x^{20} + \frac{2}{21} b d^{15} (8 b c + a d) x^{21} + \frac{1}{22} b^2 d^{16} x^{22}
 \end{aligned}$$

**Problem 172: Result more than twice size of optimal antiderivative.**

$$\int x^2 (a + b x)^2 (c + d x)^{16} dx$$

Optimal (type 1, 137 leaves, 2 steps):

$$\begin{aligned}
 & \frac{c^2 (b c - a d)^2 (c + d x)^{17}}{17 d^5} - \frac{c (b c - a d) (2 b c - a d) (c + d x)^{18}}{9 d^5} + \\
 & \frac{(6 b^2 c^2 - 6 a b c d + a^2 d^2) (c + d x)^{19}}{19 d^5} - \frac{b (2 b c - a d) (c + d x)^{20}}{10 d^5} + \frac{b^2 (c + d x)^{21}}{21 d^5}
 \end{aligned}$$

Result (type 1, 585 leaves):

$$\begin{aligned}
 & \frac{1}{3} a^2 c^{16} x^3 + \frac{1}{2} a c^{15} (b c + 8 a d) x^4 + \\
 & \frac{1}{5} c^{14} (b^2 c^2 + 32 a b c d + 120 a^2 d^2) x^5 + \frac{8}{3} c^{13} d (b^2 c^2 + 15 a b c d + 35 a^2 d^2) x^6 + \\
 & \frac{20}{7} c^{12} d^2 (6 b^2 c^2 + 56 a b c d + 91 a^2 d^2) x^7 + 7 c^{11} d^3 (10 b^2 c^2 + 65 a b c d + 78 a^2 d^2) x^8 + \\
 & \frac{364}{9} c^{10} d^4 (5 b^2 c^2 + 24 a b c d + 22 a^2 d^2) x^9 + \frac{104}{5} c^9 d^5 (21 b^2 c^2 + 77 a b c d + 55 a^2 d^2) x^{10} + \\
 & 26 c^8 d^6 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^{11} + \frac{715}{3} c^7 d^7 (4 b^2 c^2 + 9 a b c d + 4 a^2 d^2) x^{12} + \\
 & 22 c^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{13} + \frac{104}{7} c^5 d^9 (55 b^2 c^2 + 77 a b c d + 21 a^2 d^2) x^{14} + \\
 & \frac{364}{15} c^4 d^{10} (22 b^2 c^2 + 24 a b c d + 5 a^2 d^2) x^{15} + \frac{7}{2} c^3 d^{11} (78 b^2 c^2 + 65 a b c d + 10 a^2 d^2) x^{16} + \\
 & \frac{20}{17} c^2 d^{12} (91 b^2 c^2 + 56 a b c d + 6 a^2 d^2) x^{17} + \frac{8}{9} c d^{13} (35 b^2 c^2 + 15 a b c d + a^2 d^2) x^{18} + \\
 & \frac{1}{19} d^{14} (120 b^2 c^2 + 32 a b c d + a^2 d^2) x^{19} + \frac{1}{10} b d^{15} (8 b c + a d) x^{20} + \frac{1}{21} b^2 d^{16} x^{21}
 \end{aligned}$$

### Problem 173: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^2 (c + d x)^{16} dx$$

Optimal (type 1, 98 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{c (b c - a d)^2 (c + d x)^{17}}{17 d^4} + \frac{(b c - a d) (3 b c - a d) (c + d x)^{18}}{18 d^4} - \\
 & \frac{b (3 b c - 2 a d) (c + d x)^{19}}{19 d^4} + \frac{b^2 (c + d x)^{20}}{20 d^4}
 \end{aligned}$$

Result (type 1, 583 leaves):

$$\begin{aligned} & \frac{1}{2} a^2 c^{16} x^2 + \frac{2}{3} a c^{15} (b c + 8 a d) x^3 + \\ & \frac{1}{4} c^{14} (b^2 c^2 + 32 a b c d + 120 a^2 d^2) x^4 + \frac{16}{5} c^{13} d (b^2 c^2 + 15 a b c d + 35 a^2 d^2) x^5 + \\ & \frac{10}{3} c^{12} d^2 (6 b^2 c^2 + 56 a b c d + 91 a^2 d^2) x^6 + 8 c^{11} d^3 (10 b^2 c^2 + 65 a b c d + 78 a^2 d^2) x^7 + \\ & \frac{91}{2} c^{10} d^4 (5 b^2 c^2 + 24 a b c d + 22 a^2 d^2) x^8 + \frac{208}{9} c^9 d^5 (21 b^2 c^2 + 77 a b c d + 55 a^2 d^2) x^9 + \\ & \frac{143}{5} c^8 d^6 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^{10} + 260 c^7 d^7 (4 b^2 c^2 + 9 a b c d + 4 a^2 d^2) x^{11} + \\ & \frac{143}{6} c^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{12} + 16 c^5 d^9 (55 b^2 c^2 + 77 a b c d + 21 a^2 d^2) x^{13} + \\ & 26 c^4 d^{10} (22 b^2 c^2 + 24 a b c d + 5 a^2 d^2) x^{14} + \frac{56}{15} c^3 d^{11} (78 b^2 c^2 + 65 a b c d + 10 a^2 d^2) x^{15} + \\ & \frac{5}{4} c^2 d^{12} (91 b^2 c^2 + 56 a b c d + 6 a^2 d^2) x^{16} + \frac{16}{17} c d^{13} (35 b^2 c^2 + 15 a b c d + a^2 d^2) x^{17} + \\ & \frac{1}{18} d^{14} (120 b^2 c^2 + 32 a b c d + a^2 d^2) x^{18} + \frac{2}{19} b d^{15} (8 b c + a d) x^{19} + \frac{1}{20} b^2 d^{16} x^{20} \end{aligned}$$

**Problem 353: Result unnecessarily involves higher level functions.**

$$\int \frac{x^m}{(a + b x) (c + d x)^2} dx$$

Optimal (type 5, 125 leaves, 4 steps):

$$\begin{aligned} & -\frac{d x^{1+m}}{c (b c - a d) (c + d x)} + \frac{b^2 x^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, -\frac{b x}{a}\right]}{a (b c - a d)^2 (1 + m)} - \\ & \frac{d (b c (1 - m) + a d m) x^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, -\frac{d x}{c}\right]}{c^2 (b c - a d)^2 (1 + m)} \end{aligned}$$

Result (type 6, 142 leaves):

$$\begin{aligned} & \left( a c (2 + m) x^{1+m} \text{AppellF1}\left[1 + m, 2, 1, 2 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] \right) / \\ & \left( (1 + m) (a + b x) (c + d x)^2 \left( a c (2 + m) \text{AppellF1}\left[1 + m, 2, 1, 2 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] - \right. \right. \\ & \quad \left. \left. x \left( b c \text{AppellF1}\left[2 + m, 2, 2, 3 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] + \right. \right. \right. \\ & \quad \left. \left. \left. 2 a d \text{AppellF1}\left[2 + m, 3, 1, 3 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] \right) \right) \right) \end{aligned}$$

**Problem 354: Result unnecessarily involves higher level functions.**

$$\int \frac{x^m}{(a + b x) (c + d x)^3} dx$$

Optimal (type 5, 206 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{d x^{1+m}}{2 c (b c - a d) (c + d x)^2} + \frac{d (a d (1 - m) - b c (3 - m)) x^{1+m}}{2 c^2 (b c - a d)^2 (c + d x)} + \\
 & \frac{b^3 x^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, -\frac{b x}{a}\right]}{a (b c - a d)^3 (1 + m)} + \\
 & \left( d (a^2 d^2 (1 - m) m - 2 a b c d (2 - m) m - b^2 c^2 (2 - 3 m + m^2)) x^{1+m} \right. \\
 & \left. \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, -\frac{d x}{c}\right] \right) / \left( 2 c^3 (b c - a d)^3 (1 + m) \right)
 \end{aligned}$$

Result (type 6, 142 leaves):

$$\begin{aligned}
 & \left( a c (2 + m) x^{1+m} \text{AppellF1}\left[1 + m, 3, 1, 2 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] \right) / \\
 & \left( (1 + m) (a + b x) (c + d x)^3 \left( a c (2 + m) \text{AppellF1}\left[1 + m, 3, 1, 2 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] - \right. \right. \\
 & \quad \left. \left. x \left( b c \text{AppellF1}\left[2 + m, 3, 2, 3 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. 3 a d \text{AppellF1}\left[2 + m, 4, 1, 3 + m, -\frac{d x}{c}, -\frac{b x}{a}\right] \right) \right) \right)
 \end{aligned}$$

**Problem 360: Result unnecessarily involves higher level functions.**

$$\int \frac{(e x)^m}{(2 - 2 a x)^4 (1 + a x)^3} dx$$

Optimal (type 5, 86 leaves, 5 steps):

$$\frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[4, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{16 e (1 + m)} + \frac{a (e x)^{2+m} \text{Hypergeometric2F1}\left[4, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{16 e^2 (2 + m)}$$

Result (type 6, 120 leaves):

$$\begin{aligned}
 & \left( (2 + m) x (e x)^m \text{AppellF1}\left[1 + m, 4, 3, 2 + m, a x, -a x\right] \right) / \left( 16 (1 + m) (-1 + a x)^4 (1 + a x)^3 \right. \\
 & \left. \left( (2 + m) \text{AppellF1}\left[1 + m, 4, 3, 2 + m, a x, -a x\right] + a x \left( 4 \text{AppellF1}\left[2 + m, 5, 3, 3 + m, a x, -a x\right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 3 \text{HypergeometricPFQ}\left[\left\{4, 1 + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}\right\}, a^2 x^2\right]\right) \right) \right)
 \end{aligned}$$

**Problem 365: Result unnecessarily involves higher level functions.**

$$\int \frac{(e x)^m}{(a + b x)^2 (a d - b d x)^3} dx$$

Optimal (type 5, 98 leaves, 5 steps):

$$\frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[3, \frac{1+m}{2}, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^5 d^3 e (1+m)} + \frac{b (e x)^{2+m} \text{Hypergeometric2F1}\left[3, \frac{2+m}{2}, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^6 d^3 e^2 (2+m)}$$

Result (type 6, 144 leaves):

$$\left( a (2+m) x (e x)^m \text{AppellF1}\left[1+m, 3, 2, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] \right) / \left( d^3 (1+m) (a-b x)^3 (a+b x)^2 \left( a (2+m) \text{AppellF1}\left[1+m, 3, 2, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] + b x \left( 3 \text{AppellF1}\left[2+m, 4, 2, 3+m, \frac{b x}{a}, -\frac{b x}{a}\right] - 2 \text{HypergeometricPFQ}\left[\left\{3, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, \frac{b^2 x^2}{a^2}\right]\right) \right) \right)$$

**Problem 366: Result unnecessarily involves higher level functions.**

$$\int \frac{(e x)^m}{(a+b x)^3 (a d - b d x)^4} dx$$

Optimal (type 5, 98 leaves, 5 steps):

$$\frac{(e x)^{1+m} \text{Hypergeometric2F1}\left[4, \frac{1+m}{2}, \frac{3+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^7 d^4 e (1+m)} + \frac{b (e x)^{2+m} \text{Hypergeometric2F1}\left[4, \frac{2+m}{2}, \frac{4+m}{2}, \frac{b^2 x^2}{a^2}\right]}{a^8 d^4 e^2 (2+m)}$$

Result (type 6, 144 leaves):

$$\left( a (2+m) x (e x)^m \text{AppellF1}\left[1+m, 4, 3, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] \right) / \left( d^4 (1+m) (a-b x)^4 (a+b x)^3 \left( a (2+m) \text{AppellF1}\left[1+m, 4, 3, 2+m, \frac{b x}{a}, -\frac{b x}{a}\right] + b x \left( 4 \text{AppellF1}\left[2+m, 5, 3, 3+m, \frac{b x}{a}, -\frac{b x}{a}\right] - 3 \text{HypergeometricPFQ}\left[\left\{4, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, \frac{b^2 x^2}{a^2}\right]\right) \right) \right)$$

**Problem 451: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^{1/3} \sqrt{c+dx} (4c+dx)} dx$$

Optimal (type 3, 199 leaves, 2 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} (c^{1/3} + 2^{1/3} d^{1/3} x^{1/3})}{\sqrt{c+dx}}\right]}{2^{2/3} \sqrt{3} c^{5/6} d^{2/3}} + \frac{\text{ArcTan}\left[\frac{\sqrt{c+dx}}{\sqrt{3} \sqrt{c}}\right]}{2^{2/3} \sqrt{3} c^{5/6} d^{2/3}} - \frac{\text{ArcTanh}\left[\frac{c^{1/6} (c^{1/3} - 2^{1/3} d^{1/3} x^{1/3})}{\sqrt{c+dx}}\right]}{2^{2/3} c^{5/6} d^{2/3}} + \frac{\text{ArcTanh}\left[\frac{\sqrt{c+dx}}{\sqrt{c}}\right]}{3 \times 2^{2/3} c^{5/6} d^{2/3}}$$

Result (type 6, 147 leaves):

$$\left( 30 c x^{2/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{dx}{c}, -\frac{dx}{4c}\right] \right) / \left( \sqrt{c+dx} (4c+dx) \left( 20 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{dx}{c}, -\frac{dx}{4c}\right] - 3 dx \left( \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{dx}{c}, -\frac{dx}{4c}\right] + 2 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{dx}{c}, -\frac{dx}{4c}\right] \right) \right) \right)$$

**Problem 452: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^{1/3} (8c-dx) \sqrt{c+dx}} dx$$

Optimal (type 3, 143 leaves, 9 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{3} c^{1/6} (c^{1/3}+d^{1/3} x^{1/3})}{\sqrt{c+dx}}\right]}{2\sqrt{3} c^{5/6} d^{2/3}} + \frac{\text{ArcTanh}\left[\frac{(c^{1/3}+d^{1/3} x^{1/3})^2}{3 c^{1/6} \sqrt{c+dx}}\right]}{6 c^{5/6} d^{2/3}} - \frac{\text{ArcTanh}\left[\frac{\sqrt{c+dx}}{3\sqrt{c}}\right]}{6 c^{5/6} d^{2/3}}$$

Result (type 6, 148 leaves):

$$\left( 60 c x^{2/3} \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{dx}{c}, \frac{dx}{8c}\right] \right) / \left( (8c-dx) \sqrt{c+dx} \left( 40 c \text{AppellF1}\left[\frac{2}{3}, \frac{1}{2}, 1, \frac{5}{3}, -\frac{dx}{c}, \frac{dx}{8c}\right] + 3 dx \left( \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 2, \frac{8}{3}, -\frac{dx}{c}, \frac{dx}{8c}\right] - 4 \text{AppellF1}\left[\frac{5}{3}, \frac{3}{2}, 1, \frac{8}{3}, -\frac{dx}{c}, \frac{dx}{8c}\right] \right) \right) \right)$$

**Problem 716: Result more than twice size of optimal antiderivative.**

$$\int \frac{(1+x)^{3/2}}{\sqrt{1-x} x} dx$$

Optimal (type 3, 43 leaves, 6 steps):

$$-\sqrt{1-x} \sqrt{1+x} + 2 \text{ArcSin}[x] - \text{ArcTanh}\left[\frac{\sqrt{1-x} \sqrt{1+x}}{x}\right]$$

Result (type 3, 96 leaves):

$$-\sqrt{1-x^2} + 4 \text{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{2}}\right] + \text{Log}\left[1 - \sqrt{1+x}\right] - \text{Log}\left[2 + \sqrt{1-x} - \sqrt{1+x}\right] - \text{Log}\left[1 + \sqrt{1+x}\right] + \text{Log}\left[2 + \sqrt{1-x} + \sqrt{1+x}\right]$$

**Problem 746: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{x \sqrt{1-ax-bx} \sqrt{1+ax+bx}} dx$$

Optimal (type 3, 54 leaves, 2 steps):

$$\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{1-a} \sqrt{1+bx}}{\sqrt{1+a} \sqrt{1-ax}}\right]}{\sqrt{1-a^2}}$$

Result (type 3, 107 leaves):

$$\frac{i \sqrt{-1+a+bx} \sqrt{1+a+bx} \operatorname{Log}\left[\frac{2\sqrt{-1+bx} \sqrt{1+bx}}{x} + \frac{2i(-1+a^2+abx)}{\sqrt{1-a^2} x}\right]}{\sqrt{1-a^2} \sqrt{-(-1+a+bx)(1+a+bx)}}$$

**Problem 826: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-1+x} \sqrt{1+x}} dx$$

Optimal (type 3, 2 leaves, 1 step):

$$\operatorname{ArcCosh}[x]$$

Result (type 3, 16 leaves):

$$2 \operatorname{ArcSinh}\left[\frac{\sqrt{-1+x}}{\sqrt{2}}\right]$$

**Problem 843: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{x} \sqrt{2-bx} \sqrt{2+bx}} dx$$

Optimal (type 4, 30 leaves, 1 step):

$$\frac{\sqrt{2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right], -1\right]}{\sqrt{b}}$$

Result (type 4, 70 leaves):

$$\frac{2i \sqrt{-\frac{1}{b}} b \sqrt{1-\frac{4}{b^2 x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{1}{b}}}{\sqrt{x}}\right], -1\right]}{\sqrt{8-2b^2 x^2}}$$

**Problem 844: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-x} \sqrt{2-bx} \sqrt{2+bx}} dx$$

Optimal (type 4, 33 leaves, 1 step):

$$\frac{\sqrt{2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{-x}}{\sqrt{2}}\right], -1\right]}{\sqrt{b}}$$

Result (type 4, 78 leaves):

$$\frac{2 i \sqrt{-\frac{1}{b} b} \sqrt{1 - \frac{4}{b^2 x^2}} \sqrt{-x^2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{1}{b}}}{\sqrt{x}}\right], -1\right]}{\sqrt{8 - 2 b^2 x^2}}$$

**Problem 845: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{e x} \sqrt{2 - b x} \sqrt{2 + b x}} dx$$

Optimal (type 4, 42 leaves, 1 step):

$$\frac{\sqrt{2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{b}\sqrt{e x}}{\sqrt{2}\sqrt{e}}\right], -1\right]}{\sqrt{b} \sqrt{e}}$$

Result (type 4, 81 leaves):

$$\frac{2 i \sqrt{-\frac{1}{b} b} \sqrt{1 - \frac{4}{b^2 x^2}} x^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{2} \sqrt{-\frac{1}{b}}}{\sqrt{x}}\right], -1\right]}{\sqrt{e x} \sqrt{8 - 2 b^2 x^2}}$$

**Problem 849: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-x} \sqrt{x} \sqrt{1+x}} dx$$

Optimal (type 4, 10 leaves, 1 step):

$$2 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{x}\right], -1\right]$$

Result (type 4, 66 leaves):

$$\frac{2 i \sqrt{1 + \frac{1}{-1+x}} \sqrt{1 + \frac{2}{-1+x}} (-1+x)^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right], 2\right]}{\sqrt{-(-1+x) x} \sqrt{1+x}}$$

**Problem 850: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1+x} \sqrt{x-x^2}} dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{x}\right], -1\right]$$

Result (type 4, 66 leaves):

$$\frac{2 i \sqrt{1 + \frac{1}{-1+x}} \sqrt{1 + \frac{2}{-1+x}} (-1+x)^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-1+x}}\right], 2\right]}{\sqrt{-(-1+x)x} \sqrt{1+x}}$$

**Problem 851: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{b x} \sqrt{1-c x} \sqrt{1+c x}} dx$$

Optimal (type 4, 33 leaves, 1 step):

$$\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c} \sqrt{b x}}{\sqrt{b}}\right], -1\right]}{\sqrt{b} \sqrt{c}}$$

Result (type 4, 76 leaves):

$$\frac{2 i \sqrt{-\frac{1}{c}} c \sqrt{1 - \frac{1}{c^2 x^2}} x^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{1}{c}}}{\sqrt{x}}\right], -1\right]}{\sqrt{b x} \sqrt{1-c^2 x^2}}$$

**Problem 852: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{b x} \sqrt{1-c x} \sqrt{1+d x}} dx$$

Optimal (type 4, 38 leaves, 1 step):

$$\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{c} \sqrt{b x}}{\sqrt{b}}\right], -\frac{d}{c}\right]}{\sqrt{b} \sqrt{c}}$$

Result (type 4, 89 leaves):

$$\frac{2 \sqrt{\frac{c-1}{c}} \sqrt{\frac{d+1}{d}} x^{3/2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{\frac{1}{c}}}{\sqrt{x}}\right], -\frac{c}{d}\right]}{\sqrt{\frac{1}{c}} \sqrt{b x} \sqrt{1-c x} \sqrt{1+d x}}$$

**Problem 853: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x}\sqrt{x}} dx$$

Optimal (type 4, 10 leaves, 1 step):

$$2 \text{EllipticE}[\text{ArcSin}[\sqrt{x}], -1]$$

Result (type 4, 104 leaves):

$$\frac{2 \sqrt{\frac{-1+x}{1+x}} \sqrt{\frac{1+x}{-1+x}} \left( \sqrt{-1+x} x \sqrt{\frac{1+x}{-1+x}} + \frac{i \sqrt{2} x \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}}{\sqrt{-1+x}}\right], \frac{1}{2}\right]}{\sqrt{\frac{x}{-1+x}}}\right)}{\sqrt{-(-1+x)x}}$$

**Problem 854: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}} dx$$

Optimal (type 4, 10 leaves, 2 steps):

$$2 \text{EllipticE}[\text{ArcSin}[\sqrt{x}], -1]$$

Result (type 4, 104 leaves):

$$\frac{2 \sqrt{\frac{-1+x}{1+x}} \sqrt{\frac{1+x}{-1+x}} \left( \sqrt{-1+x} x \sqrt{\frac{1+x}{-1+x}} + \frac{i \sqrt{2} x \text{EllipticE}\left[i \text{ArcSinh}\left[\frac{\sqrt{2}}{\sqrt{-1+x}}\right], \frac{1}{2}\right]}{\sqrt{\frac{x}{-1+x}}}\right)}{\sqrt{-(-1+x)x}}$$

**Problem 855: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1+cx}}{\sqrt{bx}\sqrt{1-cx}} dx$$

Optimal (type 4, 33 leaves, 1 step):

$$\frac{2 \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{c}\sqrt{bx}}{\sqrt{b}}\right], -1\right]}{\sqrt{b}\sqrt{c}}$$

Result (type 4, 119 leaves):

$$\begin{aligned}
 & - \left( \left( 2 \sqrt{-\frac{1}{c}} (-1 + c x) \right. \right. \\
 & \left. \left. \left( \sqrt{-\frac{1}{c}} \sqrt{1 - \frac{1}{c x}} (1 + c x) - \sqrt{1 + \frac{1}{c x}} \sqrt{x} \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}} \right], -1 \right] \right) \right) / \right. \\
 & \left. \left( \sqrt{1 - \frac{1}{c x}} \sqrt{b x} \sqrt{1 - c^2 x^2} \right) \right)
 \end{aligned}$$

**Problem 856: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1 + c x}}{\sqrt{b x} \sqrt{1 - d x}} dx$$

Optimal (type 4, 38 leaves, 1 step):

$$\frac{2 \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{d} \sqrt{b x}}{\sqrt{b}} \right], -\frac{c}{d} \right]}{\sqrt{b} \sqrt{d}}$$

Result (type 4, 102 leaves):

$$\frac{2 \sqrt{1 - d x} \left( -1 - c x + \frac{\sqrt{1 + \frac{1}{c x}} \sqrt{x} \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{\frac{1}{c}}}{\sqrt{x}} \right], -\frac{c}{d} \right]}{\sqrt{-\frac{1}{c}} \sqrt{1 - \frac{1}{d x}}} \right)}{d \sqrt{b x} \sqrt{1 + c x}}$$

**Problem 859: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1 - c x}}{\sqrt{b x} \sqrt{1 + c x}} dx$$

Optimal (type 4, 37 leaves, 1 step):

$$- \frac{2 \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \frac{\sqrt{c} \sqrt{b x}}{\sqrt{-b}} \right], -1 \right]}{\sqrt{-b} \sqrt{c}}$$

Result (type 4, 77 leaves):

$$\frac{2c \left( \frac{1}{c^2} - x^2 - \sqrt{\frac{1}{c}} \sqrt{1 - \frac{1}{c^2 x^2}} x^{3/2} \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{1}{c}}}{\sqrt{x}} \right], -1 \right] \right)}{\sqrt{bx} \sqrt{1 - c^2 x^2}}$$

**Problem 860: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{1 - cx}}{\sqrt{bx} \sqrt{1 + dx}} dx$$

Optimal (type 4, 42 leaves, 1 step):

$$\frac{2 \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{d} \sqrt{bx}}{\sqrt{-b}} \right], -\frac{c}{d} \right]}{\sqrt{-b} \sqrt{d}}$$

Result (type 4, 112 leaves):

$$\left( \frac{2 \sqrt{\frac{1}{c}} (-1 + cx) (1 + dx)}{d} - 2 \sqrt{1 - \frac{1}{cx}} \sqrt{1 + \frac{1}{dx}} x^{3/2} \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{\frac{1}{c}}}{\sqrt{x}} \right], -\frac{c}{d} \right] \right) / \left( \sqrt{\frac{1}{c}} \sqrt{bx} \sqrt{1 - cx} \sqrt{1 + dx} \right)$$

**Problem 862: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{d + ex}}{\sqrt{2 - 3x} \sqrt{x}} dx$$

Optimal (type 4, 51 leaves, 2 steps):

$$\frac{2 \sqrt{d + ex} \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{3}{2}} \sqrt{x} \right], -\frac{2e}{3d} \right]}{\sqrt{3} \sqrt{1 + \frac{ex}{d}}}$$

Result (type 4, 125 leaves):

$$\frac{2 \sqrt{x} \left( \frac{3(d+ex)}{\sqrt{2-3x}} - \frac{(3d+2e) \sqrt{\frac{d+ex}{e(-2+3x)}} \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{2+\frac{3d}{e}}}{\sqrt{2-3x}} \right], \frac{2e}{3d+2e} \right]}{\sqrt{2+\frac{3d}{e}} \sqrt{\frac{x}{-2+3x}}} \right)}{3 \sqrt{d+ex}}$$

**Problem 863: Result unnecessarily involves higher level functions.**

$$\int \frac{x^4}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 752 leaves, 8 steps):

$$\begin{aligned} & \frac{99}{130} (1-x)^{2/3} (2-x)^{2/3} x^2 + \frac{3}{13} (1-x)^{2/3} (2-x)^{2/3} x^3 + \frac{27}{455} (1-x)^{2/3} (2-x)^{2/3} (89+34x) - \\ & \frac{891 \times 2^{2/3} \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{91 (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} + \\ & \left( 891 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} \right. \\ & \left. (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\ & \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\ & \left( 91 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) - \\ & \left( 594 \times 2^{1/6} \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\ & \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\ & \left( 91 (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 54 leaves):

$$\frac{3}{910} (1-x)^{2/3} \left( (2-x)^{2/3} (1602 + 612x + 231x^2 + 70x^3) - 2970 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right] \right)$$

**Problem 864: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 727 leaves, 7 steps):

$$\begin{aligned} & \frac{3}{10} (1-x)^{2/3} (2-x)^{2/3} x^2 + \frac{9}{70} (1-x)^{2/3} (2-x)^{2/3} (23+8x) - \\ & \frac{81 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{7 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2} + \\ & \left( 81 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} \right. \\ & \left. (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\ & \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\ & \left( 14 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) - \\ & \left( 27 \times 2^{1/6} \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\ & \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\ & \left( 7 (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 49 leaves):

$$\frac{3}{70} (1-x)^{2/3} \left( (2-x)^{2/3} (69+24x+7x^2) - 135 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right] \right)$$

**Problem 865: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 720 leaves, 7 steps):

$$\frac{45}{28} (1-x)^{2/3} (2-x)^{2/3} + \frac{3}{7} (1-x)^{2/3} (2-x)^{2/3} x - \frac{99 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{14 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} + \left( 99 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\ \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \left( 28 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) - \left( 33 \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \left( 7 \times 2^{5/6} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right)$$

Result (type 5, 44 leaves):

$$\frac{3}{28} (1-x)^{2/3} \left( (2-x)^{2/3} (15+4x) - 33 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right] \right)$$

**Problem 866: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 695 leaves, 6 steps):

$$\frac{3}{4} (1-x)^{2/3} (2-x)^{2/3} - \frac{9 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} +$$

$$\left( 9 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} \right.$$

$$\left. (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right.$$

$$\left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}} \right], -7-4\sqrt{3} \right] \right) /$$

$$\left( 4 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) -$$

$$\left( 3 \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right.$$

$$\left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right.$$

$$\left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}} \right], -7-4\sqrt{3} \right] \right) /$$

$$\left( 2^{5/6} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right)$$

Result (type 5, 38 leaves):

$$\frac{3}{4} (1-x)^{2/3} \left( (2-x)^{2/3} - 3 \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x \right] \right)$$

**Problem 867: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3}} dx$$

Optimal (type 4, 671 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{3 \sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} + \\
 & \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} \right. \\
 & \quad \left. (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
 & \quad \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left( 2 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) - \\
 & \left( 2^{1/6} \times 3^{3/4} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\
 & \quad \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
 & \quad \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left( (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right)
 \end{aligned}$$

Result (type 5, 26 leaves):

$$-\frac{3}{2} (1-x)^{2/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, -1+x\right]$$

**Problem 868: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3} x} dx$$

Optimal (type 3, 99 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{1/3} (2-x)^{2/3}}{\sqrt{3} (1-x)^{1/3}}\right]}{2 \times 2^{1/3}} + \frac{3 \operatorname{Log}\left[-(1-x)^{1/3} + \frac{(2-x)^{2/3}}{2^{2/3}}\right]}{4 \times 2^{1/3}} - \frac{\operatorname{Log}[x]}{2 \times 2^{1/3}}$$

Result (type 6, 115 leaves):

$$\left(15 (1-x)^{2/3} \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right]\right) / \left(2 (2-x)^{1/3} x \left(-5 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right] + (-1+x) \left(3 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -1+x, 1-x\right] - \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, -1+x, 1-x\right]\right)\right)\right)$$

**Problem 869: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3} x^2} dx$$

Optimal (type 4, 796 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{(1-x)^{2/3} (2-x)^{2/3}}{2x} - \frac{\sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} \\
 & \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{1/3} (2-x)^{2/3}}{\sqrt{3} (1-x)^{1/3}}\right]}{4 \times 2^{1/3}} + \left( 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} \right. \\
 & \left. (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
 & \left. \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left( 4 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) - \\
 & \left( \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\
 & \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
 & \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left( 2^{5/6} \times 3^{1/4} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) + \\
 & \frac{3 \operatorname{Log}\left[-(1-x)^{1/3} + \frac{(2-x)^{2/3}}{2^{2/3}}\right]}{8 \times 2^{1/3}} - \frac{\operatorname{Log}[x]}{4 \times 2^{1/3}}
 \end{aligned}$$

Result (type 6, 219 leaves):

$$\frac{1}{10 (2-x)^{1/3} x} (1-x)^{2/3} \left( 5(-2+x) - \left( 50 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right] \right) / \left( 5 \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x\right] - (-1+x) \left( 3 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -1+x, 1-x\right] - \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, -1+x, 1-x\right] \right) \right) + \left( 8(-1+x) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x\right] \right) / \left( -8 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x\right] + (-1+x) \left( 3 \operatorname{AppellF1}\left[\frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, -1+x, 1-x\right] - \operatorname{AppellF1}\left[\frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, -1+x, 1-x\right] \right) \right) \right)$$

**Problem 870: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/3} (2-x)^{1/3} x^3} dx$$

Optimal (type 4, 821 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{(1-x)^{2/3} (2-x)^{2/3}}{4x^2} - \frac{(1-x)^{2/3} (2-x)^{2/3}}{2x} - \\
 & \frac{\sqrt{(3-2x)^2} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3}}{2 \times 2^{1/3} (3-2x) (1-x)^{1/3} (2-x)^{1/3} (1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})} - \\
 & \frac{\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2^{1/3} (2-x)^{2/3}}{\sqrt{3} (1-x)^{1/3}}\right]}{2 \times 2^{1/3} \sqrt{3}} + \left( 3^{1/4} \sqrt{2-\sqrt{3}} \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} \right. \\
 & \left. (1+2^{2/3} (2-3x+x^2)^{1/3}) \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
 & \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left( 4 \times 2^{1/3} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) - \\
 & \left( \sqrt{(-3+2x)^2} (2-3x+x^2)^{1/3} (1+2^{2/3} (2-3x+x^2)^{1/3}) \right. \\
 & \left. \sqrt{\frac{1-2^{2/3} (2-3x+x^2)^{1/3} + 2 \times 2^{1/3} (2-3x+x^2)^{2/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{1-\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}{1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3}}\right], -7-4\sqrt{3}\right] \right) / \\
 & \left( 2^{5/6} \times 3^{1/4} (3-2x) \sqrt{(3-2x)^2} (1-x)^{1/3} (2-x)^{1/3} \sqrt{\frac{1+2^{2/3} (2-3x+x^2)^{1/3}}{(1+\sqrt{3}+2^{2/3} (2-3x+x^2)^{1/3})^2}} \right) + \\
 & \frac{\text{Log}\left[-(1-x)^{1/3} + \frac{(2-x)^{2/3}}{2^{2/3}}\right]}{4 \times 2^{1/3}} - \frac{\text{Log}[x]}{6 \times 2^{1/3}}
 \end{aligned}$$

Result (type 6, 225 leaves):

$$\frac{1}{20 (2-x)^{1/3} x^2} (1-x)^{2/3} \left( 5 (-2+x) (1+2x) + \right. \\ \left. \left( 75 x \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x \right] \right) / \left( -5 \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, -1+x, 1-x \right] + \right. \right. \\ \left. \left. (-1+x) \left( 3 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, -1+x, 1-x \right] - \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, -1+x, 1-x \right] \right) \right) \right) + \\ \left( 16 (-1+x) x \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x \right] \right) / \\ \left( -8 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, -1+x, 1-x \right] + \right. \\ \left. (-1+x) \left( 3 \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, -1+x, 1-x \right] - \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, -1+x, 1-x \right] \right) \right) \right)$$

**Problem 871: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3 (a+bx)^{1/4}}{(c+dx)^{1/4}} dx$$

Optimal (type 3, 340 leaves, 8 steps):

$$-\frac{1}{512 b^3 d^4} (195 b^3 c^3 + 135 a b^2 c^2 d + 105 a^2 b c d^2 + 77 a^3 d^3) (a+bx)^{1/4} (c+dx)^{3/4} + \\ \frac{x^2 (a+bx)^{5/4} (c+dx)^{3/4}}{4 b d} + \frac{1}{384 b^3 d^3} \\ (a+bx)^{5/4} (c+dx)^{3/4} (117 b^2 c^2 + 94 a b c d + 77 a^2 d^2 - 8 b d (13 b c + 11 a d) x) + \frac{1}{1024 b^{15/4} d^{17/4}} \\ (b c - a d) (195 b^3 c^3 + 135 a b^2 c^2 d + 105 a^2 b c d^2 + 77 a^3 d^3) \operatorname{ArcTan} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right] + \\ \frac{1}{1024 b^{15/4} d^{17/4}} (b c - a d) (195 b^3 c^3 + 135 a b^2 c^2 d + 105 a^2 b c d^2 + 77 a^3 d^3) \operatorname{ArcTanh} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]$$

Result (type 5, 221 leaves):

$$\left( (c+dx)^{3/4} \left( d (a+bx) (77 a^3 d^3 + a^2 b d^2 (61 c - 44 d x) + \right. \right. \\ \left. \left. a b^2 d (63 c^2 - 40 c d x + 32 d^2 x^2) + b^3 (-585 c^3 + 468 c^2 d x - 416 c d^2 x^2 + 384 d^3 x^3) \right) - \right. \\ \left. (-195 b^4 c^4 + 60 a b^3 c^3 d + 30 a^2 b^2 c^2 d^2 + 28 a^3 b c d^3 + 77 a^4 d^4) \left( \frac{d (a+bx)}{-b c + a d} \right)^{3/4} \right. \\ \left. \operatorname{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c+dx)}{b c - a d} \right] \right) / (1536 b^3 d^5 (a+bx)^{3/4})$$

### Problem 872: Result unnecessarily involves higher level functions.

$$\int \frac{x^2 (a+bx)^{1/4}}{(c+dx)^{1/4}} dx$$

Optimal (type 3, 268 leaves, 8 steps):

$$\frac{(15b^2c^2 + 10abcd + 7a^2d^2)(a+bx)^{1/4}(c+dx)^{3/4}}{32b^2d^3} - \frac{(9bc + 7ad)(a+bx)^{5/4}(c+dx)^{3/4}}{24b^2d^2} +$$

$$\frac{x(a+bx)^{5/4}(c+dx)^{3/4}}{3bd} - \frac{(bc-ad)(15b^2c^2 + 10abcd + 7a^2d^2) \operatorname{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{64b^{11/4}d^{13/4}} -$$

$$\frac{(bc-ad)(15b^2c^2 + 10abcd + 7a^2d^2) \operatorname{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{64b^{11/4}d^{13/4}}$$

Result (type 5, 168 leaves):

$$\left( (c+dx)^{3/4} \left( -d(a+bx)(7a^2d^2 + 2abd(3c-2dx) + b^2(-45c^2 + 36cdx - 32d^2x^2)) + \right. \right.$$

$$\left. (-15b^3c^3 + 5ab^2c^2d + 3a^2bcd^2 + 7a^3d^3) \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} \right.$$

$$\left. \left. \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / (96b^2d^4(a+bx)^{3/4})$$

### Problem 873: Result unnecessarily involves higher level functions.

$$\int \frac{x(a+bx)^{1/4}}{(c+dx)^{1/4}} dx$$

Optimal (type 3, 188 leaves, 7 steps):

$$- \frac{(5bc + 3ad)(a+bx)^{1/4}(c+dx)^{3/4}}{8bd^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2bd} +$$

$$\frac{(bc-ad)(5bc+3ad) \operatorname{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{16b^{7/4}d^{9/4}} + \frac{(bc-ad)(5bc+3ad) \operatorname{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{16b^{7/4}d^{9/4}}$$

Result (type 5, 122 leaves):

$$\left( (c+dx)^{3/4} \left( 3d(a+bx)(-5bc+ad+4bdx) + (5b^2c^2 - 2abcd - 3a^2d^2) \right. \right.$$

$$\left. \left. \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / (24bd^3(a+bx)^{3/4})$$

**Problem 874: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{(c+dx)^{1/4}} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$\frac{(a+bx)^{1/4} (c+dx)^{3/4}}{d} - \frac{(bc-ad) \operatorname{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{2b^{3/4}d^{5/4}} - \frac{(bc-ad) \operatorname{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{2b^{3/4}d^{5/4}}$$

Result (type 5, 76 leaves):

$$\frac{(a+bx)^{1/4} (c+dx)^{3/4} \left( 3 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{b-c-ad}\right]}{\left(\frac{d(a+bx)}{-bc-ad}\right)^{1/4}} \right)}{3d}$$

**Problem 875: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{x(c+dx)^{1/4}} dx$$

Optimal (type 3, 169 leaves, 11 steps):

$$-\frac{2a^{1/4} \operatorname{ArcTan}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{c^{1/4}} + \frac{2b^{1/4} \operatorname{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{d^{1/4}} -$$

$$\frac{2a^{1/4} \operatorname{ArcTanh}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{c^{1/4}} + \frac{2b^{1/4} \operatorname{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{d^{1/4}}$$

Result (type 6, 216 leaves):

$$\left( 36a(bc-ad)(a+bx)^{5/4} \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{d(a+bx)}{-bc+ad}, 1 + \frac{bx}{a}\right] \right) /$$

$$\left( 5bx(c+dx)^{1/4} \left( 9a(bc-ad) \operatorname{AppellF1}\left[\frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{d(a+bx)}{-bc+ad}, 1 + \frac{bx}{a}\right] - \right. \right.$$

$$(a+bx) \left( (-4bc+4ad) \operatorname{AppellF1}\left[\frac{9}{4}, \frac{1}{4}, 2, \frac{13}{4}, \frac{d(a+bx)}{-bc+ad}, 1 + \frac{bx}{a}\right] + \right.$$

$$\left. \left. \left. \left. ad \operatorname{AppellF1}\left[\frac{9}{4}, \frac{5}{4}, 1, \frac{13}{4}, \frac{d(a+bx)}{-bc+ad}, 1 + \frac{bx}{a}\right] \right) \right) \right) \right)$$

**Problem 876: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{x^2(c+dx)^{1/4}} dx$$

Optimal (type 3, 131 leaves, 5 steps):

$$-\frac{(a+bx)^{1/4} (c+dx)^{3/4}}{cx} - \frac{(bc-ad) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{2a^{3/4} c^{5/4}} - \frac{(bc-ad) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{2a^{3/4} c^{5/4}}$$

Result (type 6, 176 leaves):

$$\left( -(a+bx)(c+dx) + \left( 2bd(bc-ad)x^2 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / \right. \\ \left. \left( -8bdx \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] + bc \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] + \right. \right. \\ \left. \left. 3ad \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right) / \left( cx(a+bx)^{3/4} (c+dx)^{1/4} \right)$$

**Problem 877: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{x^3 (c+dx)^{1/4}} dx$$

Optimal (type 3, 194 leaves, 6 steps):

$$\frac{(3bc+5ad)(a+bx)^{1/4} (c+dx)^{3/4}}{8ac^2x} - \frac{(a+bx)^{5/4} (c+dx)^{3/4}}{2acx^2} + \\ \frac{(bc-ad)(3bc+5ad) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{16a^{7/4} c^{9/4}} + \frac{(bc-ad)(3bc+5ad) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{16a^{7/4} c^{9/4}}$$

Result (type 6, 211 leaves):

$$\left( (a+bx)(c+dx)(-4ac-bcx+5adx) + \right. \\ \left. \left( 2bd(-3b^2c^2-2abcd+5a^2d^2)x^3 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / \right. \\ \left. \left( -8bdx \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] + bc \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] + \right. \right. \\ \left. \left. 3ad \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right) / \left( 8ac^2x^2 (a+bx)^{3/4} (c+dx)^{1/4} \right)$$

**Problem 878: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{x^4 (c+dx)^{1/4}} dx$$

Optimal (type 3, 266 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{(a+bx)^{1/4} (c+dx)^{3/4}}{3cx^3} - \frac{(bc-9ad)(a+bx)^{1/4} (c+dx)^{3/4}}{24a^2c^2x^2} + \\
 & \frac{(7bc-15ad)(bc+3ad)(a+bx)^{1/4} (c+dx)^{3/4}}{96a^2c^3x} - \\
 & \frac{(bc-ad)(7b^2c^2+10abcd+15a^2d^2) \operatorname{ArcTan}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{64a^{11/4}c^{13/4}} - \\
 & \frac{(bc-ad)(7b^2c^2+10abcd+15a^2d^2) \operatorname{ArcTanh}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{64a^{11/4}c^{13/4}}
 \end{aligned}$$

Result (type 6, 260 leaves):

$$\begin{aligned}
 & \left( -(a+bx)(c+dx)(-7b^2c^2x^2+2abcx(2c-3dx)+a^2(32c^2-36cdx+45d^2x^2)) + \right. \\
 & \left. (6bd(7b^3c^3+3a^2b^2c^2d+5a^2bcd^2-15a^3d^3)x^4 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right]) \right) / \\
 & \left( -8bdx \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] + bc \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] + \right. \\
 & \left. 3ad \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / \left( 96a^2c^3x^3(a+bx)^{3/4}(c+dx)^{1/4} \right)
 \end{aligned}$$

**Problem 879: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{x^5(c+dx)^{1/4}} dx$$

Optimal (type 3, 368 leaves, 9 steps):

$$\begin{aligned}
 & - \frac{(a+bx)^{1/4} (c+dx)^{3/4}}{4cx^4} - \frac{(bc-13ad)(a+bx)^{1/4} (c+dx)^{3/4}}{48a^2c^2x^3} + \\
 & \frac{(11b^2c^2+10abcd-117a^2d^2)(a+bx)^{1/4} (c+dx)^{3/4}}{384a^2c^3x^2} - \\
 & \frac{(77b^3c^3+61a^2b^2c^2d+63a^2bcd^2-585a^3d^3)(a+bx)^{1/4} (c+dx)^{3/4}}{1536a^3c^4x} + \frac{1}{1024a^{15/4}c^{17/4}} \\
 & \frac{(bc-ad)(77b^3c^3+105a^2b^2c^2d+135a^2bcd^2+195a^3d^3) \operatorname{ArcTan}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{64a^{11/4}c^{13/4}} + \\
 & \frac{1}{1024a^{15/4}c^{17/4}} (bc-ad)(77b^3c^3+105a^2b^2c^2d+135a^2bcd^2+195a^3d^3) \operatorname{ArcTanh}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]
 \end{aligned}$$

Result (type 6, 315 leaves):

$$\left( (a + b x) (c + d x) (-77 b^3 c^3 x^3 + a b^2 c^2 x^2 (44 c - 61 d x) + a^2 b c x (-32 c^2 + 40 c d x - 63 d^2 x^2) + a^3 (-384 c^3 + 416 c^2 d x - 468 c d^2 x^2 + 585 d^3 x^3)) - (6 b d (77 b^4 c^4 + 28 a b^3 c^3 d + 30 a^2 b^2 c^2 d^2 + 60 a^3 b c d^3 - 195 a^4 d^4) x^5 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) / \left( -8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{b x}, -\frac{c}{d x}\right] + b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{b x}, -\frac{c}{d x}\right] \right) / \left( 1536 a^3 c^4 x^4 (a + b x)^{3/4} (c + d x)^{1/4} \right)$$

**Problem 880: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2 (1+x)^{1/4}}{(1-x)^{1/4}} dx$$

Optimal (type 3, 234 leaves, 14 steps):

$$-\frac{3}{8} (1-x)^{3/4} (1+x)^{1/4} - \frac{1}{12} (1-x)^{3/4} (1+x)^{5/4} - \frac{1}{3} (1-x)^{3/4} x (1+x)^{5/4} + \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{16 \sqrt{2}} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{16 \sqrt{2}}$$

Result (type 5, 57 leaves):

$$\frac{1}{24} (1+x)^{1/4} \left( - (1-x)^{3/4} (11 + 10 x + 8 x^2) + 9 \times 2^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1+x}{2}\right] \right)$$

**Problem 881: Result unnecessarily involves higher level functions.**

$$\int \frac{x (1+x)^{1/4}}{(1-x)^{1/4}} dx$$

Optimal (type 3, 213 leaves, 13 steps):

$$-\frac{1}{4} (1-x)^{3/4} (1+x)^{1/4} - \frac{1}{2} (1-x)^{3/4} (1+x)^{5/4} + \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{4 \sqrt{2}} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{4 \sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}}\right]}{8 \sqrt{2}}$$

Result (type 5, 51 leaves):

$$\frac{1}{4} (1+x)^{1/4} \left( - (1-x)^{3/4} (3+2x) + 2^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1+x}{2} \right] \right)$$

**Problem 882: Result unnecessarily involves higher level functions.**

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4}} dx$$

Optimal (type 3, 186 leaves, 12 steps):

$$- (1-x)^{3/4} (1+x)^{1/4} + \frac{\text{ArcTan} \left[ 1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right]}{\sqrt{2}} - \frac{\text{ArcTan} \left[ 1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right]}{\sqrt{2}} - \frac{\text{Log} \left[ 1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right]}{2\sqrt{2}} + \frac{\text{Log} \left[ 1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right]}{2\sqrt{2}}$$

Result (type 5, 43 leaves):

$$(1+x)^{1/4} \left( - (1-x)^{3/4} + 2^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1+x}{2} \right] \right)$$

**Problem 883: Result unnecessarily involves higher level functions.**

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4} x} dx$$

Optimal (type 3, 203 leaves, 16 steps):

$$-2 \text{ArcTan} \left[ \frac{(1+x)^{1/4}}{(1-x)^{1/4}} \right] + \sqrt{2} \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right] - \sqrt{2} \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right] - 2 \text{ArcTanh} \left[ \frac{(1+x)^{1/4}}{(1-x)^{1/4}} \right] - \frac{\text{Log} \left[ 1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} - \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right]}{\sqrt{2}} + \frac{\text{Log} \left[ 1 + \frac{\sqrt{1-x}}{\sqrt{1+x}} + \frac{\sqrt{2} (1-x)^{1/4}}{(1+x)^{1/4}} \right]}{\sqrt{2}}$$

Result (type 6, 119 leaves):

$$\left( 72 (1+x)^{5/4} \text{AppellF1} \left[ \frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{1+x}{2}, 1+x \right] \right) / \left( 5 (1-x)^{1/4} x \left( 18 \text{AppellF1} \left[ \frac{5}{4}, \frac{1}{4}, 1, \frac{9}{4}, \frac{1+x}{2}, 1+x \right] + (1+x) \left( 8 \text{AppellF1} \left[ \frac{9}{4}, \frac{1}{4}, 2, \frac{13}{4}, \frac{1+x}{2}, 1+x \right] + \text{AppellF1} \left[ \frac{9}{4}, \frac{5}{4}, 1, \frac{13}{4}, \frac{1+x}{2}, 1+x \right] \right) \right) \right)$$

**Problem 884: Result unnecessarily involves higher level functions.**

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4} x^2} dx$$

Optimal (type 3, 62 leaves, 5 steps):

$$-\frac{(1-x)^{3/4}(1+x)^{1/4}}{x} - \text{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \text{ArcTanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 106 leaves):

$$\left(-1+x^2 - \left(4x^2 \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]\right)\right) / \left(8x \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \text{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \text{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]\right) / \left((1-x)^{1/4}x(1+x)^{3/4}\right)$$

**Problem 885: Result unnecessarily involves higher level functions.**

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4}x^3} dx$$

Optimal (type 3, 91 leaves, 6 steps):

$$-\frac{(1-x)^{3/4}(1+x)^{1/4}}{4x} - \frac{(1-x)^{3/4}(1+x)^{5/4}}{2x^2} - \frac{1}{4} \text{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \frac{1}{4} \text{ArcTanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 114 leaves):

$$\left(2 - \frac{2}{x^2} - \frac{3}{x} + 3x - \left(4x \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]\right)\right) / \left(8x \text{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \text{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \text{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]\right) / \left(4(1-x)^{1/4}(1+x)^{3/4}\right)$$

**Problem 886: Result unnecessarily involves higher level functions.**

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4}x^4} dx$$

Optimal (type 3, 114 leaves, 8 steps):

$$-\frac{(1-x)^{3/4}(1+x)^{1/4}}{3x^3} - \frac{5(1-x)^{3/4}(1+x)^{1/4}}{12x^2} - \frac{11(1-x)^{3/4}(1+x)^{1/4}}{24x} - \frac{3}{8} \text{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \frac{3}{8} \text{ArcTanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 119 leaves):

$$\left(10 - \frac{8}{x^3} - \frac{10}{x^2} - \frac{3}{x} + 11x - \left(36x \operatorname{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]\right)\right) / \left(8x \operatorname{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \operatorname{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \operatorname{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]\right) / \left(24(1-x)^{1/4}(1+x)^{3/4}\right)$$

**Problem 887: Result unnecessarily involves higher level functions.**

$$\int \frac{(1+x)^{1/4}}{(1-x)^{1/4} x^5} dx$$

Optimal (type 3, 137 leaves, 9 steps):

$$\frac{(1-x)^{3/4}(1+x)^{1/4}}{4x^4} - \frac{7(1-x)^{3/4}(1+x)^{1/4}}{24x^3} - \frac{29(1-x)^{3/4}(1+x)^{1/4}}{96x^2} - \frac{83(1-x)^{3/4}(1+x)^{1/4}}{192x} - \frac{11}{64} \operatorname{ArcTan}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right] - \frac{11}{64} \operatorname{ArcTanh}\left[\frac{(1+x)^{1/4}}{(1-x)^{1/4}}\right]$$

Result (type 6, 124 leaves):

$$\left(58 - \frac{48}{x^4} - \frac{56}{x^3} - \frac{10}{x^2} - \frac{27}{x} + 83x - \left(132x \operatorname{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right]\right)\right) / \left(8x \operatorname{AppellF1}\left[1, \frac{1}{4}, \frac{3}{4}, 2, \frac{1}{x}, -\frac{1}{x}\right] - 3 \operatorname{AppellF1}\left[2, \frac{1}{4}, \frac{7}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right] + \operatorname{AppellF1}\left[2, \frac{5}{4}, \frac{3}{4}, 3, \frac{1}{x}, -\frac{1}{x}\right]\right) / \left(192(1-x)^{1/4}(1+x)^{3/4}\right)$$

**Problem 888: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3}{(a+bx)^{3/4}(c+dx)^{1/4}} dx$$

Optimal (type 3, 259 leaves, 7 steps):

$$\frac{x^2(a+bx)^{1/4}(c+dx)^{3/4}}{3bd} + \frac{1}{96b^3d^3} - \frac{(a+bx)^{1/4}(c+dx)^{3/4}(45b^2c^2 + 54abcd + 77a^2d^2 - 4bd(9bc + 11ad)x) - (15b^3c^3 + 15ab^2c^2d + 21a^2bcd^2 + 77a^3d^3) \operatorname{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{64b^{15/4}d^{13/4}} - \frac{(15b^3c^3 + 15ab^2c^2d + 21a^2bcd^2 + 77a^3d^3) \operatorname{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{64b^{15/4}d^{13/4}}$$

Result (type 5, 168 leaves):

$$\left( (c+dx)^{3/4} \left( d(a+bx) (77a^2d^2 + 2abd(27c - 22dx) + b^2(45c^2 - 36cdx + 32d^2x^2)) - (15b^3c^3 + 15ab^2c^2d + 21a^2bcd^2 + 77a^3d^3) \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} \right) \right) - \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right] \Bigg) / (96b^3d^4(a+bx)^{3/4})$$

**Problem 889: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2}{(a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 201 leaves, 7 steps):

$$-\frac{(5bc+7ad)(a+bx)^{1/4}(c+dx)^{3/4}}{8b^2d^2} + \frac{x(a+bx)^{1/4}(c+dx)^{3/4}}{2bd} + \frac{(5b^2c^2+6abcd+21a^2d^2)\text{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{16b^{11/4}d^{9/4}} + \frac{(5b^2c^2+6abcd+21a^2d^2)\text{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{16b^{11/4}d^{9/4}}$$

Result (type 5, 123 leaves):

$$\left( (c+dx)^{3/4} \left( -3d(a+bx)(5bc+7ad-4bdx) + (5b^2c^2+6abcd+21a^2d^2) \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / (24b^2d^3(a+bx)^{3/4})$$

**Problem 890: Result unnecessarily involves higher level functions.**

$$\int \frac{x}{(a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$\frac{(a+bx)^{1/4}(c+dx)^{3/4}}{bd} - \frac{(bc+3ad)\text{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{2b^{7/4}d^{5/4}} - \frac{(bc+3ad)\text{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{2b^{7/4}d^{5/4}}$$

Result (type 5, 95 leaves):

$$\frac{1}{3bd^2(a+bx)^{3/4}} (c+dx)^{3/4} \left( 3d(a+bx) - (bc+3ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 891: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{2 \operatorname{ArcTan}\left[\frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}}\right]}{b^{3/4} d^{1/4}} + \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}}\right]}{b^{3/4} d^{1/4}}$$

Result (type 5, 73 leaves):

$$\frac{4 \left(\frac{d(a+bx)}{-bc+ad}\right)^{3/4} (c+dx)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]}{3 d (a+bx)^{3/4}}$$

**Problem 892: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x (a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 85 leaves, 4 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{a^{3/4} c^{1/4}} - \frac{2 \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{a^{3/4} c^{1/4}}$$

Result (type 6, 146 leaves):

$$\left(8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right]\right) / \left(\left(a+bx\right)^{3/4} (c+dx)^{1/4} \left(-8 b d x \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] + b c \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] + 3 a d \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right]\right)\right)$$

**Problem 893: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 (a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 134 leaves, 5 steps):

$$-\frac{(a+bx)^{1/4} (c+dx)^{3/4}}{a c x} + \frac{(3 b c + a d) \operatorname{ArcTan}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{2 a^{7/4} c^{5/4}} + \frac{(3 b c + a d) \operatorname{ArcTanh}\left[\frac{c^{1/4} (a+bx)^{1/4}}{a^{1/4} (c+dx)^{1/4}}\right]}{2 a^{7/4} c^{5/4}}$$

Result (type 6, 180 leaves):

$$\left( -(a+bx)(c+dx) + \left( 2bd(3bc+ad)x^2 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right) /$$

$$\left( 8bdx \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] - bc \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] - \right.$$

$$\left. 3ad \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / (acx(a+bx)^{3/4}(c+dx)^{1/4})$$

**Problem 894: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 (a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 206 leaves, 7 steps):

$$-\frac{(a+bx)^{1/4} (c+dx)^{3/4}}{2acx^2} + \frac{(7bc+5ad)(a+bx)^{1/4} (c+dx)^{3/4}}{8a^2c^2x}$$

$$\frac{(21b^2c^2+6abcd+5a^2d^2) \operatorname{ArcTan}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{16a^{11/4}c^{9/4}}$$

$$\frac{(21b^2c^2+6abcd+5a^2d^2) \operatorname{ArcTanh}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{16a^{11/4}c^{9/4}}$$

Result (type 6, 211 leaves):

$$\left( (a+bx)(c+dx)(-4ac+7bcx+5adx) + \right.$$

$$\left( 2bd(21b^2c^2+6abcd+5a^2d^2)x^3 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) /$$

$$\left( -8bdx \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] + bc \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] + \right.$$

$$\left. 3ad \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / (8a^2c^2x^2(a+bx)^{3/4}(c+dx)^{1/4})$$

**Problem 895: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^4 (a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 288 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{(a+bx)^{1/4} (c+dx)^{3/4}}{3acx^3} + \frac{(11bc+9ad)(a+bx)^{1/4} (c+dx)^{3/4}}{24a^2c^2x^2} - \\
 & \frac{(77b^2c^2+54abcd+45a^2d^2)(a+bx)^{1/4} (c+dx)^{3/4}}{96a^3c^3x} + \\
 & \frac{(77b^3c^3+21ab^2c^2d+15a^2b^2cd^2+15a^3d^3) \operatorname{ArcTan}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{64a^{15/4}c^{13/4}} + \\
 & \frac{(77b^3c^3+21ab^2c^2d+15a^2b^2cd^2+15a^3d^3) \operatorname{ArcTanh}\left[\frac{c^{1/4}(a+bx)^{1/4}}{a^{1/4}(c+dx)^{1/4}}\right]}{64a^{15/4}c^{13/4}}
 \end{aligned}$$

Result (type 6, 259 leaves):

$$\begin{aligned}
 & - \left( (a+bx)(c+dx)(77b^2c^2x^2+2abcx(-22c+27dx)+a^2(32c^2-36cdx+45d^2x^2)) + \right. \\
 & \left. \left( 6bd(77b^3c^3+21ab^2c^2d+15a^2b^2cd^2+15a^3d^3)x^4 \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right) / \\
 & \left( -8bdx \operatorname{AppellF1}\left[1, \frac{3}{4}, \frac{1}{4}, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] + bc \operatorname{AppellF1}\left[2, \frac{3}{4}, \frac{5}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] + \right. \\
 & \left. 3ad \operatorname{AppellF1}\left[2, \frac{7}{4}, \frac{1}{4}, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / \left( 96a^3c^3x^3(a+bx)^{3/4}(c+dx)^{1/4} \right)
 \end{aligned}$$

**Problem 896: Result unnecessarily involves higher level functions.**

$$\int \frac{(ex)^{3/2}}{(1-x)^{1/4}(1+x)^{1/4}} dx$$

Optimal (type 3, 244 leaves, 13 steps):

$$\begin{aligned}
 & - \frac{1}{2} e \sqrt{ex} (1-x^2)^{3/4} - \frac{e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{ex}}{\sqrt{e}(1-x^2)^{1/4}}\right]}{4\sqrt{2}} + \frac{e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{ex}}{\sqrt{e}(1-x^2)^{1/4}}\right]}{4\sqrt{2}} - \\
 & \frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \frac{\sqrt{ex}}{\sqrt{1-x^2}} - \frac{\sqrt{2}\sqrt{ex}}{(1-x^2)^{1/4}}\right]}{8\sqrt{2}} + \frac{e^{3/2} \operatorname{Log}\left[\sqrt{e} + \frac{\sqrt{ex}}{\sqrt{1-x^2}} + \frac{\sqrt{2}\sqrt{ex}}{(1-x^2)^{1/4}}\right]}{8\sqrt{2}}
 \end{aligned}$$

Result (type 5, 39 leaves):

$$\frac{1}{2} e \sqrt{ex} \left( -(1-x^2)^{3/4} + \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, x^2\right] \right)$$

**Problem 897: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/4} \sqrt{ex} (1+x)^{1/4}} dx$$

Optimal (type 3, 216 leaves, 12 steps):

$$-\frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}\sqrt{ex}}{\sqrt{e}(1-x^2)^{1/4}}\right]}{\sqrt{2}\sqrt{e}} + \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}\sqrt{ex}}{\sqrt{e}(1-x^2)^{1/4}}\right]}{\sqrt{2}\sqrt{e}} - \frac{\text{Log}\left[\sqrt{e} + \frac{\sqrt{e}x}{\sqrt{1-x^2}} - \frac{\sqrt{2}\sqrt{ex}}{(1-x^2)^{1/4}}\right]}{2\sqrt{2}\sqrt{e}} + \frac{\text{Log}\left[\sqrt{e} + \frac{\sqrt{e}x}{\sqrt{1-x^2}} + \frac{\sqrt{2}\sqrt{ex}}{(1-x^2)^{1/4}}\right]}{2\sqrt{2}\sqrt{e}}$$

Result (type 5, 23 leaves):

$$\frac{2 \times \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, x^2\right]}{\sqrt{ex}}$$

**Problem 901: Result unnecessarily involves higher level functions.**

$$\int \frac{(ex)^{5/2}}{(1-x)^{1/4}(1+x)^{1/4}} dx$$

Optimal (type 4, 93 leaves, 6 steps):

$$-\frac{e^3(1-x^2)^{3/4}}{2\sqrt{ex}} - \frac{1}{3}e(ex)^{3/2}(1-x^2)^{3/4} + \frac{e^2\left(1 - \frac{1}{x^2}\right)^{1/4}\sqrt{ex}\text{EllipticE}\left[\frac{\text{ArcCsc}[x]}{2}, 2\right]}{2(1-x^2)^{1/4}}$$

Result (type 5, 39 leaves):

$$-\frac{1}{3}e(ex)^{3/2}\left((1-x^2)^{3/4} - \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]\right)$$

**Problem 902: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{ex}}{(1-x)^{1/4}(1+x)^{1/4}} dx$$

Optimal (type 4, 60 leaves, 5 steps):

$$-\frac{e(1-x^2)^{3/4}}{\sqrt{ex}} + \frac{\left(1 - \frac{1}{x^2}\right)^{1/4}\sqrt{ex}\text{EllipticE}\left[\frac{\text{ArcCsc}[x]}{2}, 2\right]}{(1-x^2)^{1/4}}$$

Result (type 5, 25 leaves):

$$\frac{2}{3}x\sqrt{ex}\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]$$

**Problem 903: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/4}(ex)^{3/2}(1+x)^{1/4}} dx$$

Optimal (type 4, 42 leaves, 4 steps):

$$-\frac{2 \left(1 - \frac{1}{x^2}\right)^{1/4} \sqrt{e x} \operatorname{EllipticE}\left[\frac{\operatorname{ArcCsc}[x]}{2}, 2\right]}{e^2 (1-x^2)^{1/4}}$$

Result (type 5, 44 leaves):

$$-\frac{2 x \left(3 (1-x^2)^{3/4} + 2 x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]\right)}{3 (e x)^{3/2}}$$

**Problem 904: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/4} (e x)^{7/2} (1+x)^{1/4}} dx$$

Optimal (type 4, 70 leaves, 5 steps):

$$-\frac{2 (1-x^2)^{3/4}}{5 e (e x)^{5/2}} - \frac{4 \left(1 - \frac{1}{x^2}\right)^{1/4} \sqrt{e x} \operatorname{EllipticE}\left[\frac{\operatorname{ArcCsc}[x]}{2}, 2\right]}{5 e^4 (1-x^2)^{1/4}}$$

Result (type 5, 51 leaves):

$$\frac{x \left(-6 (1-x^2)^{3/4} (1+2 x^2) - 8 x^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right]\right)}{15 (e x)^{7/2}}$$

**Problem 905: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1-x)^{1/4} (e x)^{11/2} (1+x)^{1/4}} dx$$

Optimal (type 4, 95 leaves, 6 steps):

$$-\frac{2 (1-x^2)^{3/4}}{9 e (e x)^{9/2}} - \frac{4 (1-x^2)^{3/4}}{15 e^3 (e x)^{5/2}} - \frac{8 \left(1 - \frac{1}{x^2}\right)^{1/4} \sqrt{e x} \operatorname{EllipticE}\left[\frac{\operatorname{ArcCsc}[x]}{2}, 2\right]}{15 e^6 (1-x^2)^{1/4}}$$

Result (type 5, 60 leaves):

$$-\frac{1}{45 e^6 x^5} 2 \sqrt{e x} \left( (1-x^2)^{3/4} (5+6 x^2+12 x^4) + 8 x^6 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, x^2\right] \right)$$

**Problem 930: Result unnecessarily involves higher level functions.**

$$\int \frac{x^3 (a+b x)^n}{(c+d x)^2} dx$$

Optimal (type 5, 203 leaves, 3 steps):

$$\frac{x^2 (a+bx)^{1+n}}{bd(2+n)(c+dx)} - \left( (a+bx)^{1+n} \left( \frac{c(bc(2+n)(ad+bc(3+n)) - ad(ad+bc(5+3n))) + d(bc-ad)(ad+bc(3+n))x}{b^2d^3(bc-ad)(1+n)(2+n)(c+dx)} - \frac{c^2(3ad-bc(3+n))(a+bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad}\right]}{d^3(bc-ad)^2(1+n)} \right) \right) /$$

Result (type 6, 126 leaves):

$$\left( 5acx^4(a+bx)^n \text{AppellF1}\left[4, -n, 2, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \left( 4(c+dx)^2 \left( 5ac \text{AppellF1}\left[4, -n, 2, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] + bcnx \text{AppellF1}\left[5, 1-n, 2, 6, -\frac{bx}{a}, -\frac{dx}{c}\right] - 2adx \text{AppellF1}\left[5, -n, 3, 6, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right)$$

**Problem 931: Result unnecessarily involves higher level functions.**

$$\int \frac{x^2 (a+bx)^n}{(c+dx)^2} dx$$

Optimal (type 5, 122 leaves, 3 steps):

$$\frac{(a+bx)^{1+n}}{bd^2(1+n)} + \frac{c^2(a+bx)^{1+n}}{d^2(bc-ad)(c+dx)} + \left( \frac{c(2ad-bc(2+n))(a+bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad}\right]}{d^2(bc-ad)^2(1+n)} \right) /$$

Result (type 6, 126 leaves):

$$\left( 4acx^3(a+bx)^n \text{AppellF1}\left[3, -n, 2, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \left( 3(c+dx)^2 \left( 4ac \text{AppellF1}\left[3, -n, 2, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + bcnx \text{AppellF1}\left[4, 1-n, 2, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] - 2adx \text{AppellF1}\left[4, -n, 3, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right)$$

**Problem 932: Result unnecessarily involves higher level functions.**

$$\int \frac{x(a+bx)^n}{(c+dx)^2} dx$$

Optimal (type 5, 99 leaves, 2 steps):

$$-\frac{c(a+bx)^{1+n}}{d(bc-ad)(c+dx)} - \left( \frac{(ad-bc(1+n))(a+bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad}\right]}{(d(bc-ad))^2(1+n)} \right) /$$

Result (type 6, 126 leaves):

$$\left( 3acx^2(a+bx)^n \text{AppellF1}\left[2, -n, 2, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \left( 2(c+dx)^2 \left( 3ac \text{AppellF1}\left[2, -n, 2, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + bcnx \text{AppellF1}\left[3, 1-n, 2, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] - 2adx \text{AppellF1}\left[3, -n, 3, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right)$$

**Problem 933: Unable to integrate problem.**

$$\int \frac{(a+bx)^n}{(c+dx)^2} dx$$

Optimal (type 5, 52 leaves, 1 step):

$$\frac{b(a+bx)^{1+n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)^2(1+n)}$$

Result (type 8, 17 leaves):

$$\int \frac{(a+bx)^n}{(c+dx)^2} dx$$

**Problem 934: Unable to integrate problem.**

$$\int \frac{(a+bx)^n}{x(c+dx)^2} dx$$

Optimal (type 5, 139 leaves, 4 steps):

$$-\frac{d(a+bx)^{1+n}}{c(bc-ad)(c+dx)} + \left( \frac{(d(ad-bc(1-n))(a+bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad}\right]}{(c^2(bc-ad)^2(1+n))} - \frac{(a+bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, 1+\frac{bx}{a}\right]}{ac^2(1+n)} \right) /$$

Result (type 8, 20 leaves):

$$\int \frac{(a+bx)^n}{x(c+dx)^2} dx$$

**Problem 935: Unable to integrate problem.**

$$\int \frac{(a+bx)^n}{x^2(c+dx)^2} dx$$

Optimal (type 5, 190 leaves, 5 steps):

$$\begin{aligned} & -\frac{d(bc-2ad)(a+bx)^{1+n}}{ac^2(bc-ad)(c+dx)} - \frac{(a+bx)^{1+n}}{acx(c+dx)} - \\ & \left( d^2(2ad-bc(2-n))(a+bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad}\right] \right) / \\ & \left( c^3(bc-ad)^2(1+n) \right) + \frac{(2ad-bcn)(a+bx)^{1+n} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, 1+\frac{bx}{a}\right]}{a^2c^3(1+n)} \end{aligned}$$

Result (type 8, 20 leaves):

$$\int \frac{(a+bx)^n}{x^2(c+dx)^2} dx$$

**Problem 938: Result more than twice size of optimal antiderivative.**

$$\int \frac{(bx)^{5/2}(c+dx)^n}{e+fx} dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$\frac{2(bx)^{7/2}(c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}\left[\frac{7}{2}, -n, 1, \frac{9}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right]}{7be}$$

Result (type 6, 239 leaves):

$$\begin{aligned} & \frac{1}{15f^3x^2} 2(bx)^{5/2}(c+dx)^n \left( -\left( \left( 45ce^4 \text{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) / \right. \right. \\ & \left. \left( (e+fx) \left( 3ce \text{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] + 2denx \text{AppellF1}\left[\frac{3}{2}, 1-n, \right. \right. \right. \right. \\ & \left. \left. \left. 1, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] - 2cfx \text{AppellF1}\left[\frac{3}{2}, -n, 2, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) \right) \right) + \\ & \left( 1 + \frac{dx}{c} \right)^{-n} \left( 15e^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, -\frac{dx}{c}\right] + fx \right. \\ & \left. \left( -5e \text{Hypergeometric2F1}\left[\frac{3}{2}, -n, \frac{5}{2}, -\frac{dx}{c}\right] + 3fx \text{Hypergeometric2F1}\left[\frac{5}{2}, -n, \frac{7}{2}, -\frac{dx}{c}\right] \right) \right) \end{aligned}$$

### Problem 939: Result more than twice size of optimal antiderivative.

$$\int \frac{(bx)^{5/2} (c+dx)^n}{(e+fx)^2} dx$$

Optimal (type 6, 61 leaves, 3 steps):

$$\frac{2 (bx)^{7/2} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}\left[\frac{7}{2}, -n, 2, \frac{9}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right]}{7 b e^2}$$

Result (type 6, 345 leaves):

$$\begin{aligned} & \frac{1}{3 f^3} 2 b^2 \sqrt{bx} (c+dx)^n \left( \left( 27 c e^3 \text{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) / \right. \\ & \left( (e+fx) \left( 3 c e \text{AppellF1}\left[\frac{1}{2}, -n, 1, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] + 2 d e n x \text{AppellF1}\left[\frac{3}{2}, \right. \right. \right. \\ & \left. \left. \left. 1-n, 1, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] - 2 c f x \text{AppellF1}\left[\frac{3}{2}, -n, 2, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) \right) - \\ & \left( 9 c e^4 \text{AppellF1}\left[\frac{1}{2}, -n, 2, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) / \left( (e+fx)^2 \right. \\ & \left. \left( 3 c e \text{AppellF1}\left[\frac{1}{2}, -n, 2, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] + 2 d e n x \text{AppellF1}\left[\frac{3}{2}, 1-n, 2, \frac{5}{2}, \right. \right. \right. \\ & \left. \left. \left. -\frac{dx}{c}, -\frac{fx}{e}\right] - 4 c f x \text{AppellF1}\left[\frac{3}{2}, -n, 3, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) \right) + \left( 1 + \frac{dx}{c} \right)^{-n} \\ & \left. \left( -6 e \text{Hypergeometric2F1}\left[\frac{1}{2}, -n, \frac{3}{2}, -\frac{dx}{c}\right] + f x \text{Hypergeometric2F1}\left[\frac{3}{2}, -n, \frac{5}{2}, -\frac{dx}{c}\right] \right) \right) \end{aligned}$$

### Problem 942: Result more than twice size of optimal antiderivative.

$$\int \frac{(bx)^m (c+dx)^n}{e+fx} dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$\frac{(bx)^{1+m} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}\left[1+m, -n, 1, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right]}{b e (1+m)}$$

Result (type 6, 153 leaves):

$$\begin{aligned} & \left( c e (2+m) x (bx)^m (c+dx)^n \text{AppellF1}\left[1+m, -n, 1, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) / \\ & \left( (1+m) (e+fx) \left( c e (2+m) \text{AppellF1}\left[1+m, -n, 1, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right] + \right. \right. \\ & \left. \left. x \left( d e n \text{AppellF1}\left[2+m, 1-n, 1, 3+m, -\frac{dx}{c}, -\frac{fx}{e}\right] - \right. \right. \right. \\ & \left. \left. \left. c f \text{AppellF1}\left[2+m, -n, 2, 3+m, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) \right) \right) \end{aligned}$$

**Problem 943: Result more than twice size of optimal antiderivative.**

$$\int \frac{(bx)^m (c+dx)^n}{(e+fx)^2} dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$\frac{(bx)^{1+m} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} \text{AppellF1}\left[1+m, -n, 2, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right]}{b e^2 (1+m)}$$

Result (type 6, 153 leaves):

$$\left( c e (2+m) x (bx)^m (c+dx)^n \text{AppellF1}\left[1+m, -n, 2, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) /$$

$$\left( (1+m) (e+fx)^2 \left( c e (2+m) \text{AppellF1}\left[1+m, -n, 2, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right] + \right. \right.$$

$$x \left( d e n \text{AppellF1}\left[2+m, 1-n, 2, 3+m, -\frac{dx}{c}, -\frac{fx}{e}\right] - \right.$$

$$\left. \left. \left. 2 c f \text{AppellF1}\left[2+m, -n, 3, 3+m, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) \right) \right)$$

**Problem 944: Result more than twice size of optimal antiderivative.**

$$\int (bx)^m (c+dx)^n (e+fx)^p dx$$

Optimal (type 6, 81 leaves, 3 steps):

$$\frac{1}{b (1+m)} (bx)^{1+m} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} (e+fx)^p$$

$$\left(1 + \frac{fx}{e}\right)^{-p} \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right]$$

Result (type 6, 163 leaves):

$$\left( c e (2+m) x (bx)^m (c+dx)^n (e+fx)^p \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) /$$

$$\left( (1+m) \left( c e (2+m) \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{c}, -\frac{fx}{e}\right] + \right. \right.$$

$$x \left( d e n \text{AppellF1}\left[2+m, 1-n, -p, 3+m, -\frac{dx}{c}, -\frac{fx}{e}\right] + \right.$$

$$\left. \left. \left. c f p \text{AppellF1}\left[2+m, -n, 1-p, 3+m, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) \right) \right)$$

**Problem 946: Result unnecessarily involves higher level functions.**

$$\int x^2 (a+bx)^n (c+dx)^p dx$$

Optimal (type 5, 206 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{(bc(2+n) + ad(2+p)) (a+bx)^{1+n} (c+dx)^{1+p}}{b^2 d^2 (2+n+p) (3+n+p)} + \frac{x (a+bx)^{1+n} (c+dx)^{1+p}}{bd(3+n+p)} - \\
 & \left( (b^2 c^2 (2+3n+n^2) + 2abcd(1+n)(1+p) + a^2 d^2 (2+3p+p^2)) \right. \\
 & \left. (a+bx)^{1+n} (c+dx)^{1+p} \text{Hypergeometric2F1}\left[1, 2+n+p, 2+p, \frac{b(c+dx)}{bc-ad}\right] \right) / \\
 & (b^2 d^2 (bc-ad) (1+p) (2+n+p) (3+n+p))
 \end{aligned}$$

Result (type 6, 136 leaves):

$$\begin{aligned}
 & \left( 4acx^3 (a+bx)^n (c+dx)^p \text{AppellF1}\left[3, -n, -p, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \\
 & \left( 3 \left( 4ac \text{AppellF1}\left[3, -n, -p, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + bcnx \text{AppellF1}\left[4, 1-n, -p, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] + \right. \right. \\
 & \left. \left. adpx \text{AppellF1}\left[4, -n, 1-p, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right)
 \end{aligned}$$

**Problem 947: Result unnecessarily involves higher level functions.**

$$\int x (a+bx)^n (c+dx)^p dx$$

Optimal (type 5, 117 leaves, 3 steps):

$$\begin{aligned}
 & \frac{(a+bx)^{1+n} (c+dx)^{1+p}}{bd(2+n+p)} + \left( (bc(1+n) + ad(1+p)) (a+bx)^{1+n} (c+dx)^{1+p} \right. \\
 & \left. \text{Hypergeometric2F1}\left[1, 2+n+p, 2+p, \frac{b(c+dx)}{bc-ad}\right] \right) / (bd(bc-ad)(1+p)(2+n+p))
 \end{aligned}$$

Result (type 6, 136 leaves):

$$\begin{aligned}
 & \left( 3acx^2 (a+bx)^n (c+dx)^p \text{AppellF1}\left[2, -n, -p, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \\
 & \left( 6ac \text{AppellF1}\left[2, -n, -p, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + 2bcnx \text{AppellF1}\left[3, 1-n, -p, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + \right. \\
 & \left. 2adpx \text{AppellF1}\left[3, -n, 1-p, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right)
 \end{aligned}$$

**Problem 949: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^n (c+dx)^p}{x} dx$$

Optimal (type 6, 85 leaves, 2 steps):

$$- \frac{1}{a(1+n)} (a+bx)^{1+n} (c+dx)^p \left( \frac{b(c+dx)}{bc-ad} \right)^{-p} \text{AppellF1}\left[1+n, -p, 1, 2+n, -\frac{d(a+bx)}{bc-ad}, \frac{a+bx}{a}\right]$$

Result (type 6, 214 leaves):

$$\left( b d (-1+n+p) x (a+bx)^n (c+dx)^p \text{AppellF1}\left[-n-p, -n, -p, 1-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) /$$

$$\left( (n+p) \left( b d (-1+n+p) x \text{AppellF1}\left[-n-p, -n, -p, 1-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] - \right. \right.$$

$$a d n \text{AppellF1}\left[1-n-p, 1-n, -p, 2-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] -$$

$$\left. \left. b c p \text{AppellF1}\left[1-n-p, -n, 1-p, 2-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right)$$

**Problem 950: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^n (c+dx)^p}{x^2} dx$$

Optimal (type 6, 85 leaves, 2 steps):

$$\frac{1}{a^2 (1+n)} b (a+bx)^{1+n} (c+dx)^p \left( \frac{b(c+dx)}{bc-ad} \right)^{-p} \text{AppellF1}\left[1+n, -p, 2, 2+n, -\frac{d(a+bx)}{bc-ad}, \frac{a+bx}{a}\right]$$

Result (type 6, 216 leaves):

$$\left( b d (-2+n+p) (a+bx)^n (c+dx)^p \text{AppellF1}\left[1-n-p, -n, -p, 2-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) /$$

$$\left( (-1+n+p) \left( b d (-2+n+p) x \text{AppellF1}\left[1-n-p, -n, -p, 2-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] - \right. \right.$$

$$a d n \text{AppellF1}\left[2-n-p, 1-n, -p, 3-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] -$$

$$\left. \left. b c p \text{AppellF1}\left[2-n-p, -n, 1-p, 3-n-p, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right)$$

**Problem 951: Result more than twice size of optimal antiderivative.**

$$\int (bx)^{3/2} (c+dx)^n (e+fx)^p dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\frac{1}{5b} 2 (bx)^{5/2} (c+dx)^n \left( 1 + \frac{dx}{c} \right)^{-n} (e+fx)^p \left( 1 + \frac{fx}{e} \right)^{-p} \text{AppellF1}\left[\frac{5}{2}, -n, -p, \frac{7}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right]$$

Result (type 6, 159 leaves):

$$\left( 14 c e x (bx)^{3/2} (c+dx)^n (e+fx)^p \text{AppellF1}\left[\frac{5}{2}, -n, -p, \frac{7}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) /$$

$$\left( 5 \left( 7 c e \text{AppellF1}\left[\frac{5}{2}, -n, -p, \frac{7}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] + \right. \right.$$

$$2 x \left( d e n \text{AppellF1}\left[\frac{7}{2}, 1-n, -p, \frac{9}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] + \right.$$

$$\left. \left. c f p \text{AppellF1}\left[\frac{7}{2}, -n, 1-p, \frac{9}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] \right) \right)$$

**Problem 953: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^n (e+fx)^p}{\sqrt{bx}} dx$$

Optimal (type 6, 77 leaves, 3 steps):

$$\frac{1}{b} 2 \sqrt{bx} (c+dx)^n \left(1 + \frac{dx}{c}\right)^{-n} (e+fx)^p \left(1 + \frac{fx}{e}\right)^{-p} \text{AppellF1}\left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right]$$

Result (type 6, 157 leaves):

$$\left(6 c e x (c+dx)^n (e+fx)^p \text{AppellF1}\left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right]\right) / \left(\sqrt{bx}\right) \\ \left(3 c e \text{AppellF1}\left[\frac{1}{2}, -n, -p, \frac{3}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] + 2 d e n x \text{AppellF1}\left[\frac{3}{2}, 1-n, -p, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right] + \right. \\ \left. 2 c f p x \text{AppellF1}\left[\frac{3}{2}, -n, 1-p, \frac{5}{2}, -\frac{dx}{c}, -\frac{fx}{e}\right]\right)$$

**Problem 954: Result more than twice size of optimal antiderivative.**

$$\int (bx)^m (\pi+dx)^n (e+fx)^p dx$$

Optimal (type 6, 49 leaves, 1 step):

$$\frac{e^p \pi^n (bx)^{1+m} \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right]}{b(1+m)}$$

Result (type 6, 163 leaves):

$$\left(e(2+m)\pi x (bx)^m (\pi+dx)^n (e+fx)^p \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right]\right) / \\ \left((1+m) \left(e(2+m)\pi \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right. \right. \\ \left. \left. x \left(d e n \text{AppellF1}\left[2+m, 1-n, -p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right. \right. \right. \\ \left. \left. \left. f p \pi \text{AppellF1}\left[2+m, -n, 1-p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right]\right)\right)\right)$$

**Problem 955: Result more than twice size of optimal antiderivative.**

$$\int (bx)^m (\pi+dx)^n (e+fx)^p dx$$

Optimal (type 6, 65 leaves, 2 steps):

$$\frac{1}{b(1+m)} \pi^n (bx)^{1+m} (e+fx)^p \left(1 + \frac{fx}{e}\right)^{-p} \text{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right]$$

Result (type 6, 163 leaves):

$$\left( e (2+m) \pi x (bx)^m (\pi+dx)^n (e+fx)^p \operatorname{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right] \right) /$$

$$\left( (1+m) \left( e (2+m) \pi \operatorname{AppellF1}\left[1+m, -n, -p, 2+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right. \right.$$

$$x \left( d e n \operatorname{AppellF1}\left[2+m, 1-n, -p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right.$$

$$\left. \left. \left. f p \pi \operatorname{AppellF1}\left[2+m, -n, 1-p, 3+m, -\frac{dx}{\pi}, -\frac{fx}{e}\right] \right) \right) \right)$$

**Problem 956: Result more than twice size of optimal antiderivative.**

$$\int (bx)^{5/2} (\pi+dx)^n (e+fx)^p dx$$

Optimal (type 6, 47 leaves, 1 step):

$$\frac{2 e^p \pi^n (bx)^{7/2} \operatorname{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right]}{7 b}$$

Result (type 6, 159 leaves):

$$\left( 18 e \pi x (bx)^{5/2} (\pi+dx)^n (e+fx)^p \operatorname{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] \right) /$$

$$\left( 7 \left( 9 e \pi \operatorname{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right. \right.$$

$$2 x \left( d e n \operatorname{AppellF1}\left[\frac{9}{2}, 1-n, -p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right.$$

$$\left. \left. \left. f p \pi \operatorname{AppellF1}\left[\frac{9}{2}, -n, 1-p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] \right) \right) \right)$$

**Problem 957: Result more than twice size of optimal antiderivative.**

$$\int (bx)^{5/2} (\pi+dx)^n (e+fx)^p dx$$

Optimal (type 6, 63 leaves, 2 steps):

$$\frac{2 \pi^n (bx)^{7/2} (e+fx)^p \left(1 + \frac{fx}{e}\right)^{-p} \operatorname{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right]}{7 b}$$

Result (type 6, 159 leaves):

$$\left( 18 e \pi x (bx)^{5/2} (\pi+dx)^n (e+fx)^p \operatorname{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] \right) /$$

$$\left( 7 \left( 9 e \pi \operatorname{AppellF1}\left[\frac{7}{2}, -n, -p, \frac{9}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right. \right.$$

$$2 x \left( d e n \operatorname{AppellF1}\left[\frac{9}{2}, 1-n, -p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] + \right.$$

$$\left. \left. \left. f p \pi \operatorname{AppellF1}\left[\frac{9}{2}, -n, 1-p, \frac{11}{2}, -\frac{dx}{\pi}, -\frac{fx}{e}\right] \right) \right) \right)$$

**Problem 958: Result unnecessarily involves higher level functions.**

$$\int x^3 (a+bx)^n (c+dx)^{-n} dx$$

Optimal (type 5, 295 leaves, 4 steps):

$$\begin{aligned} & \frac{x^2 (a+bx)^{1+n} (c+dx)^{1-n}}{4bd} + \frac{1}{24b^3d^3} (a+bx)^{1+n} (c+dx)^{1-n} \\ & (2abcd(3-n^2) + a^2d^2(6-5n+n^2) + b^2c^2(6+5n+n^2) - 2bd(ad(3-n) + bc(3+n))x) - \\ & \frac{1}{24b^4d^3(1+n)} (3ab^2c^2d(2+n-2n^2-n^3) + a^3d^3(6-11n+6n^2-n^3) + \\ & 3a^2bcd^2(2-n-2n^2+n^3) + b^3c^3(6+11n+6n^2+n^3)) (a+bx)^{1+n} \\ & (c+dx)^{-n} \left( \frac{b(c+dx)}{bc-ad} \right)^n \text{Hypergeometric2F1} \left[ n, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad} \right] \end{aligned}$$

Result (type 6, 130 leaves):

$$\begin{aligned} & \left( 5acx^4 (a+bx)^n (c+dx)^{-n} \text{AppellF1} \left[ 4, -n, n, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \\ & \left( 20ac \text{AppellF1} \left[ 4, -n, n, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] + \right. \\ & \left. 4bcnx \text{AppellF1} \left[ 5, 1-n, n, 6, -\frac{bx}{a}, -\frac{dx}{c} \right] - 4adnx \text{AppellF1} \left[ 5, -n, 1+n, 6, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) \end{aligned}$$

**Problem 959: Result unnecessarily involves higher level functions.**

$$\int x^2 (a+bx)^n (c+dx)^{-n} dx$$

Optimal (type 5, 199 leaves, 4 steps):

$$\begin{aligned} & -\frac{(ad(2-n) + bc(2+n)) (a+bx)^{1+n} (c+dx)^{1-n}}{6b^2d^2} + \frac{x(a+bx)^{1+n} (c+dx)^{1-n}}{3bd} + \\ & \frac{1}{6b^3d^2(1+n)} (2abcd(1-n^2) + a^2d^2(2-3n+n^2) + b^2c^2(2+3n+n^2)) (a+bx)^{1+n} \\ & (c+dx)^{-n} \left( \frac{b(c+dx)}{bc-ad} \right)^n \text{Hypergeometric2F1} \left[ n, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad} \right] \end{aligned}$$

Result (type 6, 130 leaves):

$$\begin{aligned} & \left( 4acx^3 (a+bx)^n (c+dx)^{-n} \text{AppellF1} \left[ 3, -n, n, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \\ & \left( 12ac \text{AppellF1} \left[ 3, -n, n, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] + \right. \\ & \left. 3bcnx \text{AppellF1} \left[ 4, 1-n, n, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] - 3adnx \text{AppellF1} \left[ 4, -n, 1+n, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) \end{aligned}$$

### Problem 960: Result unnecessarily involves higher level functions.

$$\int x (a+bx)^n (c+dx)^{-n} dx$$

Optimal (type 5, 124 leaves, 3 steps):

$$\frac{(a+bx)^{1+n} (c+dx)^{1-n}}{2bd} - \frac{1}{2b^2d(1+n)} (ad(1-n) + bc(1+n)) (a+bx)^{1+n} (c+dx)^{-n} \left( \frac{b(c+dx)}{bc-ad} \right)^n \text{Hypergeometric2F1}\left[n, 1+n, 2+n, -\frac{d(a+bx)}{bc-ad}\right]$$

Result (type 6, 130 leaves):

$$\left( 3acx^2 (a+bx)^n (c+dx)^{-n} \text{AppellF1}\left[2, -n, n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \left( 6ac \text{AppellF1}\left[2, -n, n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + 2nx \left( bc \text{AppellF1}\left[3, 1-n, n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] - ad \text{AppellF1}\left[3, -n, 1+n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right)$$

### Problem 962: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^n (c+dx)^{-n}}{x} dx$$

Optimal (type 5, 108 leaves, 5 steps):

$$-\frac{(a+bx)^n (c+dx)^{-n} \text{Hypergeometric2F1}\left[1, n, 1+n, \frac{c(a+bx)}{a(c+dx)}\right]}{n} + \frac{1}{n} (a+bx)^n (c+dx)^{-n} \left( \frac{b(c+dx)}{bc-ad} \right)^n \text{Hypergeometric2F1}\left[n, n, 1+n, -\frac{d(a+bx)}{bc-ad}\right]$$

Result (type 6, 216 leaves):

$$\left( a(-bc+ad)(2+n)(a+bx)^{1+n} (c+dx)^{-n} \text{AppellF1}\left[1+n, n, 1, 2+n, \frac{d(a+bx)}{-bc+ad}, 1+\frac{bx}{a}\right] \right) / \left( b(1+n)x \left( a(-bc+ad)(2+n) \text{AppellF1}\left[1+n, n, 1, 2+n, \frac{d(a+bx)}{-bc+ad}, 1+\frac{bx}{a}\right] + (a+bx) \left( (-bc+ad) \text{AppellF1}\left[2+n, n, 2, 3+n, \frac{d(a+bx)}{-bc+ad}, 1+\frac{bx}{a}\right] + adn \text{AppellF1}\left[2+n, 1+n, 1, 3+n, \frac{d(a+bx)}{-bc+ad}, 1+\frac{bx}{a}\right] \right) \right) \right)$$

**Problem 963: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^n (c+dx)^{-n}}{x^2} dx$$

Optimal (type 5, 62 leaves, 1 step):

$$\frac{1}{a^2 (1+n)} (bc-ad) (a+bx)^{1+n} (c+dx)^{-1-n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{c(a+bx)}{a(c+dx)}\right]$$

Result (type 6, 141 leaves):

$$\begin{aligned} & - \left( \left( 2bd(a+bx)^n (c+dx)^{-n} \text{AppellF1}\left[1, -n, n, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / \right. \\ & \left( 2bdx \text{AppellF1}\left[1, -n, n, 2, -\frac{a}{bx}, -\frac{c}{dx}\right] + \right. \\ & \left. \left. adn \text{AppellF1}\left[2, 1-n, n, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] - bcn \text{AppellF1}\left[2, -n, 1+n, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right) \end{aligned}$$

**Problem 964: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^n (c+dx)^{-n}}{x^3} dx$$

Optimal (type 5, 117 leaves, 2 steps):

$$\begin{aligned} & - \frac{(a+bx)^{1+n} (c+dx)^{1-n}}{2acx^2} - \frac{1}{2a^3c(1+n)} (bc-ad) (ad(1+n) + b(c-cn)) \\ & (a+bx)^{1+n} (c+dx)^{-1-n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{c(a+bx)}{a(c+dx)}\right] \end{aligned}$$

Result (type 6, 146 leaves):

$$\begin{aligned} & - \left( \left( 3bd(a+bx)^n (c+dx)^{-n} \text{AppellF1}\left[2, -n, n, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / \right. \\ & \left( 6bdx^2 \text{AppellF1}\left[2, -n, n, 3, -\frac{a}{bx}, -\frac{c}{dx}\right] + 2adnx \text{AppellF1}\left[3, 1-n, n, 4, -\frac{a}{bx}, -\frac{c}{dx}\right] - \right. \\ & \left. \left. 2bcnx \text{AppellF1}\left[3, -n, 1+n, 4, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right) \end{aligned}$$

**Problem 965: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^n (c+dx)^{-n}}{x^4} dx$$

Optimal (type 5, 194 leaves, 4 steps):

$$\begin{aligned}
& - \frac{(a+bx)^{1+n} (c+dx)^{1-n}}{3acx^3} + \frac{(bc(2-n) + ad(2+n)) (a+bx)^{1+n} (c+dx)^{1-n}}{6a^2c^2x^2} + \\
& \frac{1}{6a^4c^2(1+n)} (bc-ad) (2abcd(1-n^2) + b^2c^2(2-3n+n^2) + a^2d^2(2+3n+n^2)) \\
& (a+bx)^{1+n} (c+dx)^{-1-n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{c(a+bx)}{a(c+dx)}\right]
\end{aligned}$$

Result (type 6, 146 leaves):

$$\begin{aligned}
& - \left( \left( 4bd(a+bx)^n (c+dx)^{-n} \text{AppellF1}\left[3, -n, n, 4, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) / \right. \\
& \left. \left( 3x^2 \left( 4bdx \text{AppellF1}\left[3, -n, n, 4, -\frac{a}{bx}, -\frac{c}{dx}\right] + adn \text{AppellF1}\left[4, 1-n, n, 5, -\frac{a}{bx}, -\frac{c}{dx}\right] - \right. \right. \right. \\
& \left. \left. \left. bcn \text{AppellF1}\left[4, -n, 1+n, 5, -\frac{a}{bx}, -\frac{c}{dx}\right] \right) \right) \right)
\end{aligned}$$

**Problem 966: Result unnecessarily involves higher level functions.**

$$\int (1-x)^n x^3 (1+x)^{-n} dx$$

Optimal (type 5, 105 leaves, 3 steps):

$$\begin{aligned}
& - \frac{1}{4} (1-x)^{1+n} x^2 (1+x)^{1-n} - \frac{1}{12} (1-x)^{1+n} (1+x)^{1-n} (3+2n^2-2nx) + \\
& \frac{2^{-n} n (2+n^2) (1-x)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{1-x}{2}\right]}{3(1+n)}
\end{aligned}$$

Result (type 6, 79 leaves):

$$(5(1-x)^n x^4 (1+x)^{-n} \text{AppellF1}[4, -n, n, 5, x, -x]) / (4(5 \text{AppellF1}[4, -n, n, 5, x, -x] - nx(\text{AppellF1}[5, 1-n, n, 6, x, -x] + \text{AppellF1}[5, -n, 1+n, 6, x, -x])))$$

**Problem 967: Result unnecessarily involves higher level functions.**

$$\int (1-x)^n x^2 (1+x)^{-n} dx$$

Optimal (type 5, 94 leaves, 3 steps):

$$\begin{aligned}
& \frac{1}{3} n (1-x)^{1+n} (1+x)^{1-n} - \frac{1}{3} (1-x)^{1+n} x (1+x)^{1-n} - \\
& \frac{2^{-n} (1+2n^2) (1-x)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{1-x}{2}\right]}{3(1+n)}
\end{aligned}$$

Result (type 6, 79 leaves):

$$(4(1-x)^n x^3 (1+x)^{-n} \text{AppellF1}[3, -n, n, 4, x, -x]) / (3(4 \text{AppellF1}[3, -n, n, 4, x, -x] - nx(\text{AppellF1}[4, 1-n, n, 5, x, -x] + \text{AppellF1}[4, -n, 1+n, 5, x, -x])))$$

**Problem 968: Result unnecessarily involves higher level functions.**

$$\int (1-x)^n x (1+x)^{-n} dx$$

Optimal (type 5, 61 leaves, 2 steps):

$$-\frac{1}{2} (1-x)^{1+n} (1+x)^{1-n} + \frac{2^{-n} n (1-x)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{1-x}{2}\right]}{1+n}$$

Result (type 6, 79 leaves):

$$\left( 3 (1-x)^n x^2 (1+x)^{-n} \text{AppellF1}\left[2, -n, n, 3, x, -x\right] \right) / \left( 2 \left( 3 \text{AppellF1}\left[2, -n, n, 3, x, -x\right] - n x \left( \text{AppellF1}\left[3, 1-n, n, 4, x, -x\right] + \text{AppellF1}\left[3, -n, 1+n, 4, x, -x\right] \right) \right) \right)$$

**Problem 970: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(1-x)^n (1+x)^{-n}}{x} dx$$

Optimal (type 5, 68 leaves, 3 steps):

$$-\frac{(1-x)^n (1+x)^{-n} \text{Hypergeometric2F1}\left[1, n, 1+n, \frac{1-x}{1+x}\right]}{n} + \frac{2^{-n} (1-x)^n \text{Hypergeometric2F1}\left[n, n, 1+n, \frac{1-x}{2}\right]}{n}$$

Result (type 6, 140 leaves):

$$\left( 2 (2+n) (1-x)^{1+n} (1+x)^{-n} \text{AppellF1}\left[1+n, n, 1, 2+n, \frac{1-x}{2}, 1-x\right] \right) / \left( (1+n) x \left( -2 (2+n) \text{AppellF1}\left[1+n, n, 1, 2+n, \frac{1-x}{2}, 1-x\right] + (-1+x) \left( 2 \text{AppellF1}\left[2+n, n, 2, 3+n, \frac{1-x}{2}, 1-x\right] + n \text{AppellF1}\left[2+n, 1+n, 1, 3+n, \frac{1-x}{2}, 1-x\right] \right) \right) \right)$$

**Problem 971: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(1-x)^n (1+x)^{-n}}{x^2} dx$$

Optimal (type 5, 44 leaves, 1 step):

$$-\frac{2 (1-x)^{1+n} (1+x)^{-1-n} \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{1-x}{1+x}\right]}{1+n}$$

Result (type 6, 90 leaves):

$$-\left(\left(2(1-x)^n(1+x)^{-n}\text{AppellF1}\left[1,-n,n,2,\frac{1}{x},-\frac{1}{x}\right]\right)\right)/\left(2x\text{AppellF1}\left[1,-n,n,2,\frac{1}{x},-\frac{1}{x}\right]-n\left(\text{AppellF1}\left[2,1-n,n,3,\frac{1}{x},-\frac{1}{x}\right]+\text{AppellF1}\left[2,-n,1+n,3,\frac{1}{x},-\frac{1}{x}\right]\right)\right)$$

**Problem 972: Result unnecessarily involves higher level functions.**

$$\int \frac{(1-x)^n(1+x)^{-n}}{x^3} dx$$

Optimal (type 5, 71 leaves, 2 steps):

$$-\frac{(1-x)^{1+n}(1+x)^{1-n}}{2x^2} + \frac{2n(1-x)^{1+n}(1+x)^{-1-n}\text{Hypergeometric2F1}\left[2,1+n,2+n,\frac{1-x}{1+x}\right]}{1+n}$$

Result (type 6, 95 leaves):

$$-\left(\left(3(1-x)^n(1+x)^{-n}\text{AppellF1}\left[2,-n,n,3,\frac{1}{x},-\frac{1}{x}\right]\right)\right)/\left(2x\left(3x\text{AppellF1}\left[2,-n,n,3,\frac{1}{x},-\frac{1}{x}\right]-n\left(\text{AppellF1}\left[3,1-n,n,4,\frac{1}{x},-\frac{1}{x}\right]+\text{AppellF1}\left[3,-n,1+n,4,\frac{1}{x},-\frac{1}{x}\right]\right)\right)\right)$$

**Problem 973: Result unnecessarily involves higher level functions.**

$$\int \frac{(1-x)^n(1+x)^{-n}}{x^4} dx$$

Optimal (type 5, 105 leaves, 4 steps):

$$-\frac{(1-x)^{1+n}(1+x)^{1-n}}{3x^3} + \frac{n(1-x)^{1+n}(1+x)^{1-n}}{3x^2} - \frac{1}{3(1+n)} + \frac{2(1+2n^2)(1-x)^{1+n}(1+x)^{-1-n}\text{Hypergeometric2F1}\left[2,1+n,2+n,\frac{1-x}{1+x}\right]}{3(1+n)}$$

Result (type 6, 95 leaves):

$$-\left(\left(4(1-x)^n(1+x)^{-n}\text{AppellF1}\left[3,-n,n,4,\frac{1}{x},-\frac{1}{x}\right]\right)\right)/\left(3x^2\left(4x\text{AppellF1}\left[3,-n,n,4,\frac{1}{x},-\frac{1}{x}\right]-n\left(\text{AppellF1}\left[4,1-n,n,5,\frac{1}{x},-\frac{1}{x}\right]+\text{AppellF1}\left[4,-n,1+n,5,\frac{1}{x},-\frac{1}{x}\right]\right)\right)\right)$$

**Problem 981: Result unnecessarily involves higher level functions.**

$$\int x^m(3-2ax)^{-1+n}(6+4ax)^n dx$$

Optimal (type 5, 104 leaves, 5 steps):

$$\frac{2^n \times 3^{-1+2n} x^{1+m} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, 1-n, \frac{3+m}{2}, \frac{4a^2x^2}{9}\right]}{1+m} +$$

$$\frac{2^{1+n} \times 9^{-1+n} a x^{2+m} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, 1-n, \frac{4+m}{2}, \frac{4a^2x^2}{9}\right]}{2+m}$$

Result (type 6, 168 leaves):

$$-\left(\left(3(2+m)x^{1+m}(18-8a^2x^2)^n \text{AppellF1}\left[1+m, 1-n, -n, 2+m, \frac{2ax}{3}, -\frac{2ax}{3}\right]\right) / \right.$$

$$\left(\left(1+m\right)\left(-3+2ax\right)\left(3(2+m) \text{AppellF1}\left[1+m, 1-n, -n, 2+m, \frac{2ax}{3}, -\frac{2ax}{3}\right] + \right.\right.$$

$$\left.2ax\left(-(-1+n) \text{AppellF1}\left[2+m, 2-n, -n, 3+m, \frac{2ax}{3}, -\frac{2ax}{3}\right] + \right.\right.$$

$$\left.\left.\left.n \text{HypergeometricPFQ}\left[\left\{1+\frac{m}{2}, 1-n\right\}, \left\{2+\frac{m}{2}\right\}, \frac{4a^2x^2}{9}\right]\right)\right)\right)$$

**Problem 982: Result unnecessarily involves higher level functions.**

$$\int x^m (3-2ax)^{-2+n} (6+4ax)^n dx$$

Optimal (type 5, 158 leaves, 8 steps):

$$\frac{2^n \times 9^{-1+n} x^{1+m} \text{Hypergeometric2F1}\left[\frac{1+m}{2}, 2-n, \frac{3+m}{2}, \frac{4a^2x^2}{9}\right]}{1+m} +$$

$$\frac{2^{2+n} \times 3^{-3+2n} a x^{2+m} \text{Hypergeometric2F1}\left[\frac{2+m}{2}, 2-n, \frac{4+m}{2}, \frac{4a^2x^2}{9}\right]}{2+m} +$$

$$\frac{2^{2+n} \times 9^{-2+n} a^2 x^{3+m} \text{Hypergeometric2F1}\left[\frac{3+m}{2}, 2-n, \frac{5+m}{2}, \frac{4a^2x^2}{9}\right]}{3+m}$$

Result (type 6, 163 leaves):

$$\left(3(2+m)x^{1+m}(3-2ax)^{-2+n}(6+4ax)^n \text{AppellF1}\left[1+m, 2-n, -n, 2+m, \frac{2ax}{3}, -\frac{2ax}{3}\right]\right) /$$

$$\left(\left(1+m\right)\left(3(2+m) \text{AppellF1}\left[1+m, 2-n, -n, 2+m, \frac{2ax}{3}, -\frac{2ax}{3}\right] + \right.\right.$$

$$2ax\left(n \text{AppellF1}\left[2+m, 2-n, 1-n, 3+m, \frac{2ax}{3}, -\frac{2ax}{3}\right] - \right.$$

$$\left.\left.\left(-2+n\right) \text{AppellF1}\left[2+m, 3-n, -n, 3+m, \frac{2ax}{3}, -\frac{2ax}{3}\right]\right)\right)$$

**Problem 983: Result more than twice size of optimal antiderivative.**

$$\int x^m (a+bx)^{1+n} (c+dx)^n dx$$

Optimal (type 6, 79 leaves, 3 steps):

$$\frac{1}{1+m} a x^{1+m} (a+bx)^n \left(1+\frac{bx}{a}\right)^{-n} (c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} \text{AppellF1}\left[1+m, -1-n, -n, 2+m, -\frac{bx}{a}, -\frac{dx}{c}\right]$$

Result (type 6, 308 leaves):

$$\begin{aligned} & \frac{1}{2+m} a c x^{1+m} (a+bx)^n (c+dx)^n \left( \left( a (2+m)^2 \text{AppellF1}\left[1+m, -n, -n, 2+m, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \right. \\ & \quad \left( (1+m) \left( a c (2+m) \text{AppellF1}\left[1+m, -n, -n, 2+m, -\frac{bx}{a}, -\frac{dx}{c}\right] + n x \left( b c \text{AppellF1}\left[2+m, 1-n, \right. \right. \right. \right. \\ & \quad \quad \left. \left. \left. -n, 3+m, -\frac{bx}{a}, -\frac{dx}{c}\right] + a d \text{AppellF1}\left[2+m, -n, 1-n, 3+m, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right) \right) + \\ & \quad \left( b (3+m) x \text{AppellF1}\left[2+m, -n, -n, 3+m, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \\ & \quad \left( a c (3+m) \text{AppellF1}\left[2+m, -n, -n, 3+m, -\frac{bx}{a}, -\frac{dx}{c}\right] + \right. \\ & \quad \quad n x \left( b c \text{AppellF1}\left[3+m, 1-n, -n, 4+m, -\frac{bx}{a}, -\frac{dx}{c}\right] + \right. \\ & \quad \quad \quad \left. \left. a d \text{AppellF1}\left[3+m, -n, 1-n, 4+m, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right) \right) \end{aligned}$$

**Problem 986: Result unnecessarily involves higher level functions.**

$$\int \frac{\left(1-\frac{x}{a}\right)^{-n/2} \left(1+\frac{x}{a}\right)^{n/2}}{x^2} dx$$

Optimal (type 5, 70 leaves, 1 step):

$$\frac{4 \left(1-\frac{x}{a}\right)^{1-\frac{n}{2}} \left(1+\frac{x}{a}\right)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{a-x}{a+x}\right]}{a(2-n)}$$

Result (type 6, 139 leaves):

$$\begin{aligned} & - \left( \left( 4 \left( \frac{a+x}{a} \right)^{n/2} \left( 1 - \frac{x}{a} \right)^{-n/2} \text{AppellF1}\left[1, -\frac{n}{2}, \frac{n}{2}, 2, -\frac{a}{x}, \frac{a}{x}\right] \right) / \right. \\ & \quad \left( 4 x \text{AppellF1}\left[1, -\frac{n}{2}, \frac{n}{2}, 2, -\frac{a}{x}, \frac{a}{x}\right] + \right. \\ & \quad \quad \left. a n \left( \text{AppellF1}\left[2, 1-\frac{n}{2}, \frac{n}{2}, 3, -\frac{a}{x}, \frac{a}{x}\right] + \text{AppellF1}\left[2, -\frac{n}{2}, \frac{2+n}{2}, 3, -\frac{a}{x}, \frac{a}{x}\right] \right) \right) \right) \end{aligned}$$

**Problem 988: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(1-ax)^{-n} (1+ax)^n}{x} dx$$

Optimal (type 5, 86 leaves, 3 steps):

$$\frac{(1-ax)^{-n} (1+ax)^n \text{Hypergeometric2F1}\left[1, -n, 1-n, \frac{1-ax}{1+ax}\right]}{n} - \frac{2^n (1-ax)^{-n} \text{Hypergeometric2F1}\left[-n, -n, 1-n, \frac{1}{2}(1-ax)\right]}{n}$$

Result (type 6, 182 leaves):

$$\left( 2(-2+n)(1-ax)^{1-n}(1+ax)^n \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2}(1-ax), 1-ax\right] \right) / \left( a(1-n)x \left( -2(-2+n) \text{AppellF1}\left[1-n, -n, 1, 2-n, \frac{1}{2}(1-ax), 1-ax\right] + (-1+ax) \left( n \text{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{1}{2}(1-ax), 1-ax\right] - 2 \text{AppellF1}\left[2-n, -n, 2, 3-n, \frac{1}{2}(1-ax), 1-ax\right] \right) \right) \right)$$

**Problem 989: Result unnecessarily involves higher level functions.**

$$\int \frac{(1-ax)^{1-n} (1+ax)^{1+n}}{x^2} dx$$

Optimal (type 5, 106 leaves, 3 steps):

$$- \frac{2a(1-ax)^{1-n} (1+ax)^{-1+n} \text{Hypergeometric2F1}\left[2, 1-n, 2-n, \frac{1-ax}{1+ax}\right]}{1-n} + \frac{2^n a (1-ax)^{1-n} \text{Hypergeometric2F1}\left[1-n, -n, 2-n, \frac{1}{2}(1-ax)\right]}{1-n}$$

Result (type 6, 158 leaves):

$$a(1+ax)^n \left( - \left( \left( 2(1-ax)^{-n} \text{AppellF1}\left[1, n, -n, 2, \frac{1}{ax}, -\frac{1}{ax}\right] \right) / \left( 2ax \text{AppellF1}\left[1, n, -n, 2, \frac{1}{ax}, -\frac{1}{ax}\right] + n \left( \text{AppellF1}\left[2, n, 1-n, 3, \frac{1}{ax}, -\frac{1}{ax}\right] + \text{AppellF1}\left[2, 1+n, -n, 3, \frac{1}{ax}, -\frac{1}{ax}\right] \right) \right) \right) - \frac{2^{-n} (1+ax) \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{1}{2}(1+ax)\right]}{1+n} \right)$$

**Problem 994: Result unnecessarily involves higher level functions.**

$$\int \frac{(a-bx)^{-n} (a+bx)^{1+n}}{x} dx$$

Optimal (type 5, 142 leaves, 6 steps):

$$\frac{(a-bx)^{1-n} (a+bx)^n}{2n} - \frac{a (a-bx)^{-n} (a+bx)^n \operatorname{Hypergeometric2F1}\left[1, n, 1+n, \frac{a+bx}{a-bx}\right]}{n} + \frac{1}{n(1+n)}$$

$$2^{-1-n} (1+2n) (a-bx)^{-n} \left(\frac{a-bx}{a}\right)^n (a+bx)^{1+n} \operatorname{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{a+bx}{2a}\right]$$

Result (type 6, 262 leaves):

$$(a-bx)^{-n} (a+bx)^n$$

$$\left( \left( 2a^2 (-2+n) (a-bx) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{a-bx}{2a}, 1-\frac{bx}{a}\right] \right) / \left( b(-1+n)x \right. \right.$$

$$\left. \left. \left( 2a(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{a-bx}{2a}, 1-\frac{bx}{a}\right] + (a-bx) \left( n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{a-bx}{2a}, 1-\frac{bx}{a}\right] - 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{a-bx}{2a}, 1-\frac{bx}{a}\right] \right) \right) \right) \right) +$$

$$\left. \frac{(a+bx) \left( 1 - \frac{a+bx}{2a} \right)^n \operatorname{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{a+bx}{2a}\right]}{1+n} \right)$$

Problem 995: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a-bx)^{-n} (a+bx)^{1+n}}{x^2} dx$$

Optimal (type 5, 140 leaves, 5 steps):

$$-\frac{(a-bx)^{-n} (a+bx)^{1+n}}{x} + \frac{1}{n}$$

$$b(1+2n) (a-bx)^{-n} (a+bx)^n \operatorname{Hypergeometric2F1}\left[1, -n, 1-n, \frac{a-bx}{a+bx}\right] - \frac{1}{n}$$

$$2^n b (a-bx)^{-n} (a+bx)^n \left(\frac{a+bx}{a}\right)^{-n} \operatorname{Hypergeometric2F1}\left[-n, -n, 1-n, \frac{a-bx}{2a}\right]$$

Result (type 6, 324 leaves):

$$\begin{aligned}
 & 2 a (a - b x)^{-n} (a + b x)^n \\
 & \left( - \left( \left( b \operatorname{AppellF1} \left[ 1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x} \right] \right) / \left( 2 b x \operatorname{AppellF1} \left[ 1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x} \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. a n \left( \operatorname{AppellF1} \left[ 2, n, 1 - n, 3, \frac{a}{b x}, -\frac{a}{b x} \right] + \operatorname{AppellF1} \left[ 2, 1 + n, -n, 3, \frac{a}{b x}, -\frac{a}{b x} \right] \right) \right) \right) + \right. \\
 & \left( (-2 + n) (a - b x) \operatorname{AppellF1} \left[ 1 - n, -n, 1, 2 - n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a} \right] \right) / \\
 & \left( (-1 + n) x \left( 2 a (-2 + n) \operatorname{AppellF1} \left[ 1 - n, -n, 1, 2 - n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a} \right] + \right. \right. \\
 & \quad \left. \left. (a - b x) \left( n \operatorname{AppellF1} \left[ 2 - n, 1 - n, 1, 3 - n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a} \right] - \right. \right. \right. \\
 & \quad \left. \left. \left. 2 \operatorname{AppellF1} \left[ 2 - n, -n, 2, 3 - n, \frac{a - b x}{2 a}, 1 - \frac{b x}{a} \right] \right) \right) \right) \right)
 \end{aligned}$$

**Problem 996: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a - b x)^{-n} (a + b x)^{1+n}}{x^3} dx$$

Optimal (type 5, 62 leaves, 1 step):

$$- \frac{4 b^2 (a - b x)^{1-n} (a + b x)^{-1+n} \operatorname{Hypergeometric2F1} \left[ 3, 1 - n, 2 - n, \frac{a - b x}{a + b x} \right]}{a (1 - n)}$$

Result (type 6, 254 leaves):

$$\begin{aligned}
 & \frac{1}{2} b (a - b x)^{-n} (a + b x)^n \\
 & \left( - \left( \left( 4 b \operatorname{AppellF1} \left[ 1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x} \right] \right) / \left( 2 b x \operatorname{AppellF1} \left[ 1, n, -n, 2, \frac{a}{b x}, -\frac{a}{b x} \right] + \right. \right. \right. \\
 & \quad \left. \left. \left. a n \left( \operatorname{AppellF1} \left[ 2, n, 1 - n, 3, \frac{a}{b x}, -\frac{a}{b x} \right] + \operatorname{AppellF1} \left[ 2, 1 + n, -n, 3, \frac{a}{b x}, -\frac{a}{b x} \right] \right) \right) \right) - \right. \\
 & \left( 3 a \operatorname{AppellF1} \left[ 2, n, -n, 3, \frac{a}{b x}, -\frac{a}{b x} \right] \right) / \left( x \left( 3 b x \operatorname{AppellF1} \left[ 2, n, -n, 3, \frac{a}{b x}, -\frac{a}{b x} \right] + \right. \right. \\
 & \quad \left. \left. a n \left( \operatorname{AppellF1} \left[ 3, n, 1 - n, 4, \frac{a}{b x}, -\frac{a}{b x} \right] + \operatorname{AppellF1} \left[ 3, 1 + n, -n, 4, \frac{a}{b x}, -\frac{a}{b x} \right] \right) \right) \right)
 \end{aligned}$$

**Problem 997: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a - b x)^{-n} (a + b x)^{1+n}}{x^4} dx$$

Optimal (type 5, 101 leaves, 2 steps):

$$-\frac{(a-bx)^{1-n}(a+bx)^{2+n}}{3a^2x^3} - \frac{1}{3a^2(1-n)}$$

$$4b^3(1+2n)(a-bx)^{1-n}(a+bx)^{-1+n} \text{Hypergeometric2F1}\left[3, 1-n, 2-n, \frac{a-bx}{a+bx}\right]$$

Result (type 6, 255 leaves):

$$\frac{1}{6x^2}b(a-bx)^{-n}(a+bx)^n$$

$$\left(-\left(\left(9bx \text{AppellF1}\left[2, n, -n, 3, \frac{a}{bx}, -\frac{a}{bx}\right]\right) / \left(3bx \text{AppellF1}\left[2, n, -n, 3, \frac{a}{bx}, -\frac{a}{bx}\right] + \right.\right.\right.$$

$$\left.\left.\left. a n \left(\text{AppellF1}\left[3, n, 1-n, 4, \frac{a}{bx}, -\frac{a}{bx}\right] + \text{AppellF1}\left[3, 1+n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}\right]\right)\right)\right) - \right.$$

$$\left.\left(\left(8a \text{AppellF1}\left[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}\right]\right) / \left(4bx \text{AppellF1}\left[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}\right] + \right.\right.\right.$$

$$\left.\left.\left. a n \left(\text{AppellF1}\left[4, n, 1-n, 5, \frac{a}{bx}, -\frac{a}{bx}\right] + \text{AppellF1}\left[4, 1+n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}\right]\right)\right)\right)\right)$$

Problem 998: Result unnecessarily involves higher level functions.

$$\int \frac{(a-bx)^{-n}(a+bx)^{1+n}}{x^5} dx$$

Optimal (type 5, 139 leaves, 4 steps):

$$-\frac{(a-bx)^{1-n}(a+bx)^{2+n}}{4a^2x^4} - \frac{b(1+2n)(a-bx)^{1-n}(a+bx)^{2+n}}{12a^3x^3} - \frac{1}{3a^3(1-n)}$$

$$4b^4(1+n+n^2)(a-bx)^{1-n}(a+bx)^{-1+n} \text{Hypergeometric2F1}\left[3, 1-n, 2-n, \frac{a-bx}{a+bx}\right]$$

Result (type 6, 255 leaves):

$$\frac{1}{12x^3}b(a-bx)^{-n}(a+bx)^n$$

$$\left(-\left(\left(16bx \text{AppellF1}\left[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}\right]\right) / \left(4bx \text{AppellF1}\left[3, n, -n, 4, \frac{a}{bx}, -\frac{a}{bx}\right] + \right.\right.\right.$$

$$\left.\left.\left. a n \left(\text{AppellF1}\left[4, n, 1-n, 5, \frac{a}{bx}, -\frac{a}{bx}\right] + \text{AppellF1}\left[4, 1+n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}\right]\right)\right)\right) - \right.$$

$$\left.\left(\left(15a \text{AppellF1}\left[4, n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}\right]\right) / \left(5bx \text{AppellF1}\left[4, n, -n, 5, \frac{a}{bx}, -\frac{a}{bx}\right] + \right.\right.\right.$$

$$\left.\left.\left. a n \left(\text{AppellF1}\left[5, n, 1-n, 6, \frac{a}{bx}, -\frac{a}{bx}\right] + \text{AppellF1}\left[5, 1+n, -n, 6, \frac{a}{bx}, -\frac{a}{bx}\right]\right)\right)\right)\right)$$

Problem 999: Result more than twice size of optimal antiderivative.

$$\int (a+bx)(A+Bx)(d+ex)^4 dx$$

Optimal (type 1, 77 leaves, 2 steps):

$$\frac{(bd - ae)(Bd - Ae)(d + ex)^5}{5e^3} - \frac{(2bBd - Abe - aBe)(d + ex)^6}{6e^3} + \frac{bB(d + ex)^7}{7e^3}$$

Result (type 1, 172 leaves):

$$\begin{aligned} & aAd^4x + \frac{1}{2}d^3(Abd + aBd + 4aAe)x^2 + \\ & \frac{1}{3}d^2(2ae(2Bd + 3Ae) + bd(Bd + 4Ae))x^3 + \frac{1}{2}de(ae(3Bd + 2Ae) + bd(2Bd + 3Ae))x^4 + \\ & \frac{1}{5}e^2(ae(4Bd + Ae) + 2bd(3Bd + 2Ae))x^5 + \frac{1}{6}e^3(4bBd + Abe + aBe)x^6 + \frac{1}{7}bBe^4x^7 \end{aligned}$$

**Problem 1010: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^2 (A + Bx) (d + ex)^4 dx$$

Optimal (type 1, 120 leaves, 2 steps):

$$\begin{aligned} & -\frac{(bd - ae)^2(Bd - Ae)(d + ex)^5}{5e^4} + \frac{(bd - ae)(3bBd - 2Abe - aBe)(d + ex)^6}{6e^4} - \\ & \frac{b(3bBd - Abe - 2aBe)(d + ex)^7}{7e^4} + \frac{b^2B(d + ex)^8}{8e^4} \end{aligned}$$

Result (type 1, 283 leaves):

$$\begin{aligned} & a^2Ad^4x + \frac{1}{2}ad^3(2Abd + aBd + 4aAe)x^2 + \\ & \frac{1}{3}d^2(2aBd(bd + 2ae) + A(b^2d^2 + 8abde + 6a^2e^2))x^3 + \\ & \frac{1}{4}d(2a^2e^2(3Bd + 2Ae) + 4abde(2Bd + 3Ae) + b^2d^2(Bd + 4Ae))x^4 + \\ & \frac{1}{5}e(a^2e^2(4Bd + Ae) + 4abde(3Bd + 2Ae) + 2b^2d^2(2Bd + 3Ae))x^5 + \\ & \frac{1}{6}e^2(a^2Be^2 + 2abe(4Bd + Ae) + 2b^2d(3Bd + 2Ae))x^6 + \\ & \frac{1}{7}be^3(4bBd + Abe + 2aBe)x^7 + \frac{1}{8}b^2Be^4x^8 \end{aligned}$$

**Problem 1023: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^3 (A + Bx) (d + ex)^5 dx$$

Optimal (type 1, 163 leaves, 2 steps):

$$\frac{(bd - ae)^3 (Bd - Ae) (d + ex)^6}{6e^5} - \frac{(bd - ae)^2 (4bBd - 3Abe - aBe) (d + ex)^7}{7e^5} +$$

$$\frac{3b (bd - ae) (2bBd - Abe - aBe) (d + ex)^8}{8e^5} -$$

$$\frac{b^2 (4bBd - Abe - 3aBe) (d + ex)^9}{9e^5} + \frac{b^3 B (d + ex)^{10}}{10e^5}$$

Result (type 1, 471 leaves):

$$a^3 A d^5 x + \frac{1}{2} a^2 d^4 (3 A b d + a B d + 5 a A e) x^2 +$$

$$\frac{1}{3} a d^3 (a B d (3 b d + 5 a e) + A (3 b^2 d^2 + 15 a b d e + 10 a^2 e^2)) x^3 +$$

$$\frac{1}{4} d^2 (a B d (3 b^2 d^2 + 15 a b d e + 10 a^2 e^2) + A (b^3 d^3 + 15 a b^2 d^2 e + 30 a^2 b d e^2 + 10 a^3 e^3)) x^4 +$$

$$\frac{1}{5} d (30 a^2 b d e^2 (B d + A e) + 5 a^3 e^3 (2 B d + A e) + 15 a b^2 d^2 e (B d + 2 A e) + b^3 d^3 (B d + 5 A e)) x^5 +$$

$$\frac{1}{6} e (30 a b^2 d^2 e (B d + A e) + 15 a^2 b d e^2 (2 B d + A e) + a^3 e^3 (5 B d + A e) + 5 b^3 d^3 (B d + 2 A e)) x^6 +$$

$$\frac{1}{7} e^2 (a^3 B e^3 + 10 b^3 d^2 (B d + A e) + 15 a b^2 d e (2 B d + A e) + 3 a^2 b e^2 (5 B d + A e)) x^7 +$$

$$\frac{1}{8} b e^3 (3 a^2 B e^2 + 5 b^2 d (2 B d + A e) + 3 a b e (5 B d + A e)) x^8 +$$

$$\frac{1}{9} b^2 e^4 (5 b B d + A b e + 3 a B e) x^9 + \frac{1}{10} b^3 B e^5 x^{10}$$

**Problem 1024: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^3 (A + B x) (d + e x)^4 dx$$

Optimal (type 1, 163 leaves, 2 steps):

$$\frac{(bd - ae)^3 (Bd - Ae) (d + ex)^5}{5e^5} - \frac{(bd - ae)^2 (4bBd - 3Abe - aBe) (d + ex)^6}{6e^5} +$$

$$\frac{3b (bd - ae) (2bBd - Abe - aBe) (d + ex)^7}{7e^5} - \frac{b^2 (4bBd - Abe - 3aBe) (d + ex)^8}{8e^5} + \frac{b^3 B (d + ex)^9}{9e^5}$$

Result (type 1, 397 leaves):

$$\begin{aligned}
 & a^3 A d^4 x + \frac{1}{2} a^2 d^3 (3 A b d + a B d + 4 a A e) x^2 + \\
 & \frac{1}{3} a d^2 (a B d (3 b d + 4 a e) + 3 A (b^2 d^2 + 4 a b d e + 2 a^2 e^2)) x^3 + \\
 & \frac{1}{4} d (3 a B d (b^2 d^2 + 4 a b d e + 2 a^2 e^2) + A (b^3 d^3 + 12 a b^2 d^2 e + 18 a^2 b d e^2 + 4 a^3 e^3)) x^4 + \\
 & \frac{1}{5} (a^3 e^3 (4 B d + A e) + 6 a^2 b d e^2 (3 B d + 2 A e) + 6 a b^2 d^2 e (2 B d + 3 A e) + b^3 d^3 (B d + 4 A e)) x^5 + \\
 & \frac{1}{6} e (a^3 B e^3 + 3 a^2 b e^2 (4 B d + A e) + 6 a b^2 d e (3 B d + 2 A e) + 2 b^3 d^2 (2 B d + 3 A e)) x^6 + \\
 & \frac{1}{7} b e^2 (3 a^2 B e^2 + 3 a b e (4 B d + A e) + 2 b^2 d (3 B d + 2 A e)) x^7 + \\
 & \frac{1}{8} b^2 e^3 (4 b B d + A b e + 3 a B e) x^8 + \frac{1}{9} b^3 B e^4 x^9
 \end{aligned}$$

**Problem 1034: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^3 (A+Bx)}{(d+ex)^6} dx$$

Optimal (type 1, 86 leaves, 2 steps):

$$-\frac{(Bd - Ae)(a+bx)^4}{5e(bd - ae)(d+ex)^5} + \frac{(4bBd + ABe - 5aBe)(a+bx)^4}{20e(bd - ae)^2(d+ex)^4}$$

Result (type 1, 211 leaves):

$$\begin{aligned}
 & -\frac{1}{20e^5(d+ex)^5} (a^3 e^3 (4Ae + B(d+5ex)) + a^2 b e^2 (3Ae(d+5ex) + 2B(d^2 + 5dex + 10e^2 x^2)) + \\
 & a b^2 e (2Ae(d^2 + 5dex + 10e^2 x^2) + 3B(d^3 + 5d^2 ex + 10de^2 x^2 + 10e^3 x^3)) + \\
 & b^3 (Ae(d^3 + 5d^2 ex + 10de^2 x^2 + 10e^3 x^3) + 4B(d^4 + 5d^3 ex + 10d^2 e^2 x^2 + 10de^3 x^3 + 5e^4 x^4)))
 \end{aligned}$$

**Problem 1039: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^6 (A+Bx) (d+ex)^8 dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(bd - ae)^6 (Bd - Ae) (d + ex)^9}{9e^8} + \frac{(bd - ae)^5 (7bBd - 6Abe - aBe) (d + ex)^{10}}{10e^8} - \\
 & \frac{3b (bd - ae)^4 (7bBd - 5Abe - 2aBe) (d + ex)^{11}}{11e^8} + \\
 & \frac{5b^2 (bd - ae)^3 (7bBd - 4Abe - 3aBe) (d + ex)^{12}}{12e^8} - \\
 & \frac{5b^3 (bd - ae)^2 (7bBd - 3Abe - 4aBe) (d + ex)^{13}}{13e^8} + \\
 & \frac{3b^4 (bd - ae) (7bBd - 2Abe - 5aBe) (d + ex)^{14}}{14e^8} - \\
 & \frac{b^5 (7bBd - Abe - 6aBe) (d + ex)^{15}}{15e^8} + \frac{b^6 B (d + ex)^{16}}{16e^8}
 \end{aligned}$$

Result(type 1, 1385 leaves):

$$\begin{aligned}
 & a^6 A d^8 x + \frac{1}{2} a^5 d^7 (6 A b d + a B d + 8 a A e) x^2 + \\
 & \frac{1}{3} a^4 d^6 (2 a B d (3 b d + 4 a e) + A (15 b^2 d^2 + 48 a b d e + 28 a^2 e^2)) x^3 + \\
 & \frac{1}{4} a^3 d^5 (a B d (15 b^2 d^2 + 48 a b d e + 28 a^2 e^2) + 4 A (5 b^3 d^3 + 30 a b^2 d^2 e + 42 a^2 b d e^2 + 14 a^3 e^3)) x^4 + \\
 & \frac{1}{5} a^2 d^4 (4 a B d (5 b^3 d^3 + 30 a b^2 d^2 e + 42 a^2 b d e^2 + 14 a^3 e^3) + \\
 & \quad A (15 b^4 d^4 + 160 a b^3 d^3 e + 420 a^2 b^2 d^2 e^2 + 336 a^3 b d e^3 + 70 a^4 e^4)) x^5 + \\
 & \frac{1}{6} a d^3 (a B d (15 b^4 d^4 + 160 a b^3 d^3 e + 420 a^2 b^2 d^2 e^2 + 336 a^3 b d e^3 + 70 a^4 e^4) + \\
 & \quad 2 A (3 b^5 d^5 + 60 a b^4 d^4 e + 280 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 210 a^4 b d e^4 + 28 a^5 e^5)) x^6 + \\
 & \frac{1}{7} d^2 (2 a B d (3 b^5 d^5 + 60 a b^4 d^4 e + 280 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 210 a^4 b d e^4 + 28 a^5 e^5) + \\
 & \quad A (b^6 d^6 + 48 a b^5 d^5 e + 420 a^2 b^4 d^4 e^2 + 1120 a^3 b^3 d^3 e^3 + 1050 a^4 b^2 d^2 e^4 + 336 a^5 b d e^5 + 28 a^6 e^6)) \\
 & x^7 + \frac{1}{8} d (168 a^5 b d e^5 (2 B d + A e) + 420 a^2 b^4 d^4 e^2 (B d + 2 A e) + 4 a^6 e^6 (7 B d + 2 A e) + 210 a^4 b^2 d^2 e^4 \\
 & \quad (5 B d + 4 A e) + 280 a^3 b^3 d^3 e^3 (4 B d + 5 A e) + 24 a b^5 d^5 e (2 B d + 7 A e) + b^6 d^6 (B d + 8 A e)) x^8 + \\
 & \frac{1}{9} e (420 a^4 b^2 d^2 e^4 (2 B d + A e) + a^6 e^6 (8 B d + A e) + 168 a b^5 d^5 e (B d + 2 A e) + \\
 & \quad 24 a^5 b d e^5 (7 B d + 2 A e) + 280 a^3 b^3 d^3 e^3 (5 B d + 4 A e) + \\
 & \quad 210 a^2 b^4 d^4 e^2 (4 B d + 5 A e) + 4 b^6 d^6 (2 B d + 7 A e)) x^9 + \\
 & \frac{1}{10} e^2 (a^6 B e^6 + 560 a^3 b^3 d^2 e^3 (2 B d + A e) + 6 a^5 b e^5 (8 B d + A e) + 28 b^6 d^5 (B d + 2 A e) + \\
 & \quad 60 a^4 b^2 d e^4 (7 B d + 2 A e) + 210 a^2 b^4 d^3 e^2 (5 B d + 4 A e) + 84 a b^5 d^4 e (4 B d + 5 A e)) x^{10} + \\
 & \frac{1}{11} b e^3 (6 a^5 B e^5 + 420 a^2 b^3 d^2 e^2 (2 B d + A e) + 15 a^4 b e^4 (8 B d + A e) + \\
 & \quad 80 a^3 b^2 d e^3 (7 B d + 2 A e) + 84 a b^4 d^3 e (5 B d + 4 A e) + 14 b^5 d^4 (4 B d + 5 A e)) x^{11} + \\
 & \frac{1}{12} b^2 e^4 (15 a^4 B e^4 + 168 a b^3 d^2 e (2 B d + A e) + 20 a^3 b e^3 (8 B d + A e) + \\
 & \quad 60 a^2 b^2 d e^2 (7 B d + 2 A e) + 14 b^4 d^3 (5 B d + 4 A e)) x^{12} + \\
 & \frac{1}{13} b^3 e^5 (20 a^3 B e^3 + 28 b^3 d^2 (2 B d + A e) + 15 a^2 b e^2 (8 B d + A e) + 24 a b^2 d e (7 B d + 2 A e)) x^{13} + \\
 & \frac{1}{14} b^4 e^6 (15 a^2 B e^2 + 6 a b e (8 B d + A e) + 4 b^2 d (7 B d + 2 A e)) x^{14} + \\
 & \frac{1}{15} b^5 e^7 (8 b B d + A b e + 6 a B e) x^{15} + \frac{1}{16} b^6 B e^8 x^{16}
 \end{aligned}$$

### Problem 1040: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x)^7 dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(bd - ae)^6 (Bd - Ae) (d + ex)^8}{8 e^8} + \frac{(bd - ae)^5 (7bBd - 6Abe - aBe) (d + ex)^9}{9 e^8} - \\
 & \frac{3b (bd - ae)^4 (7bBd - 5Abe - 2aBe) (d + ex)^{10}}{10 e^8} + \\
 & \frac{5b^2 (bd - ae)^3 (7bBd - 4Abe - 3aBe) (d + ex)^{11}}{11 e^8} - \\
 & \frac{5b^3 (bd - ae)^2 (7bBd - 3Abe - 4aBe) (d + ex)^{12}}{12 e^8} + \\
 & \frac{3b^4 (bd - ae) (7bBd - 2Abe - 5aBe) (d + ex)^{13}}{13 e^8} - \\
 & \frac{b^5 (7bBd - Abe - 6aBe) (d + ex)^{14}}{14 e^8} + \frac{b^6 B (d + ex)^{15}}{15 e^8}
 \end{aligned}$$

Result(type 1, 1224 leaves):

$$\begin{aligned}
 & a^6 A d^7 x + \frac{1}{2} a^5 d^6 (6 A b d + a B d + 7 a A e) x^2 + \\
 & \frac{1}{3} a^4 d^5 (a B d (6 b d + 7 a e) + 3 A (5 b^2 d^2 + 14 a b d e + 7 a^2 e^2)) x^3 + \\
 & \frac{1}{4} a^3 d^4 (3 a B d (5 b^2 d^2 + 14 a b d e + 7 a^2 e^2) + A (20 b^3 d^3 + 105 a b^2 d^2 e + 126 a^2 b d e^2 + 35 a^3 e^3)) x^4 + \\
 & \frac{1}{5} a^2 d^3 (a B d (20 b^3 d^3 + 105 a b^2 d^2 e + 126 a^2 b d e^2 + 35 a^3 e^3) + \\
 & \quad 5 A (3 b^4 d^4 + 28 a b^3 d^3 e + 63 a^2 b^2 d^2 e^2 + 42 a^3 b d e^3 + 7 a^4 e^4)) x^5 + \\
 & \frac{1}{6} a d^2 (5 a B d (3 b^4 d^4 + 28 a b^3 d^3 e + 63 a^2 b^2 d^2 e^2 + 42 a^3 b d e^3 + 7 a^4 e^4) + \\
 & \quad 3 A (2 b^5 d^5 + 35 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 175 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5)) x^6 + \\
 & \frac{1}{7} d (3 a B d (2 b^5 d^5 + 35 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 175 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5) + \\
 & \quad A (b^6 d^6 + 42 a b^5 d^5 e + 315 a^2 b^4 d^4 e^2 + 700 a^3 b^3 d^3 e^3 + 525 a^4 b^2 d^2 e^4 + 126 a^5 b d e^5 + 7 a^6 e^6)) x^7 + \\
 & \frac{1}{8} (700 a^3 b^3 d^3 e^3 (B d + A e) + 42 a^5 b d e^5 (3 B d + A e) + a^6 e^6 (7 B d + A e) + 42 a b^5 d^5 e (B d + 3 A e) + \\
 & \quad 105 a^4 b^2 d^2 e^4 (5 B d + 3 A e) + 105 a^2 b^4 d^4 e^2 (3 B d + 5 A e) + b^6 d^6 (B d + 7 A e)) x^8 + \\
 & \frac{1}{9} e (a^6 B e^6 + 525 a^2 b^4 d^3 e^2 (B d + A e) + 105 a^4 b^2 d e^4 (3 B d + A e) + 6 a^5 b e^5 (7 B d + A e) + \\
 & \quad 7 b^6 d^5 (B d + 3 A e) + 140 a^3 b^3 d^2 e^3 (5 B d + 3 A e) + 42 a b^5 d^4 e (3 B d + 5 A e)) x^9 + \\
 & \frac{1}{10} b e^2 (6 a^5 B e^5 + 210 a b^4 d^3 e (B d + A e) + 140 a^3 b^2 d e^3 (3 B d + A e) + \\
 & \quad 15 a^4 b e^4 (7 B d + A e) + 105 a^2 b^3 d^2 e^2 (5 B d + 3 A e) + 7 b^5 d^4 (3 B d + 5 A e)) x^{10} + \\
 & \frac{1}{11} b^2 e^3 (15 a^4 B e^4 + 35 b^4 d^3 (B d + A e) + 105 a^2 b^2 d e^2 (3 B d + A e) + \\
 & \quad 20 a^3 b e^3 (7 B d + A e) + 42 a b^3 d^2 e (5 B d + 3 A e)) x^{11} + \\
 & \frac{1}{12} b^3 e^4 (20 a^3 B e^3 + 42 a b^2 d e (3 B d + A e) + 15 a^2 b e^2 (7 B d + A e) + 7 b^3 d^2 (5 B d + 3 A e)) x^{12} + \\
 & \frac{1}{13} b^4 e^5 (15 a^2 B e^2 + 7 b^2 d (3 B d + A e) + 6 a b e (7 B d + A e)) x^{13} + \\
 & \frac{1}{14} b^5 e^6 (7 b B d + A b e + 6 a B e) x^{14} + \frac{1}{15} b^6 B e^7 x^{15}
 \end{aligned}$$

**Problem 1041: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^6 (A + B x) (d + e x)^6 dx$$

Optimal (type 1, 290 leaves, 2 steps):

$$\begin{aligned} & \frac{(A b - a B) (b d - a e)^6 (a + b x)^7}{7 b^8} + \frac{(b d - a e)^5 (b B d + 6 A b e - 7 a B e) (a + b x)^8}{8 b^8} + \\ & \frac{e (b d - a e)^4 (2 b B d + 5 A b e - 7 a B e) (a + b x)^9}{3 b^8} + \\ & \frac{e^2 (b d - a e)^3 (3 b B d + 4 A b e - 7 a B e) (a + b x)^{10}}{2 b^8} + \\ & \frac{5 e^3 (b d - a e)^2 (4 b B d + 3 A b e - 7 a B e) (a + b x)^{11}}{11 b^8} + \\ & \frac{e^4 (b d - a e) (5 b B d + 2 A b e - 7 a B e) (a + b x)^{12}}{4 b^8} + \\ & \frac{e^5 (6 b B d + A b e - 7 a B e) (a + b x)^{13}}{13 b^8} + \frac{B e^6 (a + b x)^{14}}{14 b^8} \end{aligned}$$

Result (type 1, 1069 leaves):

$$\begin{aligned} & a^6 A d^6 x + \frac{1}{2} a^5 d^5 (a B d + 6 A (b d + a e)) x^2 + \\ & a^4 d^4 (2 a B d (b d + a e) + A (5 b^2 d^2 + 12 a b d e + 5 a^2 e^2)) x^3 + \\ & \frac{1}{4} a^3 d^3 (3 a B d (5 b^2 d^2 + 12 a b d e + 5 a^2 e^2) + 10 A (2 b^3 d^3 + 9 a b^2 d^2 e + 9 a^2 b d e^2 + 2 a^3 e^3)) x^4 + \\ & a^2 d^2 (2 a B d (2 b^3 d^3 + 9 a b^2 d^2 e + 9 a^2 b d e^2 + 2 a^3 e^3) + \\ & \quad 3 A (b^4 d^4 + 8 a b^3 d^3 e + 15 a^2 b^2 d^2 e^2 + 8 a^3 b d e^3 + a^4 e^4)) x^5 + \\ & \frac{1}{2} a d (5 a B d (b^4 d^4 + 8 a b^3 d^3 e + 15 a^2 b^2 d^2 e^2 + 8 a^3 b d e^3 + a^4 e^4) + \\ & \quad 2 A (b^5 d^5 + 15 a b^4 d^4 e + 50 a^2 b^3 d^3 e^2 + 50 a^3 b^2 d^2 e^3 + 15 a^4 b d e^4 + a^5 e^5)) x^6 + \\ & \frac{1}{7} (6 a B d (b^5 d^5 + 15 a b^4 d^4 e + 50 a^2 b^3 d^3 e^2 + 50 a^3 b^2 d^2 e^3 + 15 a^4 b d e^4 + a^5 e^5) + \\ & \quad A (b^6 d^6 + 36 a b^5 d^5 e + 225 a^2 b^4 d^4 e^2 + 400 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 36 a^5 b d e^5 + a^6 e^6)) x^7 + \\ & \frac{1}{8} (a^6 B e^6 + 6 a^5 b e^5 (6 B d + A e) + 45 a^4 b^2 d e^4 (5 B d + 2 A e) + 100 a^3 b^3 d^2 e^3 (4 B d + 3 A e) + \\ & \quad 75 a^2 b^4 d^3 e^2 (3 B d + 4 A e) + 18 a b^5 d^4 e (2 B d + 5 A e) + b^6 d^5 (B d + 6 A e)) x^8 + \\ & \frac{1}{3} b e (2 a^5 B e^5 + 5 a^4 b e^4 (6 B d + A e) + 20 a^3 b^2 d e^3 (5 B d + 2 A e) + \\ & \quad 25 a^2 b^3 d^2 e^2 (4 B d + 3 A e) + 10 a b^4 d^3 e (3 B d + 4 A e) + b^5 d^4 (2 B d + 5 A e)) x^9 + \\ & \frac{1}{2} b^2 e^2 (3 a^4 B e^4 + 4 a^3 b e^3 (6 B d + A e) + 9 a^2 b^2 d e^2 (5 B d + 2 A e) + \\ & \quad 6 a b^3 d^2 e (4 B d + 3 A e) + b^4 d^3 (3 B d + 4 A e)) x^{10} + \\ & \frac{1}{11} b^3 e^3 (20 a^3 B e^3 + 15 a^2 b e^2 (6 B d + A e) + 18 a b^2 d e (5 B d + 2 A e) + 5 b^3 d^2 (4 B d + 3 A e)) x^{11} + \\ & \frac{1}{4} b^4 e^4 (5 a^2 B e^2 + 2 a b e (6 B d + A e) + b^2 d (5 B d + 2 A e)) x^{12} + \\ & \frac{1}{13} b^5 e^5 (6 b B d + A b e + 6 a B e) x^{13} + \frac{1}{14} b^6 B e^6 x^{14} \end{aligned}$$

### Problem 1042: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^6 (A+Bx) (d+ex)^5 dx$$

Optimal (type 1, 240 leaves, 2 steps):

$$\begin{aligned} & \frac{(Ab-aB)(bd-ae)^5(a+bx)^7}{7b^7} + \frac{(bd-ae)^4(bBd+5Abe-6aBe)(a+bx)^8}{8b^7} + \\ & \frac{5e(bd-ae)^3(bBd+2Abe-3aBe)(a+bx)^9}{9b^7} + \frac{e^2(bd-ae)^2(bBd+Abe-2aBe)(a+bx)^{10}}{b^7} + \\ & \frac{5e^3(bd-ae)(2bBd+Abe-3aBe)(a+bx)^{11}}{11b^7} + \\ & \frac{e^4(5bBd+Abe-6aBe)(a+bx)^{12}}{12b^7} + \frac{Be^5(a+bx)^{13}}{13b^7} \end{aligned}$$

Result (type 1, 907 leaves):

$$\begin{aligned} & a^6 A d^5 x + \frac{1}{2} a^5 d^4 (6 A b d + a B d + 5 a A e) x^2 + \\ & \frac{1}{3} a^4 d^3 (a B d (6 b d + 5 a e) + 5 A (3 b^2 d^2 + 6 a b d e + 2 a^2 e^2)) x^3 + \\ & \frac{5}{4} a^3 d^2 (a B d (3 b^2 d^2 + 6 a b d e + 2 a^2 e^2) + A (4 b^3 d^3 + 15 a b^2 d^2 e + 12 a^2 b d e^2 + 2 a^3 e^3)) x^4 + \\ & a^2 d (a B d (4 b^3 d^3 + 15 a b^2 d^2 e + 12 a^2 b d e^2 + 2 a^3 e^3) + \\ & \quad A (3 b^4 d^4 + 20 a b^3 d^3 e + 30 a^2 b^2 d^2 e^2 + 12 a^3 b d e^3 + a^4 e^4)) x^5 + \\ & \frac{1}{6} a (5 a B d (3 b^4 d^4 + 20 a b^3 d^3 e + 30 a^2 b^2 d^2 e^2 + 12 a^3 b d e^3 + a^4 e^4) + \\ & \quad A (6 b^5 d^5 + 75 a b^4 d^4 e + 200 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 30 a^4 b d e^4 + a^5 e^5)) x^6 + \\ & \frac{1}{7} (a B (6 b^5 d^5 + 75 a b^4 d^4 e + 200 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 30 a^4 b d e^4 + a^5 e^5) + \\ & \quad A b (b^5 d^5 + 30 a b^4 d^4 e + 150 a^2 b^3 d^3 e^2 + 200 a^3 b^2 d^2 e^3 + 75 a^4 b d e^4 + 6 a^5 e^5)) x^7 + \\ & \frac{1}{8} b (6 a^5 B e^5 + 150 a^2 b^3 d^2 e^2 (B d + A e) + 100 a^3 b^2 d e^3 (2 B d + A e) + \\ & \quad 15 a^4 b e^4 (5 B d + A e) + 30 a b^4 d^3 e (B d + 2 A e) + b^5 d^4 (B d + 5 A e)) x^8 + \\ & \frac{5}{9} b^2 e (3 a^4 B e^4 + 12 a b^3 d^2 e (B d + A e) + 15 a^2 b^2 d e^2 (2 B d + A e) + \\ & \quad 4 a^3 b e^3 (5 B d + A e) + b^4 d^3 (B d + 2 A e)) x^9 + \\ & \frac{1}{2} b^3 e^2 (4 a^3 B e^3 + 2 b^3 d^2 (B d + A e) + 6 a b^2 d e (2 B d + A e) + 3 a^2 b e^2 (5 B d + A e)) x^{10} + \\ & \frac{1}{11} b^4 e^3 (15 a^2 B e^2 + 5 b^2 d (2 B d + A e) + 6 a b e (5 B d + A e)) x^{11} + \\ & \frac{1}{12} b^5 e^4 (5 b B d + A b e + 6 a B e) x^{12} + \frac{1}{13} b^6 B e^5 x^{13} \end{aligned}$$

### Problem 1043: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^6 (A+Bx) (d+ex)^4 dx$$

Optimal (type 1, 204 leaves, 2 steps):

$$\frac{(Ab - aB)(bd - ae)^4 (a+bx)^7}{7b^6} + \frac{(bd - ae)^3 (bBd + 4Abe - 5aBe)(a+bx)^8}{8b^6} +$$

$$\frac{2e(bd - ae)^2 (2bBd + 3Abe - 5aBe)(a+bx)^9}{9b^6} +$$

$$\frac{e^2(bd - ae)(3bBd + 2Abe - 5aBe)(a+bx)^{10}}{5b^6} +$$

$$\frac{e^3(4bBd + Abe - 5aBe)(a+bx)^{11}}{11b^6} + \frac{Be^4(a+bx)^{12}}{12b^6}$$

Result (type 1, 762 leaves):

$$a^6 A d^4 x + \frac{1}{2} a^5 d^3 (6 A b d + a B d + 4 a A e) x^2 +$$

$$\frac{1}{3} a^4 d^2 (2 a B d (3 b d + 2 a e) + 3 A (5 b^2 d^2 + 8 a b d e + 2 a^2 e^2)) x^3 +$$

$$\frac{1}{4} a^3 d (3 a B d (5 b^2 d^2 + 8 a b d e + 2 a^2 e^2) + 4 A (5 b^3 d^3 + 15 a b^2 d^2 e + 9 a^2 b d e^2 + a^3 e^3)) x^4 +$$

$$\frac{1}{5} a^2 (4 a B d (5 b^3 d^3 + 15 a b^2 d^2 e + 9 a^2 b d e^2 + a^3 e^3) +$$

$$A (15 b^4 d^4 + 80 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 24 a^3 b d e^3 + a^4 e^4)) x^5 +$$

$$\frac{1}{6} a (6 A b (b^4 d^4 + 10 a b^3 d^3 e + 20 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4) +$$

$$a B (15 b^4 d^4 + 80 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 24 a^3 b d e^3 + a^4 e^4)) x^6 +$$

$$\frac{1}{7} b (6 a B (b^4 d^4 + 10 a b^3 d^3 e + 20 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4) +$$

$$A b (b^4 d^4 + 24 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 80 a^3 b d e^3 + 15 a^4 e^4)) x^7 +$$

$$\frac{1}{8} b^2 (15 a^4 B e^4 + 20 a^3 b e^3 (4 B d + A e) + 30 a^2 b^2 d e^2 (3 B d + 2 A e) +$$

$$12 a b^3 d^2 e (2 B d + 3 A e) + b^4 d^3 (B d + 4 A e)) x^8 +$$

$$\frac{1}{9} b^3 e (20 a^3 B e^3 + 15 a^2 b e^2 (4 B d + A e) + 12 a b^2 d e (3 B d + 2 A e) + 2 b^3 d^2 (2 B d + 3 A e)) x^9 +$$

$$\frac{1}{10} b^4 e^2 (15 a^2 B e^2 + 6 a b e (4 B d + A e) + 2 b^2 d (3 B d + 2 A e)) x^{10} +$$

$$\frac{1}{11} b^5 e^3 (4 b B d + A b e + 6 a B e) x^{11} + \frac{1}{12} b^6 B e^4 x^{12}$$

### Problem 1044: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^6 (A+Bx) (d+ex)^3 dx$$

Optimal (type 1, 159 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e)^3 (a + b x)^7}{7 b^5} + \frac{(b d - a e)^2 (b B d + 3 A b e - 4 a B e) (a + b x)^8}{8 b^5} + \frac{e (b d - a e) (b B d + A b e - 2 a B e) (a + b x)^9}{3 b^5} + \frac{e^2 (3 b B d + A b e - 4 a B e) (a + b x)^{10}}{10 b^5} + \frac{B e^3 (a + b x)^{11}}{11 b^5}$$

Result (type 1, 586 leaves):

$$\begin{aligned} & a^6 A d^3 x + \frac{1}{2} a^5 d^2 (6 A b d + a B d + 3 a A e) x^2 + a^4 d (a B d (2 b d + a e) + A (5 b^2 d^2 + 6 a b d e + a^2 e^2)) x^3 + \\ & \frac{1}{4} a^3 (3 a B d (5 b^2 d^2 + 6 a b d e + a^2 e^2) + A (20 b^3 d^3 + 45 a b^2 d^2 e + 18 a^2 b d e^2 + a^3 e^3)) x^4 + \frac{1}{5} a^2 \\ & (a B (20 b^3 d^3 + 45 a b^2 d^2 e + 18 a^2 b d e^2 + a^3 e^3) + 3 A b (5 b^3 d^3 + 20 a b^2 d^2 e + 15 a^2 b d e^2 + 2 a^3 e^3)) \\ & x^5 + \frac{1}{2} a b (a B (5 b^3 d^3 + 20 a b^2 d^2 e + 15 a^2 b d e^2 + 2 a^3 e^3) + \\ & A b (2 b^3 d^3 + 15 a b^2 d^2 e + 20 a^2 b d e^2 + 5 a^3 e^3)) x^6 + \frac{1}{7} b^2 \\ & (3 a B (2 b^3 d^3 + 15 a b^2 d^2 e + 20 a^2 b d e^2 + 5 a^3 e^3) + A b (b^3 d^3 + 18 a b^2 d^2 e + 45 a^2 b d e^2 + 20 a^3 e^3)) \\ & x^7 + \frac{1}{8} b^3 (20 a^3 B e^3 + 18 a b^2 d e (B d + A e) + 15 a^2 b e^2 (3 B d + A e) + b^3 d^2 (B d + 3 A e)) x^8 + \\ & \frac{1}{3} b^4 e (5 a^2 B e^2 + b^2 d (B d + A e) + 2 a b e (3 B d + A e)) x^9 + \\ & \frac{1}{10} b^5 e^2 (3 b B d + A b e + 6 a B e) x^{10} + \frac{1}{11} b^6 B e^3 x^{11} \end{aligned}$$

### Problem 1045: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x)^2 dx$$

Optimal (type 1, 118 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e)^2 (a + b x)^7}{7 b^4} + \frac{(b d - a e) (b B d + 2 A b e - 3 a B e) (a + b x)^8}{8 b^4} + \frac{e (2 b B d + A b e - 3 a B e) (a + b x)^9}{9 b^4} + \frac{B e^2 (a + b x)^{10}}{10 b^4}$$

Result (type 1, 386 leaves):

$$\begin{aligned} & \frac{1}{2520} x (210 a^6 (4 A (3 d^2 + 3 d e x + e^2 x^2) + B x (6 d^2 + 8 d e x + 3 e^2 x^2)) + \\ & 252 a^5 b x (5 A (6 d^2 + 8 d e x + 3 e^2 x^2) + 2 B x (10 d^2 + 15 d e x + 6 e^2 x^2)) + \\ & 630 a^4 b^2 x^2 (2 A (10 d^2 + 15 d e x + 6 e^2 x^2) + B x (15 d^2 + 24 d e x + 10 e^2 x^2)) + \\ & 120 a^3 b^3 x^3 (7 A (15 d^2 + 24 d e x + 10 e^2 x^2) + 4 B x (21 d^2 + 35 d e x + 15 e^2 x^2)) + \\ & 45 a^2 b^4 x^4 (8 A (21 d^2 + 35 d e x + 15 e^2 x^2) + 5 B x (28 d^2 + 48 d e x + 21 e^2 x^2)) + \\ & 30 a b^5 x^5 (3 A (28 d^2 + 48 d e x + 21 e^2 x^2) + 2 B x (36 d^2 + 63 d e x + 28 e^2 x^2)) + \\ & b^6 x^6 (10 A (36 d^2 + 63 d e x + 28 e^2 x^2) + 7 B x (45 d^2 + 80 d e x + 36 e^2 x^2))) \end{aligned}$$

### Problem 1046: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) (d + e x) dx$$

Optimal (type 1, 75 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e) (a + b x)^7}{7 b^3} + \frac{(b B d + A b e - 2 a B e) (a + b x)^8}{8 b^3} + \frac{B e (a + b x)^9}{9 b^3}$$

Result (type 1, 231 leaves):

$$\frac{1}{504} x (84 a^6 (3 A (2 d + e x) + B x (3 d + 2 e x)) + 126 a^4 b^2 x^2 (5 A (4 d + 3 e x) + 3 B x (5 d + 4 e x)) + 252 a^5 b x (B x (4 d + 3 e x) + A (6 d + 4 e x)) + 168 a^3 b^3 x^3 (3 A (5 d + 4 e x) + 2 B x (6 d + 5 e x)) + 36 a^2 b^4 x^4 (7 A (6 d + 5 e x) + 5 B x (7 d + 6 e x)) + 18 a b^5 x^5 (4 A (7 d + 6 e x) + 3 B x (8 d + 7 e x)) + b^6 x^6 (9 A (8 d + 7 e x) + 7 B x (9 d + 8 e x)))$$

### Problem 1047: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (A + B x) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(A b - a B) (a + b x)^7}{7 b^2} + \frac{B (a + b x)^8}{8 b^2}$$

Result (type 1, 122 leaves):

$$\frac{1}{56} x (28 a^6 (2 A + B x) + 56 a^5 b x (3 A + 2 B x) + 70 a^4 b^2 x^2 (4 A + 3 B x) + 56 a^3 b^3 x^3 (5 A + 4 B x) + 28 a^2 b^4 x^4 (6 A + 5 B x) + 8 a b^5 x^5 (7 A + 6 B x) + b^6 x^6 (8 A + 7 B x))$$

### Problem 1048: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^6 (A + B x)}{d + e x} dx$$

Optimal (type 3, 220 leaves, 2 steps):

$$\frac{b (b d - a e)^5 (B d - A e) x}{e^7} - \frac{(b d - a e)^4 (B d - A e) (a + b x)^2}{2 e^6} + \frac{(b d - a e)^3 (B d - A e) (a + b x)^3}{3 e^5} - \frac{(b d - a e)^2 (B d - A e) (a + b x)^4}{4 e^4} + \frac{(b d - a e) (B d - A e) (a + b x)^5}{5 e^3} - \frac{(B d - A e) (a + b x)^6}{6 e^2} + \frac{B (a + b x)^7}{7 b e} - \frac{(b d - a e)^6 (B d - A e) \text{Log}[d + e x]}{e^8}$$

Result (type 3, 501 leaves):

$$\begin{aligned} & \frac{1}{420 e^8} \left( e x \left( 420 a^6 B e^6 + 1260 a^5 b e^5 \left( -2 B d + 2 A e + B e x \right) + \right. \right. \\ & 1050 a^4 b^2 e^4 \left( 3 A e \left( -2 d + e x \right) + B \left( 6 d^2 - 3 d e x + 2 e^2 x^2 \right) \right) + \\ & 700 a^3 b^3 e^3 \left( 2 A e \left( 6 d^2 - 3 d e x + 2 e^2 x^2 \right) + B \left( -12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3 \right) \right) + \\ & 105 a^2 b^4 e^2 \left( 5 A e \left( -12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3 \right) + \right. \\ & \quad \left. B \left( 60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4 \right) \right) + \\ & 42 a b^5 e \left( A e \left( 60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4 \right) + \right. \\ & \quad \left. B \left( -60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5 \right) \right) + \\ & b^6 \left( 7 A e \left( -60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5 \right) + \right. \\ & \quad \left. B \left( 420 d^6 - 210 d^5 e x + 140 d^4 e^2 x^2 - 105 d^3 e^3 x^3 + 84 d^2 e^4 x^4 - 70 d e^5 x^5 + 60 e^6 x^6 \right) \right) \Big) - \\ & 420 (b d - a e)^6 (B d - A e) \operatorname{Log}[d + e x] \Big) \end{aligned}$$

**Problem 1049: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^6 (A + B x)}{(d + e x)^2} dx$$

Optimal (type 3, 277 leaves, 2 steps):

$$\begin{aligned} & - \frac{3 b (b d - a e)^4 (7 b B d - 5 A b e - 2 a B e) x}{e^7} + \\ & \frac{(b d - a e)^6 (B d - A e)}{e^8 (d + e x)} + \frac{5 b^2 (b d - a e)^3 (7 b B d - 4 A b e - 3 a B e) (d + e x)^2}{2 e^8} - \\ & \frac{5 b^3 (b d - a e)^2 (7 b B d - 3 A b e - 4 a B e) (d + e x)^3}{3 e^8} + \\ & \frac{3 b^4 (b d - a e) (7 b B d - 2 A b e - 5 a B e) (d + e x)^4}{4 e^8} - \frac{b^5 (7 b B d - A b e - 6 a B e) (d + e x)^5}{5 e^8} + \\ & \frac{b^6 B (d + e x)^6}{6 e^8} + \frac{(b d - a e)^5 (7 b B d - 6 A b e - a B e) \operatorname{Log}[d + e x]}{e^8} \end{aligned}$$

Result (type 3, 643 leaves):

$$\begin{aligned} & \frac{1}{60 e^8 (d + e x)} \left( 60 a^6 e^6 (B d - A e) + 360 a^5 b e^5 (A d e + B (-d^2 + d e x + e^2 x^2)) + \right. \\ & 450 a^4 b^2 e^4 \left( 2 A e (-d^2 + d e x + e^2 x^2) + B (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3) \right) + 200 a^3 b^3 e^3 \\ & \left( 3 A e (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3) + 2 B (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4) \right) + \\ & 75 a^2 b^4 e^2 \left( 4 A e (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4) + \right. \\ & \quad \left. B (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5) \right) + \\ & 6 a b^5 e \left( 5 A e (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5) - \right. \\ & \quad \left. 6 B (10 d^6 - 50 d^5 e x - 30 d^4 e^2 x^2 + 10 d^3 e^3 x^3 - 5 d^2 e^4 x^4 + 3 d e^5 x^5 - 2 e^6 x^6) \right) + \\ & b^6 \left( 6 A e (-10 d^6 + 50 d^5 e x + 30 d^4 e^2 x^2 - 10 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 3 d e^5 x^5 + 2 e^6 x^6) + \right. \\ & \quad \left. B (60 d^7 - 360 d^6 e x - 210 d^5 e^2 x^2 + 70 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 21 d^2 e^5 x^5 - 14 d e^6 x^6 + 10 e^7 x^7) \right) \Big) + \\ & 60 (b d - a e)^5 (7 b B d - 6 A b e - a B e) (d + e x) \operatorname{Log}[d + e x] \Big) \end{aligned}$$

### Problem 1053: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^6} dx$$

Optimal (type 3, 272 leaves, 2 steps):

$$\begin{aligned} & -\frac{b^5 (6bBd - Abe - 6aBe) x}{e^7} + \frac{b^6 B x^2}{2e^6} + \frac{(bd - ae)^6 (Bd - Ae)}{5e^8 (d+ex)^5} - \\ & \frac{(bd - ae)^5 (7bBd - 6aBe - aBe)}{4e^8 (d+ex)^4} + \frac{b (bd - ae)^4 (7bBd - 5aBe - 2aBe)}{e^8 (d+ex)^3} - \\ & \frac{5b^2 (bd - ae)^3 (7bBd - 4aBe - 3aBe)}{2e^8 (d+ex)^2} + \frac{5b^3 (bd - ae)^2 (7bBd - 3aBe - 4aBe)}{e^8 (d+ex)} + \\ & \frac{3b^4 (bd - ae) (7bBd - 2aBe - 5aBe) \operatorname{Log}[d+ex]}{e^8} \end{aligned}$$

Result (type 3, 633 leaves):

$$\begin{aligned} & \frac{1}{20e^8 (d+ex)^5} \left( -a^6 e^6 (4Ae + B(d+5ex)) - 2a^5 b e^5 (3Ae(d+5ex) + 2B(d^2 + 5dex + 10e^2 x^2)) - \right. \\ & 5a^4 b^2 e^4 (2Ae(d^2 + 5dex + 10e^2 x^2) + 3B(d^3 + 5d^2 ex + 10de^2 x^2 + 10e^3 x^3)) - 20a^3 b^3 e^3 \\ & (Ae(d^3 + 5d^2 ex + 10de^2 x^2 + 10e^3 x^3) + 4B(d^4 + 5d^3 ex + 10d^2 e^2 x^2 + 10de^3 x^3 + 5e^4 x^4)) + \\ & 5a^2 b^4 e^2 (-12Ae(d^4 + 5d^3 ex + 10d^2 e^2 x^2 + 10de^3 x^3 + 5e^4 x^4) + \\ & B d (137d^4 + 625d^3 ex + 1100d^2 e^2 x^2 + 900de^3 x^3 + 300e^4 x^4)) + \\ & 2ab^5 e (Ade(137d^4 + 625d^3 ex + 1100d^2 e^2 x^2 + 900de^3 x^3 + 300e^4 x^4) - \\ & 6B(87d^6 + 375d^5 ex + 600d^4 e^2 x^2 + 400d^3 e^3 x^3 + 50d^2 e^4 x^4 - 50de^5 x^5 - 10e^6 x^6)) + \\ & b^6 (-2Ae(87d^6 + 375d^5 ex + 600d^4 e^2 x^2 + 400d^3 e^3 x^3 + 50d^2 e^4 x^4 - 50de^5 x^5 - 10e^6 x^6) + \\ & B(459d^7 + 1875d^6 ex + 2700d^5 e^2 x^2 + 1300d^4 e^3 x^3 - 400d^3 e^4 x^4 - 500d^2 e^5 x^5 - 70de^6 x^6 + \\ & \left. 10e^7 x^7)) + 60b^4 (bd - ae) (7bBd - 2aBe - 5aBe) (d+ex)^5 \operatorname{Log}[d+ex] \right) \end{aligned}$$

### Problem 1054: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^7} dx$$

Optimal (type 3, 278 leaves, 2 steps):

$$\frac{b^6 B x}{e^7} + \frac{(bd - ae)^6 (Bd - Ae)}{6e^8 (d+ex)^6} -$$

$$\frac{(bd - ae)^5 (7bBd - 6Abe - aBe)}{5e^8 (d+ex)^5} + \frac{3b (bd - ae)^4 (7bBd - 5Abe - 2aBe)}{4e^8 (d+ex)^4} -$$

$$\frac{5b^2 (bd - ae)^3 (7bBd - 4Abe - 3aBe)}{3e^8 (d+ex)^3} + \frac{5b^3 (bd - ae)^2 (7bBd - 3Abe - 4aBe)}{2e^8 (d+ex)^2} -$$

$$\frac{3b^4 (bd - ae) (7bBd - 2Abe - 5aBe)}{e^8 (d+ex)} - \frac{b^5 (7bBd - Abe - 6aBe) \text{Log}[d+ex]}{e^8}$$

Result (type 3, 619 leaves):

$$-\frac{1}{60e^8 (d+ex)^6} \left( 2a^6 e^6 (5Ae + B(d+6ex)) + 6a^5 b e^5 (2Ae(d+6ex) + B(d^2 + 6dex + 15e^2 x^2)) + \right.$$

$$15a^4 b^2 e^4 (Ae(d^2 + 6dex + 15e^2 x^2) + B(d^3 + 6d^2 ex + 15de^2 x^2 + 20e^3 x^3)) + 20a^3 b^3 e^3$$

$$(Ae(d^3 + 6d^2 ex + 15de^2 x^2 + 20e^3 x^3) + 2B(d^4 + 6d^3 ex + 15d^2 e^2 x^2 + 20de^3 x^3 + 15e^4 x^4)) +$$

$$30a^2 b^4 e^2 (Ae(d^4 + 6d^3 ex + 15d^2 e^2 x^2 + 20de^3 x^3 + 15e^4 x^4) +$$

$$5B(d^5 + 6d^4 ex + 15d^3 e^2 x^2 + 20d^2 e^3 x^3 + 15de^4 x^4 + 6e^5 x^5)) -$$

$$6ab^5 e (-10Ae(d^5 + 6d^4 ex + 15d^3 e^2 x^2 + 20d^2 e^3 x^3 + 15de^4 x^4 + 6e^5 x^5) +$$

$$Bd(147d^5 + 822d^4 ex + 1875d^3 e^2 x^2 + 2200d^2 e^3 x^3 + 1350de^4 x^4 + 360e^5 x^5)) -$$

$$b^6 (Ade(147d^5 + 822d^4 ex + 1875d^3 e^2 x^2 + 2200d^2 e^3 x^3 + 1350de^4 x^4 + 360e^5 x^5) -$$

$$B(669d^7 + 3594d^6 ex + 7725d^5 e^2 x^2 + 8200d^4 e^3 x^3 + 4050d^3 e^4 x^4 + 360d^2 e^5 x^5 -$$

$$360de^6 x^6 - 60e^7 x^7)) + 60b^5 (7bBd - Abe - 6aBe) (d+ex)^6 \text{Log}[d+ex]$$

**Problem 1055: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^8} dx$$

Optimal (type 3, 213 leaves, 3 steps):

$$-\frac{(Bd - Ae) (a+bx)^7}{7e (bd - ae) (d+ex)^7} - \frac{B (bd - ae)^6}{6e^8 (d+ex)^6} + \frac{6bB (bd - ae)^5}{5e^8 (d+ex)^5} - \frac{15b^2 B (bd - ae)^4}{4e^8 (d+ex)^4} +$$

$$\frac{20b^3 B (bd - ae)^3}{3e^8 (d+ex)^3} - \frac{15b^4 B (bd - ae)^2}{2e^8 (d+ex)^2} + \frac{6b^5 B (bd - ae)}{e^8 (d+ex)} + \frac{b^6 B \text{Log}[d+ex]}{e^8}$$

Result (type 3, 615 leaves):

$$\begin{aligned}
& - \frac{1}{420 e^8 (d+e x)^7} \\
& \left( 10 a^6 e^6 (6 A e + B (d+7 e x)) + 12 a^5 b e^5 (5 A e (d+7 e x) + 2 B (d^2+7 d e x+21 e^2 x^2)) + \right. \\
& \quad 15 a^4 b^2 e^4 (4 A e (d^2+7 d e x+21 e^2 x^2) + 3 B (d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3)) + 20 a^3 b^3 e^3 \\
& \quad (3 A e (d^3+7 d^2 e x+21 d e^2 x^2+35 e^3 x^3) + 4 B (d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4)) + \\
& \quad 30 a^2 b^4 e^2 (2 A e (d^4+7 d^3 e x+21 d^2 e^2 x^2+35 d e^3 x^3+35 e^4 x^4) + \\
& \quad \quad 5 B (d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5)) + \\
& \quad 60 a b^5 e (A e (d^5+7 d^4 e x+21 d^3 e^2 x^2+35 d^2 e^3 x^3+35 d e^4 x^4+21 e^5 x^5) + \\
& \quad \quad 6 B (d^6+7 d^5 e x+21 d^4 e^2 x^2+35 d^3 e^3 x^3+35 d^2 e^4 x^4+21 d e^5 x^5+7 e^6 x^6)) + \\
& \quad b^6 (60 A e (d^6+7 d^5 e x+21 d^4 e^2 x^2+35 d^3 e^3 x^3+35 d^2 e^4 x^4+21 d e^5 x^5+7 e^6 x^6) - \\
& \quad \quad B d (1089 d^6+7203 d^5 e x+20139 d^4 e^2 x^2+30625 d^3 e^3 x^3+26950 d^2 e^4 x^4+ \\
& \quad \quad \quad 13230 d e^5 x^5+2940 e^6 x^6)) - 420 b^6 B (d+e x)^7 \text{Log}[d+e x] \left. \right)
\end{aligned}$$

**Problem 1056: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^9} dx$$

Optimal (type 1, 86 leaves, 2 steps):

$$-\frac{(Bd-Ae)(a+bx)^7}{8e(bd-ae)(d+ex)^8} + \frac{(7bBd+Abe-8aBe)(a+bx)^7}{56e(bd-ae)^2(d+ex)^7}$$

Result (type 1, 597 leaves):

$$\begin{aligned}
& - \frac{1}{56 e^8 (d+e x)^8} \left( a^6 e^6 (7 A e + B (d+8 e x)) + 2 a^5 b e^5 (3 A e (d+8 e x) + B (d^2+8 d e x+28 e^2 x^2)) + \right. \\
& \quad a^4 b^2 e^4 (5 A e (d^2+8 d e x+28 e^2 x^2) + 3 B (d^3+8 d^2 e x+28 d e^2 x^2+56 e^3 x^3)) + 4 a^3 b^3 e^3 \\
& \quad (A e (d^3+8 d^2 e x+28 d e^2 x^2+56 e^3 x^3) + B (d^4+8 d^3 e x+28 d^2 e^2 x^2+56 d e^3 x^3+70 e^4 x^4)) + \\
& \quad a^2 b^4 e^2 (3 A e (d^4+8 d^3 e x+28 d^2 e^2 x^2+56 d e^3 x^3+70 e^4 x^4) + \\
& \quad \quad 5 B (d^5+8 d^4 e x+28 d^3 e^2 x^2+56 d^2 e^3 x^3+70 d e^4 x^4+56 e^5 x^5)) + \\
& \quad 2 a b^5 e (A e (d^5+8 d^4 e x+28 d^3 e^2 x^2+56 d^2 e^3 x^3+70 d e^4 x^4+56 e^5 x^5) + \\
& \quad \quad 3 B (d^6+8 d^5 e x+28 d^4 e^2 x^2+56 d^3 e^3 x^3+70 d^2 e^4 x^4+56 d e^5 x^5+28 e^6 x^6)) + \\
& \quad b^6 (A e (d^6+8 d^5 e x+28 d^4 e^2 x^2+56 d^3 e^3 x^3+70 d^2 e^4 x^4+56 d e^5 x^5+28 e^6 x^6) + \\
& \quad \quad 7 B (d^7+8 d^6 e x+28 d^5 e^2 x^2+56 d^4 e^3 x^3+70 d^3 e^4 x^4+56 d^2 e^5 x^5+28 d e^6 x^6+8 e^7 x^7)) \left. \right)
\end{aligned}$$

**Problem 1057: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^{10}} dx$$

Optimal (type 1, 135 leaves, 3 steps):

$$\begin{aligned}
 & - \frac{(Bd - Ae)(a + bx)^7}{9e(bd - ae)(d + ex)^9} + \\
 & \frac{(7bBd + 2Abe - 9aBe)(a + bx)^7}{72e(bd - ae)^2(d + ex)^8} + \frac{b(7bBd + 2Abe - 9aBe)(a + bx)^7}{504e(bd - ae)^3(d + ex)^7}
 \end{aligned}$$

Result (type 1, 603 leaves):

$$\begin{aligned}
 & - \frac{1}{504e^8(d + ex)^9} \\
 & (7a^6e^6(8Ae + B(d + 9ex)) + 6a^5be^5(7Ae(d + 9ex) + 2B(d^2 + 9dex + 36e^2x^2)) + \\
 & 15a^4b^2e^4(2Ae(d^2 + 9dex + 36e^2x^2) + B(d^3 + 9d^2ex + 36de^2x^2 + 84e^3x^3)) + 4a^3b^3e^3 \\
 & (5Ae(d^3 + 9d^2ex + 36de^2x^2 + 84e^3x^3) + 4B(d^4 + 9d^3ex + 36d^2e^2x^2 + 84de^3x^3 + 126e^4x^4)) + \\
 & 3a^2b^4e^2(4Ae(d^4 + 9d^3ex + 36d^2e^2x^2 + 84de^3x^3 + 126e^4x^4) + \\
 & 5B(d^5 + 9d^4ex + 36d^3e^2x^2 + 84d^2e^3x^3 + 126de^4x^4 + 126e^5x^5)) + \\
 & 6ab^5e(Ae(d^5 + 9d^4ex + 36d^3e^2x^2 + 84d^2e^3x^3 + 126de^4x^4 + 126e^5x^5) + \\
 & 2B(d^6 + 9d^5ex + 36d^4e^2x^2 + 84d^3e^3x^3 + 126d^2e^4x^4 + 126de^5x^5 + 84e^6x^6)) + \\
 & b^6(2Ae(d^6 + 9d^5ex + 36d^4e^2x^2 + 84d^3e^3x^3 + 126d^2e^4x^4 + 126de^5x^5 + 84e^6x^6) + \\
 & 7B(d^7 + 9d^6ex + 36d^5e^2x^2 + 84d^4e^3x^3 + 126d^3e^4x^4 + 126d^2e^5x^5 + 84de^6x^6 + 36e^7x^7)))
 \end{aligned}$$

### Problem 1058: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^6 (A + Bx)}{(d + ex)^{11}} dx$$

Optimal (type 1, 185 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{(Bd - Ae)(a + bx)^7}{10e(bd - ae)(d + ex)^{10}} + \frac{(7bBd + 3Abe - 10aBe)(a + bx)^7}{90e(bd - ae)^2(d + ex)^9} + \\
 & \frac{b(7bBd + 3Abe - 10aBe)(a + bx)^7}{360e(bd - ae)^3(d + ex)^8} + \frac{b^2(7bBd + 3Abe - 10aBe)(a + bx)^7}{2520e(bd - ae)^4(d + ex)^7}
 \end{aligned}$$

Result (type 1, 602 leaves):

$$\begin{aligned}
 & - \frac{1}{2520e^8(d + ex)^{10}} \\
 & (28a^6e^6(9Ae + B(d + 10ex)) + 42a^5be^5(4Ae(d + 10ex) + B(d^2 + 10dex + 45e^2x^2)) + \\
 & 15a^4b^2e^4(7Ae(d^2 + 10dex + 45e^2x^2) + 3B(d^3 + 10d^2ex + 45de^2x^2 + 120e^3x^3)) + \\
 & 20a^3b^3e^3(3Ae(d^3 + 10d^2ex + 45de^2x^2 + 120e^3x^3) + \\
 & 2B(d^4 + 10d^3ex + 45d^2e^2x^2 + 120de^3x^3 + 210e^4x^4)) + \\
 & 30a^2b^4e^2(Ae(d^4 + 10d^3ex + 45d^2e^2x^2 + 120de^3x^3 + 210e^4x^4) + \\
 & B(d^5 + 10d^4ex + 45d^3e^2x^2 + 120d^2e^3x^3 + 210de^4x^4 + 252e^5x^5)) + \\
 & 6ab^5e(2Ae(d^5 + 10d^4ex + 45d^3e^2x^2 + 120d^2e^3x^3 + 210de^4x^4 + 252e^5x^5) + \\
 & 3B(d^6 + 10d^5ex + 45d^4e^2x^2 + 120d^3e^3x^3 + 210d^2e^4x^4 + 252de^5x^5 + 210e^6x^6)) + \\
 & b^6(3Ae(d^6 + 10d^5ex + 45d^4e^2x^2 + 120d^3e^3x^3 + 210d^2e^4x^4 + 252de^5x^5 + 210e^6x^6) + 7B \\
 & (d^7 + 10d^6ex + 45d^5e^2x^2 + 120d^4e^3x^3 + 210d^3e^4x^4 + 252d^2e^5x^5 + 210de^6x^6 + 120e^7x^7)))
 \end{aligned}$$

### Problem 1059: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^{12}} dx$$

Optimal (type 1, 235 leaves, 5 steps):

$$\begin{aligned} & - \frac{(Bd - Ae) (a+bx)^7}{11e (bd - ae) (d+ex)^{11}} + \\ & \frac{(7bBd + 4Abe - 11aBe) (a+bx)^7}{110e (bd - ae)^2 (d+ex)^{10}} + \frac{b (7bBd + 4Abe - 11aBe) (a+bx)^7}{330e (bd - ae)^3 (d+ex)^9} + \\ & \frac{b^2 (7bBd + 4Abe - 11aBe) (a+bx)^7}{1320e (bd - ae)^4 (d+ex)^8} + \frac{b^3 (7bBd + 4Abe - 11aBe) (a+bx)^7}{9240e (bd - ae)^5 (d+ex)^7} \end{aligned}$$

Result (type 1, 605 leaves):

$$\begin{aligned} & - \frac{1}{9240e^8 (d+ex)^{11}} \\ & (84a^6e^6 (10Ae + B(d+11ex)) + 56a^5be^5 (9Ae(d+11ex) + 2B(d^2+11dex+55e^2x^2)) + \\ & 35a^4b^2e^4 (8Ae(d^2+11dex+55e^2x^2) + 3B(d^3+11d^2ex+55de^2x^2+165e^3x^3)) + \\ & 20a^3b^3e^3 (7Ae(d^3+11d^2ex+55de^2x^2+165e^3x^3) + \\ & 4B(d^4+11d^3ex+55d^2e^2x^2+165de^3x^3+330e^4x^4)) + \\ & 10a^2b^4e^2 (6Ae(d^4+11d^3ex+55d^2e^2x^2+165de^3x^3+330e^4x^4) + \\ & 5B(d^5+11d^4ex+55d^3e^2x^2+165d^2e^3x^3+330de^4x^4+462e^5x^5)) + \\ & 4ab^5e (5Ae(d^5+11d^4ex+55d^3e^2x^2+165d^2e^3x^3+330de^4x^4+462e^5x^5) + \\ & 6B(d^6+11d^5ex+55d^4e^2x^2+165d^3e^3x^3+330d^2e^4x^4+462de^5x^5+462e^6x^6)) + \\ & b^6 (4Ae(d^6+11d^5ex+55d^4e^2x^2+165d^3e^3x^3+330d^2e^4x^4+462de^5x^5+462e^6x^6) + 7B \\ & (d^7+11d^6ex+55d^5e^2x^2+165d^4e^3x^3+330d^3e^4x^4+462d^2e^5x^5+462de^6x^6+330e^7x^7))) \end{aligned}$$

### Problem 1060: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^{13}} dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned} & \frac{(bd-ae)^6 (Bd-Ae)}{12e^8 (d+ex)^{12}} - \frac{(bd-ae)^5 (7bBd-6Abe-aBe)}{11e^8 (d+ex)^{11}} + \\ & \frac{3b (bd-ae)^4 (7bBd-5Abe-2aBe)}{10e^8 (d+ex)^{10}} - \frac{5b^2 (bd-ae)^3 (7bBd-4Abe-3aBe)}{9e^8 (d+ex)^9} + \\ & \frac{5b^3 (bd-ae)^2 (7bBd-3Abe-4aBe)}{8e^8 (d+ex)^8} - \\ & \frac{3b^4 (bd-ae) (7bBd-2Abe-5aBe)}{7e^8 (d+ex)^7} + \frac{b^5 (7bBd-Abe-6aBe)}{6e^8 (d+ex)^6} - \frac{b^6 B}{5e^8 (d+ex)^5} \end{aligned}$$

Result (type 1, 600 leaves):

$$\begin{aligned} & - \frac{1}{27720e^8 (d+ex)^{12}} \\ & (210a^6e^6 (11Ae+B(d+12ex)) + 252a^5be^5 (5Ae(d+12ex) + B(d^2+12dex+66e^2x^2)) + \\ & 210a^4b^2e^4 (3Ae(d^2+12dex+66e^2x^2) + B(d^3+12d^2ex+66de^2x^2+220e^3x^3)) + \\ & 140a^3b^3e^3 (2Ae(d^3+12d^2ex+66de^2x^2+220e^3x^3) + \\ & B(d^4+12d^3ex+66d^2e^2x^2+220de^3x^3+495e^4x^4)) + \\ & 15a^2b^4e^2 (7Ae(d^4+12d^3ex+66d^2e^2x^2+220de^3x^3+495e^4x^4) + \\ & 5B(d^5+12d^4ex+66d^3e^2x^2+220d^2e^3x^3+495de^4x^4+792e^5x^5)) + \\ & 30ab^5e (Ae(d^5+12d^4ex+66d^3e^2x^2+220d^2e^3x^3+495de^4x^4+792e^5x^5) + \\ & B(d^6+12d^5ex+66d^4e^2x^2+220d^3e^3x^3+495d^2e^4x^4+792de^5x^5+924e^6x^6)) + \\ & b^6 (5Ae(d^6+12d^5ex+66d^4e^2x^2+220d^3e^3x^3+495d^2e^4x^4+792de^5x^5+924e^6x^6) + 7B \\ & (d^7+12d^6ex+66d^5e^2x^2+220d^4e^3x^3+495d^3e^4x^4+792d^2e^5x^5+924de^6x^6+792e^7x^7))) \end{aligned}$$

**Problem 1061: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^{14}} dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\begin{aligned} & \frac{(bd-ae)^6 (Bd-Ae)}{13e^8 (d+ex)^{13}} - \frac{(bd-ae)^5 (7bBd-6Abe-aBe)}{12e^8 (d+ex)^{12}} + \\ & \frac{3b (bd-ae)^4 (7bBd-5Abe-2aBe)}{11e^8 (d+ex)^{11}} - \frac{b^2 (bd-ae)^3 (7bBd-4Abe-3aBe)}{2e^8 (d+ex)^{10}} + \\ & \frac{5b^3 (bd-ae)^2 (7bBd-3Abe-4aBe)}{9e^8 (d+ex)^9} - \\ & \frac{3b^4 (bd-ae) (7bBd-2Abe-5aBe)}{8e^8 (d+ex)^8} + \frac{b^5 (7bBd-Abe-6aBe)}{7e^8 (d+ex)^7} - \frac{b^6 B}{6e^8 (d+ex)^6} \end{aligned}$$

Result (type 1, 605 leaves):

$$\frac{1}{72072 e^8 (d+ex)^{13}} \left( 462 a^6 e^6 (12 A e + B (d+13 ex)) + 252 a^5 b e^5 (11 A e (d+13 ex) + 2 B (d^2+13 d e x+78 e^2 x^2)) + 126 a^4 b^2 e^4 (10 A e (d^2+13 d e x+78 e^2 x^2) + 3 B (d^3+13 d^2 e x+78 d e^2 x^2+286 e^3 x^3)) + 56 a^3 b^3 e^3 (9 A e (d^3+13 d^2 e x+78 d e^2 x^2+286 e^3 x^3) + 4 B (d^4+13 d^3 e x+78 d^2 e^2 x^2+286 d e^3 x^3+715 e^4 x^4)) + 21 a^2 b^4 e^2 (8 A e (d^4+13 d^3 e x+78 d^2 e^2 x^2+286 d e^3 x^3+715 e^4 x^4) + 5 B (d^5+13 d^4 e x+78 d^3 e^2 x^2+286 d^2 e^3 x^3+715 d e^4 x^4+1287 e^5 x^5)) + 6 a b^5 e (7 A e (d^5+13 d^4 e x+78 d^3 e^2 x^2+286 d^2 e^3 x^3+715 d e^4 x^4+1287 e^5 x^5) + 6 B (d^6+13 d^5 e x+78 d^4 e^2 x^2+286 d^3 e^3 x^3+715 d^2 e^4 x^4+1287 d e^5 x^5+1716 e^6 x^6)) + b^6 (6 A e (d^6+13 d^5 e x+78 d^4 e^2 x^2+286 d^3 e^3 x^3+715 d^2 e^4 x^4+1287 d e^5 x^5+1716 e^6 x^6) + 7 B (d^7+13 d^6 e x+78 d^5 e^2 x^2+286 d^4 e^3 x^3+715 d^3 e^4 x^4+1287 d^2 e^5 x^5+1716 d e^6 x^6+1716 e^7 x^7)) \right)$$

**Problem 1062: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^6 (A+Bx)}{(d+ex)^{15}} dx$$

Optimal (type 1, 292 leaves, 2 steps):

$$\frac{(bd-ae)^6 (Bd-Ae)}{14 e^8 (d+ex)^{14}} - \frac{(bd-ae)^5 (7bBd-6Abe-aBe)}{13 e^8 (d+ex)^{13}} + \frac{b (bd-ae)^4 (7bBd-5Abe-2aBe)}{4 e^8 (d+ex)^{12}} - \frac{5 b^2 (bd-ae)^3 (7bBd-4Abe-3aBe)}{11 e^8 (d+ex)^{11}} + \frac{b^3 (bd-ae)^2 (7bBd-3Abe-4aBe)}{2 e^8 (d+ex)^{10}} - \frac{b^4 (bd-ae) (7bBd-2Abe-5aBe)}{3 e^8 (d+ex)^9} + \frac{b^5 (7bBd-Abe-6aBe)}{8 e^8 (d+ex)^8} - \frac{b^6 B}{7 e^8 (d+ex)^7}$$

Result (type 1, 602 leaves):

$$\frac{1}{24024 e^8 (d+ex)^{14}} \left( 132 a^6 e^6 (13 A e + B (d+14 ex)) + 132 a^5 b e^5 (6 A e (d+14 ex) + B (d^2+14 d e x+91 e^2 x^2)) + 30 a^4 b^2 e^4 (11 A e (d^2+14 d e x+91 e^2 x^2) + 3 B (d^3+14 d^2 e x+91 d e^2 x^2+364 e^3 x^3)) + 24 a^3 b^3 e^3 (5 A e (d^3+14 d^2 e x+91 d e^2 x^2+364 e^3 x^3) + 2 B (d^4+14 d^3 e x+91 d^2 e^2 x^2+364 d e^3 x^3+1001 e^4 x^4)) + 4 a^2 b^4 e^2 (9 A e (d^4+14 d^3 e x+91 d^2 e^2 x^2+364 d e^3 x^3+1001 e^4 x^4) + 5 B (d^5+14 d^4 e x+91 d^3 e^2 x^2+364 d^2 e^3 x^3+1001 d e^4 x^4+2002 e^5 x^5)) + 2 a b^5 e (4 A e (d^5+14 d^4 e x+91 d^3 e^2 x^2+364 d^2 e^3 x^3+1001 d e^4 x^4+2002 e^5 x^5) + 3 B (d^6+14 d^5 e x+91 d^4 e^2 x^2+364 d^3 e^3 x^3+1001 d^2 e^4 x^4+2002 d e^5 x^5+3003 e^6 x^6)) + b^6 (A e (d^6+14 d^5 e x+91 d^4 e^2 x^2+364 d^3 e^3 x^3+1001 d^2 e^4 x^4+2002 d e^5 x^5+3003 e^6 x^6) + B (d^7+14 d^6 e x+91 d^5 e^2 x^2+364 d^4 e^3 x^3+1001 d^3 e^4 x^4+2002 d^2 e^5 x^5+3003 d e^6 x^6+3432 e^7 x^7)) \right)$$

### Problem 1063: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (A + B x) (d + e x)^{13} dx$$

Optimal (type 1, 464 leaves, 2 steps):

$$\begin{aligned} & - \frac{(b d - a e)^{10} (B d - A e) (d + e x)^{14}}{14 e^{12}} + \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e) (d + e x)^{15}}{15 e^{12}} - \\ & \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) (d + e x)^{16}}{16 e^{12}} + \\ & \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e) (d + e x)^{17}}{17 e^{12}} - \\ & \frac{5 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e) (d + e x)^{18}}{3 e^{12}} + \\ & \frac{42 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e) (d + e x)^{19}}{19 e^{12}} - \\ & \frac{21 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e) (d + e x)^{20}}{10 e^{12}} + \\ & \frac{10 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e) (d + e x)^{21}}{7 e^{12}} - \\ & \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e) (d + e x)^{22}}{22 e^{12}} + \\ & \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^{23}}{23 e^{12}} - \\ & \frac{b^9 (11 b B d - A b e - 10 a B e) (d + e x)^{24}}{24 e^{12}} + \frac{b^{10} B (d + e x)^{25}}{25 e^{12}} \end{aligned}$$

Result (type 1, 3532 leaves):

$$\begin{aligned} & a^{10} A d^{13} x + \frac{1}{2} a^9 d^{12} (10 A b d + a B d + 13 a A e) x^2 + \\ & \frac{1}{3} a^8 d^{11} (a B d (10 b d + 13 a e) + A (45 b^2 d^2 + 130 a b d e + 78 a^2 e^2)) x^3 + \\ & \frac{1}{4} a^7 d^{10} (a B d (45 b^2 d^2 + 130 a b d e + 78 a^2 e^2) + A (120 b^3 d^3 + 585 a b^2 d^2 e + 780 a^2 b d e^2 + 286 a^3 e^3)) \\ & x^4 + \frac{1}{5} a^6 d^9 (a B d (120 b^3 d^3 + 585 a b^2 d^2 e + 780 a^2 b d e^2 + 286 a^3 e^3) + \\ & 5 A (42 b^4 d^4 + 312 a b^3 d^3 e + 702 a^2 b^2 d^2 e^2 + 572 a^3 b d e^3 + 143 a^4 e^4)) x^5 + \\ & \frac{1}{6} a^5 d^8 (5 a B d (42 b^4 d^4 + 312 a b^3 d^3 e + 702 a^2 b^2 d^2 e^2 + 572 a^3 b d e^3 + 143 a^4 e^4) + \\ & A (252 b^5 d^5 + 2730 a b^4 d^4 e + 9360 a^2 b^3 d^3 e^2 + 12870 a^3 b^2 d^2 e^3 + 7150 a^4 b d e^4 + 1287 a^5 e^5)) x^6 + \\ & \frac{1}{7} a^4 d^7 (a B d (252 b^5 d^5 + 2730 a b^4 d^4 e + 9360 a^2 b^3 d^3 e^2 + 12870 a^3 b^2 d^2 e^3 + \\ & 7150 a^4 b d e^4 + 1287 a^5 e^5) + 3 A (70 b^6 d^6 + 1092 a b^5 d^5 e + 5460 a^2 b^4 d^4 e^2 + \\ & 11440 a^3 b^3 d^3 e^3 + 10725 a^4 b^2 d^2 e^4 + 4290 a^5 b d e^5 + 572 a^6 e^6)) x^7 + \end{aligned}$$

$$\begin{aligned}
& \frac{3}{8} a^3 d^6 (a B d (70 b^6 d^6 + 1092 a b^5 d^5 e + 5460 a^2 b^4 d^4 e^2 + 11440 a^3 b^3 d^3 e^3 + 10725 a^4 b^2 d^2 e^4 + \\
& \quad 4290 a^5 b d e^5 + 572 a^6 e^6) + A (40 b^7 d^7 + 910 a b^6 d^6 e + 6552 a^2 b^5 d^5 e^2 + \\
& \quad 20020 a^3 b^4 d^4 e^3 + 28600 a^4 b^3 d^3 e^4 + 19305 a^5 b^2 d^2 e^5 + 5720 a^6 b d e^6 + 572 a^7 e^7)) x^8 + \\
& \frac{1}{3} a^2 d^5 (a B d (40 b^7 d^7 + 910 a b^6 d^6 e + 6552 a^2 b^5 d^5 e^2 + 20020 a^3 b^4 d^4 e^3 + \\
& \quad 28600 a^4 b^3 d^3 e^4 + 19305 a^5 b^2 d^2 e^5 + 5720 a^6 b d e^6 + 572 a^7 e^7) + \\
& \quad A (15 b^8 d^8 + 520 a b^7 d^7 e + 5460 a^2 b^6 d^6 e^2 + 24024 a^3 b^5 d^5 e^3 + 50050 a^4 b^4 d^4 e^4 + \\
& \quad 51480 a^5 b^3 d^3 e^5 + 25740 a^6 b^2 d^2 e^6 + 5720 a^7 b d e^7 + 429 a^8 e^8)) x^9 + \\
& \frac{1}{10} a d^4 (3 a B d (15 b^8 d^8 + 520 a b^7 d^7 e + 5460 a^2 b^6 d^6 e^2 + 24024 a^3 b^5 d^5 e^3 + \\
& \quad 50050 a^4 b^4 d^4 e^4 + 51480 a^5 b^3 d^3 e^5 + 25740 a^6 b^2 d^2 e^6 + 5720 a^7 b d e^7 + 429 a^8 e^8) + \\
& \quad 5 A (2 b^9 d^9 + 117 a b^8 d^8 e + 1872 a^2 b^7 d^7 e^2 + 12012 a^3 b^6 d^6 e^3 + 36036 a^4 b^5 d^5 e^4 + \\
& \quad 54054 a^5 b^4 d^4 e^5 + 41184 a^6 b^3 d^3 e^6 + 15444 a^7 b^2 d^2 e^7 + 2574 a^8 b d e^8 + 143 a^9 e^9)) x^{10} + \\
& \frac{1}{11} d^3 (5 a B d (2 b^9 d^9 + 117 a b^8 d^8 e + 1872 a^2 b^7 d^7 e^2 + 12012 a^3 b^6 d^6 e^3 + 36036 a^4 b^5 d^5 e^4 + \\
& \quad 54054 a^5 b^4 d^4 e^5 + 41184 a^6 b^3 d^3 e^6 + 15444 a^7 b^2 d^2 e^7 + 2574 a^8 b d e^8 + 143 a^9 e^9) + \\
& \quad A (b^{10} d^{10} + 130 a b^9 d^9 e + 3510 a^2 b^8 d^8 e^2 + 34320 a^3 b^7 d^7 e^3 + 150150 a^4 b^6 d^6 e^4 + 324324 a^5 b^5 \\
& \quad d^5 e^5 + 360360 a^6 b^4 d^4 e^6 + 205920 a^7 b^3 d^3 e^7 + 57915 a^8 b^2 d^2 e^8 + 7150 a^9 b d e^9 + 286 a^{10} e^{10})) \\
& x^{11} + \frac{1}{12} d^2 (360360 a^6 b^4 d^4 e^6 (B d + A e) + 1430 a^9 b d e^9 (5 B d + 2 A e) + \\
& \quad 51480 a^7 b^3 d^3 e^7 (4 B d + 3 A e) + 26 a^{10} e^{10} (11 B d + 3 A e) + 108108 a^5 b^5 d^5 e^5 (3 B d + 4 A e) + \\
& \quad 17160 a^3 b^7 d^7 e^3 (2 B d + 5 A e) + 6435 a^8 b^2 d^2 e^8 (9 B d + 5 A e) + 130 a b^9 d^9 e (B d + 6 A e) + \\
& \quad 30030 a^4 b^6 d^6 e^4 (5 B d + 9 A e) + 1170 a^2 b^8 d^8 e^2 (3 B d + 11 A e) + b^{10} d^{10} (B d + 13 A e)) x^{12} + \\
& d e (33264 a^5 b^5 d^5 e^5 (B d + A e) + a^{10} e^{10} (6 B d + A e) + 495 a^8 b^2 d^2 e^8 (5 B d + 2 A e) + \\
& \quad 6930 a^6 b^4 d^4 e^6 (4 B d + 3 A e) + 20 a^9 b d e^9 (11 B d + 3 A e) + 6930 a^4 b^6 d^6 e^4 (3 B d + 4 A e) + \\
& \quad 495 a^2 b^8 d^8 e^2 (2 B d + 5 A e) + 1320 a^7 b^3 d^3 e^7 (9 B d + 5 A e) + b^{10} d^{10} (B d + 6 A e) + \\
& \quad 1320 a^3 b^7 d^7 e^3 (5 B d + 9 A e) + 20 a b^9 d^9 e (3 B d + 11 A e)) x^{13} + \\
& \frac{1}{14} e^2 (360360 a^4 b^6 d^6 e^4 (B d + A e) + 130 a^9 b d e^9 (6 B d + A e) + a^{10} e^{10} (13 B d + A e) + \\
& \quad 17160 a^7 b^3 d^3 e^7 (5 B d + 2 A e) + 108108 a^5 b^5 d^5 e^5 (4 B d + 3 A e) + \\
& \quad 1170 a^8 b^2 d^2 e^8 (11 B d + 3 A e) + 51480 a^3 b^7 d^7 e^3 (3 B d + 4 A e) + 1430 a b^9 d^9 e (2 B d + 5 A e) + \\
& \quad 30030 a^6 b^4 d^4 e^6 (9 B d + 5 A e) + 6435 a^2 b^8 d^8 e^2 (5 B d + 9 A e) + 26 b^{10} d^{10} (3 B d + 11 A e)) x^{14} + \\
& \frac{1}{15} e^3 (a^{10} B e^{10} + 205920 a^3 b^7 d^6 e^3 (B d + A e) + 585 a^8 b^2 d e^8 (6 B d + A e) + \\
& \quad 10 a^9 b e^9 (13 B d + A e) + 30030 a^6 b^4 d^3 e^6 (5 B d + 2 A e) + 90090 a^4 b^6 d^5 e^4 (4 B d + 3 A e) + \\
& \quad 3120 a^7 b^3 d^2 e^7 (11 B d + 3 A e) + 19305 a^2 b^8 d^7 e^2 (3 B d + 4 A e) + 143 b^{10} d^9 (2 B d + 5 A e) + \\
& \quad 36036 a^5 b^5 d^4 e^5 (9 B d + 5 A e) + 1430 a b^9 d^8 e (5 B d + 9 A e)) x^{15} + \\
& \frac{1}{16} b e^4 (10 a^9 B e^9 + 77220 a^2 b^7 d^6 e^2 (B d + A e) + 1560 a^7 b^2 d e^7 (6 B d + A e) + \\
& \quad 45 a^8 b e^8 (13 B d + A e) + 36036 a^5 b^4 d^3 e^5 (5 B d + 2 A e) + 51480 a^3 b^6 d^5 e^3 (4 B d + 3 A e) + \\
& \quad 5460 a^6 b^3 d^2 e^6 (11 B d + 3 A e) + 4290 a b^8 d^7 e (3 B d + 4 A e) + \\
& \quad 30030 a^4 b^5 d^4 e^4 (9 B d + 5 A e) + 143 b^9 d^8 (5 B d + 9 A e)) x^{16} + \\
& \frac{3}{17} b^2 e^5 (15 a^8 B e^8 + 5720 a b^7 d^6 e (B d + A e) + 910 a^6 b^2 d e^6 (6 B d + A e) + \\
& \quad 40 a^7 b e^7 (13 B d + A e) + 10010 a^4 b^4 d^3 e^4 (5 B d + 2 A e) + 6435 a^2 b^6 d^5 e^2 (4 B d + 3 A e) + \\
& \quad 2184 a^5 b^3 d^2 e^5 (11 B d + 3 A e) + 143 b^8 d^7 (3 B d + 4 A e) + 5720 a^3 b^5 d^4 e^3 (9 B d + 5 A e)) x^{17} +
\end{aligned}$$

$$\begin{aligned}
 & \frac{1}{6} b^3 e^6 (40 a^7 B e^7 + 572 b^7 d^6 (B d + A e) + 1092 a^5 b^2 d e^5 (6 B d + A e) + \\
 & \quad 70 a^6 b e^6 (13 B d + A e) + 5720 a^3 b^4 d^3 e^3 (5 B d + 2 A e) + 1430 a b^6 d^5 e (4 B d + 3 A e) + \\
 & \quad 1820 a^4 b^3 d^2 e^4 (11 B d + 3 A e) + 2145 a^2 b^5 d^4 e^2 (9 B d + 5 A e)) x^{18} + \frac{1}{19} b^4 e^7 \\
 & \quad (210 a^6 B e^6 + 2730 a^4 b^2 d e^4 (6 B d + A e) + 252 a^5 b e^5 (13 B d + A e) + 6435 a^2 b^4 d^3 e^2 (5 B d + 2 A e) + \\
 & \quad 429 b^6 d^5 (4 B d + 3 A e) + 3120 a^3 b^3 d^2 e^3 (11 B d + 3 A e) + 1430 a b^5 d^4 e (9 B d + 5 A e)) x^{19} + \\
 & \frac{1}{20} b^5 e^8 (252 a^5 B e^5 + 1560 a^3 b^2 d e^3 (6 B d + A e) + 210 a^4 b e^4 (13 B d + A e) + \\
 & \quad 1430 a b^4 d^3 e (5 B d + 2 A e) + 1170 a^2 b^3 d^2 e^2 (11 B d + 3 A e) + 143 b^5 d^4 (9 B d + 5 A e)) x^{20} + \\
 & \frac{1}{21} b^6 e^9 (210 a^4 B e^4 + 585 a^2 b^2 d e^2 (6 B d + A e) + 120 a^3 b e^3 (13 B d + A e) + \\
 & \quad 143 b^4 d^3 (5 B d + 2 A e) + 260 a b^3 d^2 e (11 B d + 3 A e)) x^{21} + \\
 & \frac{1}{22} b^7 e^{10} (120 a^3 B e^3 + 130 a b^2 d e (6 B d + A e) + 45 a^2 b e^2 (13 B d + A e) + 26 b^3 d^2 (11 B d + 3 A e)) \\
 & \quad x^{22} + \\
 & \frac{1}{23} b^8 e^{11} (45 a^2 B e^2 + 13 b^2 d (6 B d + A e) + 10 a b e (13 B d + A e)) x^{23} + \\
 & \frac{1}{24} b^9 e^{12} (13 b B d + A b e + 10 a B e) x^{24} + \\
 & \frac{1}{25} b^{10} B e^{13} x^{25}
 \end{aligned}$$

**Problem 1064: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^{12} dx$$

Optimal (type 1, 464 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(bd - ae)^{10} (Bd - Ae) (d + ex)^{13}}{13 e^{12}} + \frac{(bd - ae)^9 (11 b B d - 10 A b e - a B e) (d + ex)^{14}}{14 e^{12}} \\
 & - \frac{b (bd - ae)^8 (11 b B d - 9 A b e - 2 a B e) (d + ex)^{15}}{3 e^{12}} + \\
 & - \frac{15 b^2 (bd - ae)^7 (11 b B d - 8 A b e - 3 a B e) (d + ex)^{16}}{16 e^{12}} - \\
 & + \frac{30 b^3 (bd - ae)^6 (11 b B d - 7 A b e - 4 a B e) (d + ex)^{17}}{17 e^{12}} + \\
 & - \frac{7 b^4 (bd - ae)^5 (11 b B d - 6 A b e - 5 a B e) (d + ex)^{18}}{3 e^{12}} - \\
 & + \frac{42 b^5 (bd - ae)^4 (11 b B d - 5 A b e - 6 a B e) (d + ex)^{19}}{19 e^{12}} + \\
 & - \frac{3 b^6 (bd - ae)^3 (11 b B d - 4 A b e - 7 a B e) (d + ex)^{20}}{2 e^{12}} - \\
 & + \frac{5 b^7 (bd - ae)^2 (11 b B d - 3 A b e - 8 a B e) (d + ex)^{21}}{7 e^{12}} + \\
 & - \frac{5 b^8 (bd - ae) (11 b B d - 2 A b e - 9 a B e) (d + ex)^{22}}{22 e^{12}} - \\
 & + \frac{b^9 (11 b B d - A b e - 10 a B e) (d + ex)^{23}}{23 e^{12}} + \frac{b^{10} B (d + ex)^{24}}{24 e^{12}}
 \end{aligned}$$

Result (type 1, 3320 leaves):

$$\begin{aligned}
 & a^{10} A d^{12} x + \frac{1}{2} a^9 d^{11} (a B d + 2 A (5 b d + 6 a e)) x^2 + \\
 & \frac{1}{3} a^8 d^{10} (2 a B d (5 b d + 6 a e) + 3 A (15 b^2 d^2 + 40 a b d e + 22 a^2 e^2)) x^3 + \\
 & \frac{1}{4} a^7 d^9 (3 a B d (15 b^2 d^2 + 40 a b d e + 22 a^2 e^2) + 20 A (6 b^3 d^3 + 27 a b^2 d^2 e + 33 a^2 b d e^2 + 11 a^3 e^3)) x^4 + \\
 & a^6 d^8 (4 a B d (6 b^3 d^3 + 27 a b^2 d^2 e + 33 a^2 b d e^2 + 11 a^3 e^3) + \\
 & \quad A (42 b^4 d^4 + 288 a b^3 d^3 e + 594 a^2 b^2 d^2 e^2 + 440 a^3 b d e^3 + 99 a^4 e^4)) x^5 + \\
 & \frac{1}{6} a^5 d^7 (5 a B d (42 b^4 d^4 + 288 a b^3 d^3 e + 594 a^2 b^2 d^2 e^2 + 440 a^3 b d e^3 + 99 a^4 e^4) + \\
 & \quad 18 A (14 b^5 d^5 + 140 a b^4 d^4 e + 440 a^2 b^3 d^3 e^2 + 550 a^3 b^2 d^2 e^3 + 275 a^4 b d e^4 + 44 a^5 e^5)) x^6 + \\
 & \frac{3}{7} a^4 d^6 (6 a B d (14 b^5 d^5 + 140 a b^4 d^4 e + 440 a^2 b^3 d^3 e^2 + 550 a^3 b^2 d^2 e^3 + 275 a^4 b d e^4 + 44 a^5 e^5) + \\
 & \quad A (70 b^6 d^6 + 1008 a b^5 d^5 e + 4620 a^2 b^4 d^4 e^2 + \\
 & \quad \quad 8800 a^3 b^3 d^3 e^3 + 7425 a^4 b^2 d^2 e^4 + 2640 a^5 b d e^5 + 308 a^6 e^6)) x^7 + \\
 & \frac{3}{8} a^3 d^5 (a B d (70 b^6 d^6 + 1008 a b^5 d^5 e + 4620 a^2 b^4 d^4 e^2 + 8800 a^3 b^3 d^3 e^3 + 7425 a^4 b^2 d^2 e^4 + \\
 & \quad 2640 a^5 b d e^5 + 308 a^6 e^6) + 8 A (5 b^7 d^7 + 105 a b^6 d^6 e + 693 a^2 b^5 d^5 e^2 + \\
 & \quad 1925 a^3 b^4 d^4 e^3 + 2475 a^4 b^3 d^3 e^4 + 1485 a^5 b^2 d^2 e^5 + 385 a^6 b d e^6 + 33 a^7 e^7)) x^8 + \\
 & \frac{1}{3} a^2 d^4 (8 a B d (5 b^7 d^7 + 105 a b^6 d^6 e + 693 a^2 b^5 d^5 e^2 + 1925 a^3 b^4 d^4 e^3 + 2475 a^4 b^3 d^3 e^4 + \\
 & \quad 1485 a^5 b^2 d^2 e^5 + 385 a^6 b d e^6 + 33 a^7 e^7) + 15 A (b^8 d^8 + 32 a b^7 d^7 e + 308 a^2 b^6 d^6 e^2 +
 \end{aligned}$$

$$\begin{aligned}
 & 1232 a^3 b^5 d^5 e^3 + 2310 a^4 b^4 d^4 e^4 + 2112 a^5 b^3 d^3 e^5 + 924 a^6 b^2 d^2 e^6 + 176 a^7 b d e^7 + 11 a^8 e^8) ) \\
 x^9 + \frac{1}{2} a d^3 (9 a B d (b^8 d^8 + 32 a b^7 d^7 e + 308 a^2 b^6 d^6 e^2 + 1232 a^3 b^5 d^5 e^3 + 2310 a^4 b^4 d^4 e^4 + \\
 & 2112 a^5 b^3 d^3 e^5 + 924 a^6 b^2 d^2 e^6 + 176 a^7 b d e^7 + 11 a^8 e^8) + \\
 & 2 A (b^9 d^9 + 54 a b^8 d^8 e + 792 a^2 b^7 d^7 e^2 + 4620 a^3 b^6 d^6 e^3 + 12474 a^4 b^5 d^5 e^4 + \\
 & 16632 a^5 b^4 d^4 e^5 + 11088 a^6 b^3 d^3 e^6 + 3564 a^7 b^2 d^2 e^7 + 495 a^8 b d e^8 + 22 a^9 e^9) ) x^{10} + \\
 \frac{1}{11} d^2 (10 a B d (b^9 d^9 + 54 a b^8 d^8 e + 792 a^2 b^7 d^7 e^2 + 4620 a^3 b^6 d^6 e^3 + 12474 a^4 b^5 d^5 e^4 + \\
 & 16632 a^5 b^4 d^4 e^5 + 11088 a^6 b^3 d^3 e^6 + 3564 a^7 b^2 d^2 e^7 + 495 a^8 b d e^8 + 22 a^9 e^9) + \\
 & A (b^{10} d^{10} + 120 a b^9 d^9 e + 2970 a^2 b^8 d^8 e^2 + 26400 a^3 b^7 d^7 e^3 + 103950 a^4 b^6 d^6 e^4 + 199584 a^5 b^5 d^5 \\
 & e^5 + 194040 a^6 b^4 d^4 e^6 + 95040 a^7 b^3 d^3 e^7 + 22275 a^8 b^2 d^2 e^8 + 2200 a^9 b d e^9 + 66 a^{10} e^{10}) ) x^{11} + \\
 \frac{1}{12} d (6 a^{10} e^{10} (11 B d + 2 A e) + 220 a^9 b d e^9 (10 B d + 3 A e) + 2475 a^8 b^2 d^2 e^8 (9 B d + 4 A e) + \\
 & 11880 a^7 b^3 d^3 e^7 (8 B d + 5 A e) + 27720 a^6 b^4 d^4 e^6 (7 B d + 6 A e) + \\
 & 33264 a^5 b^5 d^5 e^5 (6 B d + 7 A e) + 20790 a^4 b^6 d^6 e^4 (5 B d + 8 A e) + 6600 a^3 b^7 d^7 e^3 (4 B d + 9 A e) + \\
 & 990 a^2 b^8 d^8 e^2 (3 B d + 10 A e) + 60 a b^9 d^9 e (2 B d + 11 A e) + b^{10} d^{10} (B d + 12 A e) ) x^{12} + \\
 \frac{1}{13} e (a^{10} e^{10} (12 B d + A e) + 60 a^9 b d e^9 (11 B d + 2 A e) + 990 a^8 b^2 d^2 e^8 (10 B d + 3 A e) + \\
 & 6600 a^7 b^3 d^3 e^7 (9 B d + 4 A e) + 20790 a^6 b^4 d^4 e^6 (8 B d + 5 A e) + 33264 a^5 b^5 d^5 e^5 (7 B d + 6 A e) + \\
 & 27720 a^4 b^6 d^6 e^4 (6 B d + 7 A e) + 11880 a^3 b^7 d^7 e^3 (5 B d + 8 A e) + \\
 & 2475 a^2 b^8 d^8 e^2 (4 B d + 9 A e) + 220 a b^9 d^9 e (3 B d + 10 A e) + 6 b^{10} d^{10} (2 B d + 11 A e) ) x^{13} + \\
 \frac{1}{14} e^2 (a^{10} B e^{10} + 10 a^9 b e^9 (12 B d + A e) + 270 a^8 b^2 d e^8 (11 B d + 2 A e) + \\
 & 2640 a^7 b^3 d^2 e^7 (10 B d + 3 A e) + 11550 a^6 b^4 d^3 e^6 (9 B d + 4 A e) + 24948 a^5 b^5 d^4 e^5 (8 B d + 5 A e) + \\
 & 27720 a^4 b^6 d^5 e^4 (7 B d + 6 A e) + 15840 a^3 b^7 d^6 e^3 (6 B d + 7 A e) + \\
 & 4455 a^2 b^8 d^7 e^2 (5 B d + 8 A e) + 550 a b^9 d^8 e (4 B d + 9 A e) + 22 b^{10} d^9 (3 B d + 10 A e) ) x^{14} + \\
 \frac{1}{3} b e^3 (2 a^9 B e^9 + 9 a^8 b e^8 (12 B d + A e) + 144 a^7 b^2 d e^7 (11 B d + 2 A e) + \\
 & 924 a^6 b^3 d^2 e^6 (10 B d + 3 A e) + 2772 a^5 b^4 d^3 e^5 (9 B d + 4 A e) + \\
 & 4158 a^4 b^5 d^4 e^4 (8 B d + 5 A e) + 3168 a^3 b^6 d^5 e^3 (7 B d + 6 A e) + \\
 & 1188 a^2 b^7 d^6 e^2 (6 B d + 7 A e) + 198 a b^8 d^7 e (5 B d + 8 A e) + 11 b^9 d^8 (4 B d + 9 A e) ) x^{15} + \\
 \frac{3}{16} b^2 e^4 (15 a^8 B e^8 + 40 a^7 b e^7 (12 B d + A e) + 420 a^6 b^2 d e^6 (11 B d + 2 A e) + \\
 & 1848 a^5 b^3 d^2 e^5 (10 B d + 3 A e) + 3850 a^4 b^4 d^3 e^4 (9 B d + 4 A e) + 3960 a^3 b^5 d^4 e^3 (8 B d + 5 A e) + \\
 & 1980 a^2 b^6 d^5 e^2 (7 B d + 6 A e) + 440 a b^7 d^6 e (6 B d + 7 A e) + 33 b^8 d^7 (5 B d + 8 A e) ) x^{16} + \\
 \frac{3}{17} b^3 e^5 (40 a^7 B e^7 + 70 a^6 b e^6 (12 B d + A e) + 504 a^5 b^2 d e^5 (11 B d + 2 A e) + \\
 & 1540 a^4 b^3 d^2 e^4 (10 B d + 3 A e) + 2200 a^3 b^4 d^3 e^3 (9 B d + 4 A e) + 1485 a^2 b^5 d^4 e^2 (8 B d + 5 A e) + \\
 & 440 a b^6 d^5 e (7 B d + 6 A e) + 44 b^7 d^6 (6 B d + 7 A e) ) x^{17} + \frac{1}{6} b^4 e^6 \\
 & (70 a^6 B e^6 + 84 a^5 b e^5 (12 B d + A e) + 420 a^4 b^2 d e^4 (11 B d + 2 A e) + 880 a^3 b^3 d^2 e^3 (10 B d + 3 A e) + \\
 & 825 a^2 b^4 d^3 e^2 (9 B d + 4 A e) + 330 a b^5 d^4 e (8 B d + 5 A e) + 44 b^6 d^5 (7 B d + 6 A e) ) x^{18} + \\
 \frac{1}{19} b^5 e^7 (252 a^5 B e^5 + 210 a^4 b e^4 (12 B d + A e) + 720 a^3 b^2 d e^3 (11 B d + 2 A e) + \\
 & 990 a^2 b^3 d^2 e^2 (10 B d + 3 A e) + 550 a b^4 d^3 e (9 B d + 4 A e) + 99 b^5 d^4 (8 B d + 5 A e) ) x^{19} + \\
 \frac{1}{4} b^6 e^8 (42 a^4 B e^4 + 24 a^3 b e^3 (12 B d + A e) + 54 a^2 b^2 d e^2 (11 B d + 2 A e) +
 \end{aligned}$$

$$\begin{aligned}
 & 44 a b^3 d^2 e (10 B d + 3 A e) + 11 b^4 d^3 (9 B d + 4 A e) x^{20} + \frac{1}{21} b^7 e^9 \\
 & (120 a^3 B e^3 + 45 a^2 b e^2 (12 B d + A e) + 60 a b^2 d e (11 B d + 2 A e) + 22 b^3 d^2 (10 B d + 3 A e)) x^{21} + \\
 & \frac{1}{22} b^8 e^{10} (45 a^2 B e^2 + 10 a b e (12 B d + A e) + 6 b^2 d (11 B d + 2 A e)) x^{22} + \\
 & \frac{1}{23} b^9 e^{11} (12 b B d + A b e + 10 a B e) x^{23} + \\
 & \frac{1}{24} b^{10} B e^{12} x^{24}
 \end{aligned}$$

**Problem 1065: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^{11} dx$$

Optimal (type 1, 461 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(b d - a e)^{10} (B d - A e) (d + e x)^{12}}{12 e^{12}} + \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e) (d + e x)^{13}}{13 e^{12}} - \\
 & \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) (d + e x)^{14}}{14 e^{12}} + \\
 & \frac{b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e) (d + e x)^{15}}{e^{12}} - \\
 & \frac{15 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e) (d + e x)^{16}}{8 e^{12}} + \\
 & \frac{42 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e) (d + e x)^{17}}{17 e^{12}} - \\
 & \frac{7 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e) (d + e x)^{18}}{3 e^{12}} + \\
 & \frac{30 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e) (d + e x)^{19}}{19 e^{12}} - \\
 & \frac{3 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e) (d + e x)^{20}}{4 e^{12}} + \\
 & \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^{21}}{21 e^{12}} - \\
 & \frac{b^9 (11 b B d - A b e - 10 a B e) (d + e x)^{22}}{22 e^{12}} + \frac{b^{10} B (d + e x)^{23}}{23 e^{12}}
 \end{aligned}$$

Result (type 1, 3018 leaves):

$$\begin{aligned}
 & a^{10} A d^{11} x + \frac{1}{2} a^9 d^{10} (10 A b d + a B d + 11 a A e) x^2 + \\
 & \frac{1}{3} a^8 d^9 (a B d (10 b d + 11 a e) + 5 A (9 b^2 d^2 + 22 a b d e + 11 a^2 e^2)) x^3 + \\
 & \frac{5}{4} a^7 d^8 (a B d (9 b^2 d^2 + 22 a b d e + 11 a^2 e^2) + A (24 b^3 d^3 + 99 a b^2 d^2 e + 110 a^2 b d e^2 + 33 a^3 e^3)) x^4 +
 \end{aligned}$$

$$\begin{aligned}
 & a^6 d^7 (a B d (24 b^3 d^3 + 99 a b^2 d^2 e + 110 a^2 b d e^2 + 33 a^3 e^3) + \\
 & \quad 3 A (14 b^4 d^4 + 88 a b^3 d^3 e + 165 a^2 b^2 d^2 e^2 + 110 a^3 b d e^3 + 22 a^4 e^4)) x^5 + \\
 & \frac{1}{2} a^5 d^6 (5 a B d (14 b^4 d^4 + 88 a b^3 d^3 e + 165 a^2 b^2 d^2 e^2 + 110 a^3 b d e^3 + 22 a^4 e^4) + \\
 & \quad A (84 b^5 d^5 + 770 a b^4 d^4 e + 2200 a^2 b^3 d^3 e^2 + 2475 a^3 b^2 d^2 e^3 + 1100 a^4 b d e^4 + 154 a^5 e^5)) x^6 + \\
 & \frac{3}{7} a^4 d^5 (a B d (84 b^5 d^5 + 770 a b^4 d^4 e + 2200 a^2 b^3 d^3 e^2 + 2475 a^3 b^2 d^2 e^3 + 1100 a^4 b d e^4 + 154 a^5 e^5) + \\
 & \quad 2 A (35 b^6 d^6 + 462 a b^5 d^5 e + 1925 a^2 b^4 d^4 e^2 + \\
 & \quad \quad 3300 a^3 b^3 d^3 e^3 + 2475 a^4 b^2 d^2 e^4 + 770 a^5 b d e^5 + 77 a^6 e^6)) x^7 + \\
 & \frac{3}{4} a^3 d^4 (a B d (35 b^6 d^6 + 462 a b^5 d^5 e + 1925 a^2 b^4 d^4 e^2 + 3300 a^3 b^3 d^3 e^3 + 2475 a^4 b^2 d^2 e^4 + \\
 & \quad 770 a^5 b d e^5 + 77 a^6 e^6) + 5 A (4 b^7 d^7 + 77 a b^6 d^6 e + 462 a^2 b^5 d^5 e^2 + \\
 & \quad 1155 a^3 b^4 d^4 e^3 + 1320 a^4 b^3 d^3 e^4 + 693 a^5 b^2 d^2 e^5 + 154 a^6 b d e^6 + 11 a^7 e^7)) x^8 + \\
 & \frac{5}{3} a^2 d^3 (2 a B d (4 b^7 d^7 + 77 a b^6 d^6 e + 462 a^2 b^5 d^5 e^2 + 1155 a^3 b^4 d^4 e^3 + 1320 a^4 b^3 d^3 e^4 + \\
 & \quad 693 a^5 b^2 d^2 e^5 + 154 a^6 b d e^6 + 11 a^7 e^7) + A (3 b^8 d^8 + 88 a b^7 d^7 e + 770 a^2 b^6 d^6 e^2 + \\
 & \quad 2772 a^3 b^5 d^5 e^3 + 4620 a^4 b^4 d^4 e^4 + 3696 a^5 b^3 d^3 e^5 + 1386 a^6 b^2 d^2 e^6 + 220 a^7 b d e^7 + 11 a^8 e^8)) \\
 & x^9 + \frac{1}{2} a d^2 (3 a B d (3 b^8 d^8 + 88 a b^7 d^7 e + 770 a^2 b^6 d^6 e^2 + 2772 a^3 b^5 d^5 e^3 + \\
 & \quad 4620 a^4 b^4 d^4 e^4 + 3696 a^5 b^3 d^3 e^5 + 1386 a^6 b^2 d^2 e^6 + 220 a^7 b d e^7 + 11 a^8 e^8) + \\
 & \quad A (2 b^9 d^9 + 99 a b^8 d^8 e + 1320 a^2 b^7 d^7 e^2 + 6930 a^3 b^6 d^6 e^3 + 16632 a^4 b^5 d^5 e^4 + \\
 & \quad 19404 a^5 b^4 d^4 e^5 + 11088 a^6 b^3 d^3 e^6 + 2970 a^7 b^2 d^2 e^7 + 330 a^8 b d e^8 + 11 a^9 e^9)) x^{10} + \\
 & \frac{1}{11} d (5 a B d (2 b^9 d^9 + 99 a b^8 d^8 e + 1320 a^2 b^7 d^7 e^2 + 6930 a^3 b^6 d^6 e^3 + 16632 a^4 b^5 d^5 e^4 + \\
 & \quad 19404 a^5 b^4 d^4 e^5 + 11088 a^6 b^3 d^3 e^6 + 2970 a^7 b^2 d^2 e^7 + 330 a^8 b d e^8 + 11 a^9 e^9) + \\
 & \quad A (b^{10} d^{10} + 110 a b^9 d^9 e + 2475 a^2 b^8 d^8 e^2 + 19800 a^3 b^7 d^7 e^3 + 69300 a^4 b^6 d^6 e^4 + 116424 a^5 b^5 d^5 e^5 + \\
 & \quad 97020 a^6 b^4 d^4 e^6 + 39600 a^7 b^3 d^3 e^7 + 7425 a^8 b^2 d^2 e^8 + 550 a^9 b d e^9 + 11 a^{10} e^{10})) x^{11} + \\
 & \frac{1}{12} (116424 a^5 b^5 d^5 e^5 (B d + A e) + 19800 a^7 b^3 d^3 e^7 (2 B d + A e) + 2475 a^8 b^2 d^2 e^8 (3 B d + A e) + \\
 & \quad 110 a^9 b d e^9 (5 B d + A e) + a^{10} e^{10} (11 B d + A e) + 19800 a^3 b^7 d^7 e^3 (B d + 2 A e) + \\
 & \quad 2475 a^2 b^8 d^8 e^2 (B d + 3 A e) + 110 a b^9 d^9 e (B d + 5 A e) + 13860 a^6 b^4 d^4 e^6 (7 B d + 5 A e) + \\
 & \quad 13860 a^4 b^6 d^6 e^4 (5 B d + 7 A e) + b^{10} d^{10} (B d + 11 A e)) x^{12} + \\
 & \frac{1}{13} e (a^{10} B e^{10} + 97020 a^4 b^6 d^5 e^4 (B d + A e) + 34650 a^6 b^4 d^3 e^6 (2 B d + A e) + \\
 & \quad 6600 a^7 b^3 d^2 e^7 (3 B d + A e) + 495 a^8 b^2 d e^8 (5 B d + A e) + 10 a^9 b e^9 (11 B d + A e) + \\
 & \quad 7425 a^2 b^8 d^7 e^2 (B d + 2 A e) + 550 a b^9 d^8 e (B d + 3 A e) + 11 b^{10} d^9 (B d + 5 A e) + \\
 & \quad 16632 a^5 b^5 d^4 e^5 (7 B d + 5 A e) + 7920 a^3 b^7 d^6 e^3 (5 B d + 7 A e)) x^{13} + \\
 & \frac{5}{14} b e^2 (2 a^9 B e^9 + 11088 a^3 b^6 d^5 e^3 (B d + A e) + 8316 a^5 b^4 d^3 e^5 (2 B d + A e) + \\
 & \quad 2310 a^6 b^3 d^2 e^6 (3 B d + A e) + 264 a^7 b^2 d e^7 (5 B d + A e) + \\
 & \quad 9 a^8 b e^8 (11 B d + A e) + 330 a b^8 d^7 e (B d + 2 A e) + 11 b^9 d^8 (B d + 3 A e) + \\
 & \quad 2772 a^4 b^5 d^4 e^4 (7 B d + 5 A e) + 594 a^2 b^7 d^6 e^2 (5 B d + 7 A e)) x^{14} + \\
 & b^2 e^3 (3 a^8 B e^8 + 1386 a^2 b^6 d^5 e^2 (B d + A e) + 2310 a^4 b^4 d^3 e^4 (2 B d + A e) + \\
 & \quad 924 a^5 b^3 d^2 e^5 (3 B d + A e) + 154 a^6 b^2 d e^6 (5 B d + A e) + 8 a^7 b e^7 (11 B d + A e) + \\
 & \quad 11 b^8 d^7 (B d + 2 A e) + 528 a^3 b^5 d^4 e^3 (7 B d + 5 A e) + 44 a b^7 d^6 e (5 B d + 7 A e)) x^{15} + \\
 & \frac{3}{8} b^3 e^4 (20 a^7 B e^7 + 770 a b^6 d^5 e (B d + A e) + 3300 a^3 b^4 d^3 e^3 (2 B d + A e) +
 \end{aligned}$$

$$\begin{aligned}
& 1925 a^4 b^3 d^2 e^4 (3 B d + A e) + 462 a^5 b^2 d e^5 (5 B d + A e) + 35 a^6 b e^6 (11 B d + A e) + \\
& 495 a^2 b^5 d^4 e^2 (7 B d + 5 A e) + 11 b^7 d^6 (5 B d + 7 A e) x^{16} + \\
& \frac{3}{17} b^4 e^5 (70 a^6 B e^6 + 154 b^6 d^5 (B d + A e) + 2475 a^2 b^4 d^3 e^2 (2 B d + A e) + 2200 a^3 b^3 d^2 e^3 (3 B d + A e) + \\
& 770 a^4 b^2 d e^4 (5 B d + A e) + 84 a^5 b e^5 (11 B d + A e) + 220 a b^5 d^4 e (7 B d + 5 A e)) x^{17} + \\
& \frac{1}{6} b^5 e^6 (84 a^5 B e^5 + 550 a b^4 d^3 e (2 B d + A e) + 825 a^2 b^3 d^2 e^2 (3 B d + A e) + \\
& 440 a^3 b^2 d e^3 (5 B d + A e) + 70 a^4 b e^4 (11 B d + A e) + 22 b^5 d^4 (7 B d + 5 A e)) x^{18} + \\
& \frac{5}{19} b^6 e^7 (42 a^4 B e^4 + 33 b^4 d^3 (2 B d + A e) + 110 a b^3 d^2 e (3 B d + A e) + \\
& 99 a^2 b^2 d e^2 (5 B d + A e) + 24 a^3 b e^3 (11 B d + A e)) x^{19} + \\
& \frac{1}{4} b^7 e^8 (24 a^3 B e^3 + 11 b^3 d^2 (3 B d + A e) + 22 a b^2 d e (5 B d + A e) + 9 a^2 b e^2 (11 B d + A e)) x^{20} + \\
& \frac{1}{21} b^8 e^9 (45 a^2 B e^2 + 11 b^2 d (5 B d + A e) + 10 a b e (11 B d + A e)) x^{21} + \\
& \frac{1}{22} b^9 e^{10} (11 b B d + A b e + 10 a B e) x^{22} + \\
& \frac{1}{23} b^{10} B e^{11} x^{23}
\end{aligned}$$

**Problem 1066: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^{10} dx$$

Optimal (type 1, 460 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(A b - a B) (b d - a e)^{10} (a + b x)^{11}}{11 b^{12}} + \frac{(b d - a e)^9 (b B d + 10 A b e - 11 a B e) (a + b x)^{12}}{12 b^{12}} + \\
 & \frac{5 e (b d - a e)^8 (2 b B d + 9 A b e - 11 a B e) (a + b x)^{13}}{13 b^{12}} + \\
 & \frac{15 e^2 (b d - a e)^7 (3 b B d + 8 A b e - 11 a B e) (a + b x)^{14}}{14 b^{12}} + \\
 & \frac{2 e^3 (b d - a e)^6 (4 b B d + 7 A b e - 11 a B e) (a + b x)^{15}}{b^{12}} + \\
 & \frac{21 e^4 (b d - a e)^5 (5 b B d + 6 A b e - 11 a B e) (a + b x)^{16}}{8 b^{12}} + \\
 & \frac{42 e^5 (b d - a e)^4 (6 b B d + 5 A b e - 11 a B e) (a + b x)^{17}}{17 b^{12}} + \\
 & \frac{5 e^6 (b d - a e)^3 (7 b B d + 4 A b e - 11 a B e) (a + b x)^{18}}{3 b^{12}} + \\
 & \frac{15 e^7 (b d - a e)^2 (8 b B d + 3 A b e - 11 a B e) (a + b x)^{19}}{19 b^{12}} + \\
 & \frac{e^8 (b d - a e) (9 b B d + 2 A b e - 11 a B e) (a + b x)^{20}}{4 b^{12}} + \\
 & \frac{e^9 (10 b B d + A b e - 11 a B e) (a + b x)^{21}}{21 b^{12}} + \frac{B e^{10} (a + b x)^{22}}{22 b^{12}}
 \end{aligned}$$

Result (type 1, 2815 leaves):

$$\begin{aligned}
 & a^{10} A d^{10} x + \frac{1}{2} a^9 d^9 (a B d + 10 A (b d + a e)) x^2 + \\
 & \frac{5}{3} a^8 d^8 (2 a B d (b d + a e) + A (9 b^2 d^2 + 20 a b d e + 9 a^2 e^2)) x^3 + \\
 & \frac{5}{4} a^7 d^7 (a B d (9 b^2 d^2 + 20 a b d e + 9 a^2 e^2) + 6 A (4 b^3 d^3 + 15 a b^2 d^2 e + 15 a^2 b d e^2 + 4 a^3 e^3)) x^4 + \\
 & 3 a^6 d^6 (2 a B d (4 b^3 d^3 + 15 a b^2 d^2 e + 15 a^2 b d e^2 + 4 a^3 e^3) + \\
 & \quad A (14 b^4 d^4 + 80 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 80 a^3 b d e^3 + 14 a^4 e^4)) x^5 + \\
 & \frac{1}{2} a^5 d^5 (5 a B d (14 b^4 d^4 + 80 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 80 a^3 b d e^3 + 14 a^4 e^4) + \\
 & \quad 4 A (21 b^5 d^5 + 175 a b^4 d^4 e + 450 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 175 a^4 b d e^4 + 21 a^5 e^5)) x^6 + \\
 & \frac{6}{7} a^4 d^4 (2 a B d (21 b^5 d^5 + 175 a b^4 d^4 e + 450 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 175 a^4 b d e^4 + 21 a^5 e^5) + \\
 & \quad 5 A (7 b^6 d^6 + 84 a b^5 d^5 e + 315 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + 315 a^4 b^2 d^2 e^4 + 84 a^5 b d e^5 + 7 a^6 e^6)) \\
 & x^7 + \frac{15}{4} a^3 d^3 (a B d (7 b^6 d^6 + 84 a b^5 d^5 e + 315 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + \\
 & \quad 315 a^4 b^2 d^2 e^4 + 84 a^5 b d e^5 + 7 a^6 e^6) + A (4 b^7 d^7 + 70 a b^6 d^6 e + 378 a^2 b^5 d^5 e^2 + \\
 & \quad 840 a^3 b^4 d^4 e^3 + 840 a^4 b^3 d^3 e^4 + 378 a^5 b^2 d^2 e^5 + 70 a^6 b d e^6 + 4 a^7 e^7)) x^8 + \\
 & \frac{5}{3} a^2 d^2 (4 a B d (2 b^7 d^7 + 35 a b^6 d^6 e + 189 a^2 b^5 d^5 e^2 + 420 a^3 b^4 d^4 e^3 + 420 a^4 b^3 d^3 e^4 + \\
 & \quad 189 a^5 b^2 d^2 e^5 + 35 a^6 b d e^6 + 2 a^7 e^7) + A (3 b^8 d^8 + 80 a b^7 d^7 e + 630 a^2 b^6 d^6 e^2 + \\
 & \quad 2016 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + 2016 a^5 b^3 d^3 e^5 + 630 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + 3 a^8 e^8)) x^9 +
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} a d \left( 3 a B d \left( 3 b^8 d^8 + 80 a b^7 d^7 e + 630 a^2 b^6 d^6 e^2 + 2016 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + \right. \right. \\
& \quad \left. \left. 2016 a^5 b^3 d^3 e^5 + 630 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + 3 a^8 e^8 \right) + \right. \\
& \quad \left. 2 A \left( b^9 d^9 + 45 a b^8 d^8 e + 540 a^2 b^7 d^7 e^2 + 2520 a^3 b^6 d^6 e^3 + 5292 a^4 b^5 d^5 e^4 + \right. \right. \\
& \quad \left. \left. 5292 a^5 b^4 d^4 e^5 + 2520 a^6 b^3 d^3 e^6 + 540 a^7 b^2 d^2 e^7 + 45 a^8 b d e^8 + a^9 e^9 \right) \right) x^{10} + \\
& \frac{1}{11} \left( 10 a B d \left( b^9 d^9 + 45 a b^8 d^8 e + 540 a^2 b^7 d^7 e^2 + 2520 a^3 b^6 d^6 e^3 + 5292 a^4 b^5 d^5 e^4 + \right. \right. \\
& \quad \left. \left. 5292 a^5 b^4 d^4 e^5 + 2520 a^6 b^3 d^3 e^6 + 540 a^7 b^2 d^2 e^7 + 45 a^8 b d e^8 + a^9 e^9 \right) + \right. \\
& \quad \left. A \left( b^{10} d^{10} + 100 a b^9 d^9 e + 2025 a^2 b^8 d^8 e^2 + 14400 a^3 b^7 d^7 e^3 + 44100 a^4 b^6 d^6 e^4 + 63504 a^5 b^5 d^5 e^5 + \right. \right. \\
& \quad \left. \left. 44100 a^6 b^4 d^4 e^6 + 14400 a^7 b^3 d^3 e^7 + 2025 a^8 b^2 d^2 e^8 + 100 a^9 b d e^9 + a^{10} e^{10} \right) \right) x^{11} + \\
& \frac{1}{12} \left( a^{10} B e^{10} + 10 a^9 b e^9 \left( 10 B d + A e \right) + 225 a^8 b^2 d e^8 \left( 9 B d + 2 A e \right) + 1800 a^7 b^3 d^2 e^7 \left( 8 B d + 3 A e \right) + \right. \\
& \quad 6300 a^6 b^4 d^3 e^6 \left( 7 B d + 4 A e \right) + 10584 a^5 b^5 d^4 e^5 \left( 6 B d + 5 A e \right) + \\
& \quad 8820 a^4 b^6 d^5 e^4 \left( 5 B d + 6 A e \right) + 3600 a^3 b^7 d^6 e^3 \left( 4 B d + 7 A e \right) + \\
& \quad \left. 675 a^2 b^8 d^7 e^2 \left( 3 B d + 8 A e \right) + 50 a b^9 d^8 e \left( 2 B d + 9 A e \right) + b^{10} d^9 \left( B d + 10 A e \right) \right) x^{12} + \\
& \frac{5}{13} b e \left( 2 a^9 B e^9 + 9 a^8 b e^8 \left( 10 B d + A e \right) + 120 a^7 b^2 d e^7 \left( 9 B d + 2 A e \right) + 630 a^6 b^3 d^2 e^6 \left( 8 B d + 3 A e \right) + \right. \\
& \quad 1512 a^5 b^4 d^3 e^5 \left( 7 B d + 4 A e \right) + 1764 a^4 b^5 d^4 e^4 \left( 6 B d + 5 A e \right) + 1008 a^3 b^6 d^5 e^3 \left( 5 B d + 6 A e \right) + \\
& \quad \left. 270 a^2 b^7 d^6 e^2 \left( 4 B d + 7 A e \right) + 30 a b^8 d^7 e \left( 3 B d + 8 A e \right) + b^9 d^8 \left( 2 B d + 9 A e \right) \right) x^{13} + \\
& \frac{15}{14} b^2 e^2 \left( 3 a^8 B e^8 + 8 a^7 b e^7 \left( 10 B d + A e \right) + 70 a^6 b^2 d e^6 \left( 9 B d + 2 A e \right) + \right. \\
& \quad 252 a^5 b^3 d^2 e^5 \left( 8 B d + 3 A e \right) + 420 a^4 b^4 d^3 e^4 \left( 7 B d + 4 A e \right) + 336 a^3 b^5 d^4 e^3 \left( 6 B d + 5 A e \right) + \\
& \quad 126 a^2 b^6 d^5 e^2 \left( 5 B d + 6 A e \right) + 20 a b^7 d^6 e \left( 4 B d + 7 A e \right) + b^8 d^7 \left( 3 B d + 8 A e \right) \right) x^{14} + \\
& 2 b^3 e^3 \left( 4 a^7 B e^7 + 7 a^6 b e^6 \left( 10 B d + A e \right) + 42 a^5 b^2 d e^5 \left( 9 B d + 2 A e \right) + \right. \\
& \quad 105 a^4 b^3 d^2 e^4 \left( 8 B d + 3 A e \right) + 120 a^3 b^4 d^3 e^3 \left( 7 B d + 4 A e \right) + 63 a^2 b^5 d^4 e^2 \left( 6 B d + 5 A e \right) + \\
& \quad \left. 14 a b^6 d^5 e \left( 5 B d + 6 A e \right) + b^7 d^6 \left( 4 B d + 7 A e \right) \right) x^{15} + \frac{3}{8} b^4 e^4 \\
& \quad \left( 35 a^6 B e^6 + 42 a^5 b e^5 \left( 10 B d + A e \right) + 175 a^4 b^2 d e^4 \left( 9 B d + 2 A e \right) + 300 a^3 b^3 d^2 e^3 \left( 8 B d + 3 A e \right) + \right. \\
& \quad \left. 225 a^2 b^4 d^3 e^2 \left( 7 B d + 4 A e \right) + 70 a b^5 d^4 e \left( 6 B d + 5 A e \right) + 7 b^6 d^5 \left( 5 B d + 6 A e \right) \right) x^{16} + \\
& \frac{3}{17} b^5 e^5 \left( 84 a^5 B e^5 + 70 a^4 b e^4 \left( 10 B d + A e \right) + 200 a^3 b^2 d e^3 \left( 9 B d + 2 A e \right) + \right. \\
& \quad \left. 225 a^2 b^3 d^2 e^2 \left( 8 B d + 3 A e \right) + 100 a b^4 d^3 e \left( 7 B d + 4 A e \right) + 14 b^5 d^4 \left( 6 B d + 5 A e \right) \right) x^{17} + \\
& \frac{5}{6} b^6 e^6 \left( 14 a^4 B e^4 + 8 a^3 b e^3 \left( 10 B d + A e \right) + 15 a^2 b^2 d e^2 \left( 9 B d + 2 A e \right) + \right. \\
& \quad \left. 10 a b^3 d^2 e \left( 8 B d + 3 A e \right) + 2 b^4 d^3 \left( 7 B d + 4 A e \right) \right) x^{18} + \\
& \frac{5}{19} b^7 e^7 \left( 24 a^3 B e^3 + 9 a^2 b e^2 \left( 10 B d + A e \right) + 10 a b^2 d e \left( 9 B d + 2 A e \right) + 3 b^3 d^2 \left( 8 B d + 3 A e \right) \right) x^{19} + \\
& \frac{1}{4} b^8 e^8 \left( 9 a^2 B e^2 + 2 a b e \left( 10 B d + A e \right) + b^2 d \left( 9 B d + 2 A e \right) \right) x^{20} + \\
& \frac{1}{21} b^9 e^9 \left( 10 b B d + A b e + 10 a B e \right) x^{21} + \\
& \frac{1}{22} b^{10} B e^{10} x^{22}
\end{aligned}$$

**Problem 1067:** Result more than twice size of optimal antiderivative.

$$\int (a+bx)^{10} (A+Bx) (d+ex)^9 dx$$

Optimal (type 1, 415 leaves, 2 steps):

$$\begin{aligned} & \frac{(Ab - aB) (bd - ae)^9 (a+bx)^{11}}{11 b^{11}} + \frac{(bd - ae)^8 (bBd + 9Abe - 10aBe) (a+bx)^{12}}{12 b^{11}} + \\ & \frac{9e (bd - ae)^7 (bBd + 4Abe - 5aBe) (a+bx)^{13}}{13 b^{11}} + \\ & \frac{6e^2 (bd - ae)^6 (3bBd + 7Abe - 10aBe) (a+bx)^{14}}{7 b^{11}} + \\ & \frac{14e^3 (bd - ae)^5 (2bBd + 3Abe - 5aBe) (a+bx)^{15}}{5 b^{11}} + \\ & \frac{63e^4 (bd - ae)^4 (bBd + Abe - 2aBe) (a+bx)^{16}}{8 b^{11}} + \\ & \frac{42e^5 (bd - ae)^3 (3bBd + 2Abe - 5aBe) (a+bx)^{17}}{17 b^{11}} + \\ & \frac{2e^6 (bd - ae)^2 (7bBd + 3Abe - 10aBe) (a+bx)^{18}}{3 b^{11}} + \\ & \frac{9e^7 (bd - ae) (4bBd + Abe - 5aBe) (a+bx)^{19}}{19 b^{11}} + \\ & \frac{e^8 (9bBd + Abe - 10aBe) (a+bx)^{20}}{20 b^{11}} + \frac{Be^9 (a+bx)^{21}}{21 b^{11}} \end{aligned}$$

Result (type 1, 2553 leaves):

$$\begin{aligned} & a^{10} A d^9 x + \frac{1}{2} a^9 d^8 (10 A b d + a B d + 9 a A e) x^2 + \\ & \frac{1}{3} a^8 d^7 (a B d (10 b d + 9 a e) + 9 A (5 b^2 d^2 + 10 a b d e + 4 a^2 e^2)) x^3 + \\ & \frac{3}{4} a^7 d^6 (3 a B d (5 b^2 d^2 + 10 a b d e + 4 a^2 e^2) + A (40 b^3 d^3 + 135 a b^2 d^2 e + 120 a^2 b d e^2 + 28 a^3 e^3)) x^4 + \\ & \frac{3}{5} a^6 d^5 (a B d (40 b^3 d^3 + 135 a b^2 d^2 e + 120 a^2 b d e^2 + 28 a^3 e^3) + \\ & A (70 b^4 d^4 + 360 a b^3 d^3 e + 540 a^2 b^2 d^2 e^2 + 280 a^3 b d e^3 + 42 a^4 e^4)) x^5 + \\ & a^5 d^4 (a B d (35 b^4 d^4 + 180 a b^3 d^3 e + 270 a^2 b^2 d^2 e^2 + 140 a^3 b d e^3 + 21 a^4 e^4) + \\ & 3 A (14 b^5 d^5 + 105 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 210 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5)) x^6 + \\ & \frac{6}{7} a^4 d^3 (3 a B d (14 b^5 d^5 + 105 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 210 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 7 a^5 e^5) + \\ & 7 A (5 b^6 d^6 + 54 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 240 a^3 b^3 d^3 e^3 + 135 a^4 b^2 d^2 e^4 + 30 a^5 b d e^5 + 2 a^6 e^6)) \\ & x^7 + \frac{3}{4} a^3 d^2 (7 a B d (5 b^6 d^6 + 54 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 240 a^3 b^3 d^3 e^3 + 135 a^4 b^2 d^2 e^4 + \\ & 30 a^5 b d e^5 + 2 a^6 e^6) + A (20 b^7 d^7 + 315 a b^6 d^6 e + 1512 a^2 b^5 d^5 e^2 + \\ & 2940 a^3 b^4 d^4 e^3 + 2520 a^4 b^3 d^3 e^4 + 945 a^5 b^2 d^2 e^5 + 140 a^6 b d e^6 + 6 a^7 e^7)) x^8 + \\ & \frac{1}{3} a^2 d (2 a B d (20 b^7 d^7 + 315 a b^6 d^6 e + 1512 a^2 b^5 d^5 e^2 + 2940 a^3 b^4 d^4 e^3 + 2520 a^4 b^3 d^3 e^4 + \\ & 945 a^5 b^2 d^2 e^5 + 140 a^6 b d e^6 + 6 a^7 e^7) + 3 A (5 b^8 d^8 + 120 a b^7 d^7 e + 840 a^2 b^6 d^6 e^2 + \end{aligned}$$

$$\begin{aligned}
 & 2352 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + 1680 a^5 b^3 d^3 e^5 + 420 a^6 b^2 d^2 e^6 + 40 a^7 b d e^7 + a^8 e^8) x^9 + \\
 & \frac{1}{10} a (9 a B d (5 b^8 d^8 + 120 a b^7 d^7 e + 840 a^2 b^6 d^6 e^2 + 2352 a^3 b^5 d^5 e^3 + 2940 a^4 b^4 d^4 e^4 + \\
 & \quad 1680 a^5 b^3 d^3 e^5 + 420 a^6 b^2 d^2 e^6 + 40 a^7 b d e^7 + a^8 e^8) + \\
 & \quad A (10 b^9 d^9 + 405 a b^8 d^8 e + 4320 a^2 b^7 d^7 e^2 + 17640 a^3 b^6 d^6 e^3 + 31752 a^4 b^5 d^5 e^4 + \\
 & \quad 26460 a^5 b^4 d^4 e^5 + 10080 a^6 b^3 d^3 e^6 + 1620 a^7 b^2 d^2 e^7 + 90 a^8 b d e^8 + a^9 e^9)) x^{10} + \\
 & \frac{1}{11} (a B (10 b^9 d^9 + 405 a b^8 d^8 e + 4320 a^2 b^7 d^7 e^2 + 17640 a^3 b^6 d^6 e^3 + 31752 a^4 b^5 d^5 e^4 + \\
 & \quad 26460 a^5 b^4 d^4 e^5 + 10080 a^6 b^3 d^3 e^6 + 1620 a^7 b^2 d^2 e^7 + 90 a^8 b d e^8 + a^9 e^9) + \\
 & \quad A b (b^9 d^9 + 90 a b^8 d^8 e + 1620 a^2 b^7 d^7 e^2 + 10080 a^3 b^6 d^6 e^3 + 26460 a^4 b^5 d^5 e^4 + \\
 & \quad 31752 a^5 b^4 d^4 e^5 + 17640 a^6 b^3 d^3 e^6 + 4320 a^7 b^2 d^2 e^7 + 405 a^8 b d e^8 + 10 a^9 e^9)) x^{11} + \\
 & \frac{1}{12} b (10 a^9 B e^9 + 26460 a^4 b^5 d^4 e^4 (B d + A e) + 1080 a^7 b^2 d e^7 (4 B d + A e) + 45 a^8 b e^8 (9 B d + A e) + \\
 & \quad 10584 a^5 b^4 d^3 e^5 (3 B d + 2 A e) + 5040 a^3 b^6 d^5 e^3 (2 B d + 3 A e) + 2520 a^6 b^3 d^2 e^6 (7 B d + 3 A e) + \\
 & \quad 90 a b^8 d^7 e (B d + 4 A e) + 540 a^2 b^7 d^6 e^2 (3 B d + 7 A e) + b^9 d^8 (B d + 9 A e)) x^{12} + \\
 & \frac{3}{13} b^2 e (15 a^8 B e^8 + 5040 a^3 b^5 d^4 e^3 (B d + A e) + 630 a^6 b^2 d e^6 (4 B d + A e) + \\
 & \quad 40 a^7 b e^7 (9 B d + A e) + 2940 a^4 b^4 d^3 e^4 (3 B d + 2 A e) + 630 a^2 b^6 d^5 e^2 (2 B d + 3 A e) + \\
 & \quad 1008 a^5 b^3 d^2 e^5 (7 B d + 3 A e) + 3 b^8 d^7 (B d + 4 A e) + 40 a b^7 d^6 e (3 B d + 7 A e)) x^{13} + \\
 & \frac{3}{7} b^3 e^2 (20 a^7 B e^7 + 945 a^2 b^5 d^4 e^2 (B d + A e) + 378 a^5 b^2 d e^5 (4 B d + A e) + \\
 & \quad 35 a^6 b e^6 (9 B d + A e) + 840 a^3 b^4 d^3 e^3 (3 B d + 2 A e) + 70 a b^6 d^5 e (2 B d + 3 A e) + \\
 & \quad 420 a^4 b^3 d^2 e^4 (7 B d + 3 A e) + 2 b^7 d^6 (3 B d + 7 A e)) x^{14} + \\
 & \frac{2}{5} b^4 e^3 (35 a^6 B e^6 + 210 a b^5 d^4 e (B d + A e) + 315 a^4 b^2 d e^4 (4 B d + A e) + 42 a^5 b e^5 (9 B d + A e) + \\
 & \quad 315 a^2 b^4 d^3 e^2 (3 B d + 2 A e) + 7 b^6 d^5 (2 B d + 3 A e) + 240 a^3 b^3 d^2 e^3 (7 B d + 3 A e)) x^{15} + \\
 & \frac{3}{8} b^5 e^4 (42 a^5 B e^5 + 21 b^5 d^4 (B d + A e) + 180 a^3 b^2 d e^3 (4 B d + A e) + 35 a^4 b e^4 (9 B d + A e) + \\
 & \quad 70 a b^4 d^3 e (3 B d + 2 A e) + 90 a^2 b^3 d^2 e^2 (7 B d + 3 A e)) x^{16} + \\
 & \frac{3}{17} b^6 e^5 (70 a^4 B e^4 + 135 a^2 b^2 d e^2 (4 B d + A e) + 40 a^3 b e^3 (9 B d + A e) + \\
 & \quad 14 b^4 d^3 (3 B d + 2 A e) + 40 a b^3 d^2 e (7 B d + 3 A e)) x^{17} + \\
 & \frac{1}{6} b^7 e^6 (40 a^3 B e^3 + 30 a b^2 d e (4 B d + A e) + 15 a^2 b e^2 (9 B d + A e) + 4 b^3 d^2 (7 B d + 3 A e)) x^{18} + \\
 & \frac{1}{19} b^8 e^7 (45 a^2 B e^2 + 9 b^2 d (4 B d + A e) + 10 a b e (9 B d + A e)) x^{19} + \\
 & \frac{1}{20} b^9 e^8 (9 b B d + A b e + 10 a B e) x^{20} + \\
 & \frac{1}{21} b^{10} B e^9 x^{21}
 \end{aligned}$$

**Problem 1068: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^8 dx$$

Optimal (type 1, 372 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(A b - a B) (b d - a e)^8 (a + b x)^{11}}{11 b^{10}} + \frac{(b d - a e)^7 (b B d + 8 A b e - 9 a B e) (a + b x)^{12}}{12 b^{10}} + \\
 & \frac{4 e (b d - a e)^6 (2 b B d + 7 A b e - 9 a B e) (a + b x)^{13}}{13 b^{10}} + \\
 & \frac{2 e^2 (b d - a e)^5 (b B d + 2 A b e - 3 a B e) (a + b x)^{14}}{b^{10}} + \\
 & \frac{14 e^3 (b d - a e)^4 (4 b B d + 5 A b e - 9 a B e) (a + b x)^{15}}{15 b^{10}} + \\
 & \frac{7 e^4 (b d - a e)^3 (5 b B d + 4 A b e - 9 a B e) (a + b x)^{16}}{8 b^{10}} + \\
 & \frac{28 e^5 (b d - a e)^2 (2 b B d + A b e - 3 a B e) (a + b x)^{17}}{17 b^{10}} + \\
 & \frac{2 e^6 (b d - a e) (7 b B d + 2 A b e - 9 a B e) (a + b x)^{18}}{9 b^{10}} + \\
 & \frac{e^7 (8 b B d + A b e - 9 a B e) (a + b x)^{19}}{19 b^{10}} + \frac{B e^8 (a + b x)^{20}}{20 b^{10}}
 \end{aligned}$$

Result (type 1, 2307 leaves):

$$\begin{aligned}
 & a^{10} A d^8 x + \frac{1}{2} a^9 d^7 (10 A b d + a B d + 8 a A e) x^2 + \\
 & \frac{1}{3} a^8 d^6 (2 a B d (5 b d + 4 a e) + A (45 b^2 d^2 + 80 a b d e + 28 a^2 e^2)) x^3 + \\
 & \frac{1}{4} a^7 d^5 (a B d (45 b^2 d^2 + 80 a b d e + 28 a^2 e^2) + 8 A (15 b^3 d^3 + 45 a b^2 d^2 e + 35 a^2 b d e^2 + 7 a^3 e^3)) x^4 + \\
 & \frac{2}{5} a^6 d^4 (4 a B d (15 b^3 d^3 + 45 a b^2 d^2 e + 35 a^2 b d e^2 + 7 a^3 e^3) + \\
 & \quad 5 A (21 b^4 d^4 + 96 a b^3 d^3 e + 126 a^2 b^2 d^2 e^2 + 56 a^3 b d e^3 + 7 a^4 e^4)) x^5 + \\
 & \frac{1}{3} a^5 d^3 (5 a B d (21 b^4 d^4 + 96 a b^3 d^3 e + 126 a^2 b^2 d^2 e^2 + 56 a^3 b d e^3 + 7 a^4 e^4) + \\
 & \quad 14 A (9 b^5 d^5 + 60 a b^4 d^4 e + 120 a^2 b^3 d^3 e^2 + 90 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + 2 a^5 e^5)) x^6 + \\
 & 2 a^4 d^2 (2 a B d (9 b^5 d^5 + 60 a b^4 d^4 e + 120 a^2 b^3 d^3 e^2 + 90 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + 2 a^5 e^5) + \\
 & \quad A (15 b^6 d^6 + 144 a b^5 d^5 e + 420 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 40 a^5 b d e^5 + 2 a^6 e^6)) \\
 & x^7 + \frac{1}{4} a^3 d (7 a B d (15 b^6 d^6 + 144 a b^5 d^5 e + 420 a^2 b^4 d^4 e^2 + 480 a^3 b^3 d^3 e^3 + \\
 & \quad 225 a^4 b^2 d^2 e^4 + 40 a^5 b d e^5 + 2 a^6 e^6) + 4 A (15 b^7 d^7 + 210 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + \\
 & \quad 1470 a^3 b^4 d^4 e^3 + 1050 a^4 b^3 d^3 e^4 + 315 a^5 b^2 d^2 e^5 + 35 a^6 b d e^6 + a^7 e^7)) x^8 + \\
 & \frac{1}{9} a^2 (8 a B d (15 b^7 d^7 + 210 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + 1470 a^3 b^4 d^4 e^3 + 1050 a^4 b^3 d^3 e^4 + \\
 & \quad 315 a^5 b^2 d^2 e^5 + 35 a^6 b d e^6 + a^7 e^7) + A (45 b^8 d^8 + 960 a b^7 d^7 e + 5880 a^2 b^6 d^6 e^2 + \\
 & \quad 14112 a^3 b^5 d^5 e^3 + 14700 a^4 b^4 d^4 e^4 + 6720 a^5 b^3 d^3 e^5 + 1260 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + a^8 e^8)) \\
 & x^9 + \frac{1}{10} a (10 A b (b^8 d^8 + 36 a b^7 d^7 e + 336 a^2 b^6 d^6 e^2 + 1176 a^3 b^5 d^5 e^3 + 1764 a^4 b^4 d^4 e^4 + \\
 & \quad 1176 a^5 b^3 d^3 e^5 + 336 a^6 b^2 d^2 e^6 + 36 a^7 b d e^7 + a^8 e^8) + \\
 & \quad a B (45 b^8 d^8 + 960 a b^7 d^7 e + 5880 a^2 b^6 d^6 e^2 + 14112 a^3 b^5 d^5 e^3 + 14700 a^4 b^4 d^4 e^4 +
 \end{aligned}$$

$$\begin{aligned}
& 6720 a^5 b^3 d^3 e^5 + 1260 a^6 b^2 d^2 e^6 + 80 a^7 b d e^7 + a^8 e^8) x^{10} + \\
\frac{1}{11} b & (10 a B (b^8 d^8 + 36 a b^7 d^7 e + 336 a^2 b^6 d^6 e^2 + 1176 a^3 b^5 d^5 e^3 + 1764 a^4 b^4 d^4 e^4 + 1176 a^5 b^3 d^3 e^5 + \\
& 336 a^6 b^2 d^2 e^6 + 36 a^7 b d e^7 + a^8 e^8) + A b (b^8 d^8 + 80 a b^7 d^7 e + 1260 a^2 b^6 d^6 e^2 + 6720 a^3 b^5 d^5 e^3 + \\
& 14700 a^4 b^4 d^4 e^4 + 14112 a^5 b^3 d^3 e^5 + 5880 a^6 b^2 d^2 e^6 + 960 a^7 b d e^7 + 45 a^8 e^8)) x^{11} + \\
\frac{1}{12} b^2 & (45 a^8 B e^8 + 7056 a^5 b^3 d^2 e^5 (2 B d + A e) + 120 a^7 b e^7 (8 B d + A e) + \\
& 1260 a^2 b^6 d^5 e^2 (B d + 2 A e) + 840 a^6 b^2 d e^6 (7 B d + 2 A e) + 2940 a^4 b^4 d^3 e^4 (5 B d + 4 A e) + \\
& 1680 a^3 b^5 d^4 e^3 (4 B d + 5 A e) + 40 a b^7 d^6 e (2 B d + 7 A e) + b^8 d^7 (B d + 8 A e)) x^{12} + \\
\frac{2}{13} b^3 e & (60 a^7 B e^7 + 2940 a^4 b^3 d^2 e^4 (2 B d + A e) + 105 a^6 b e^6 (8 B d + A e) + \\
& 140 a b^6 d^5 e (B d + 2 A e) + 504 a^5 b^2 d e^5 (7 B d + 2 A e) + 840 a^3 b^4 d^3 e^3 (5 B d + 4 A e) + \\
& 315 a^2 b^5 d^4 e^2 (4 B d + 5 A e) + 2 b^7 d^6 (2 B d + 7 A e)) x^{13} + \\
b^4 e^2 & (15 a^6 B e^6 + 240 a^3 b^3 d^2 e^3 (2 B d + A e) + 18 a^5 b e^5 (8 B d + A e) + 2 b^6 d^5 (B d + 2 A e) + \\
& 60 a^4 b^2 d e^4 (7 B d + 2 A e) + 45 a^2 b^4 d^3 e^2 (5 B d + 4 A e) + 10 a b^5 d^4 e (4 B d + 5 A e)) x^{14} + \\
\frac{2}{15} b^5 e^3 & (126 a^5 B e^5 + 630 a^2 b^3 d^2 e^2 (2 B d + A e) + 105 a^4 b e^4 (8 B d + A e) + \\
& 240 a^3 b^2 d e^3 (7 B d + 2 A e) + 70 a b^4 d^3 e (5 B d + 4 A e) + 7 b^5 d^4 (4 B d + 5 A e)) x^{15} + \\
\frac{1}{8} b^6 e^4 & (105 a^4 B e^4 + 140 a b^3 d^2 e (2 B d + A e) + 60 a^3 b e^3 (8 B d + A e) + \\
& 90 a^2 b^2 d e^2 (7 B d + 2 A e) + 7 b^4 d^3 (5 B d + 4 A e)) x^{16} + \\
\frac{1}{17} b^7 e^5 & (120 a^3 B e^3 + 28 b^3 d^2 (2 B d + A e) + 45 a^2 b e^2 (8 B d + A e) + 40 a b^2 d e (7 B d + 2 A e)) x^{17} + \\
\frac{1}{18} b^8 e^6 & (45 a^2 B e^2 + 10 a b e (8 B d + A e) + 4 b^2 d (7 B d + 2 A e)) x^{18} + \\
\frac{1}{19} b^9 e^7 & (8 b B d + A b e + 10 a B e) x^{19} + \\
\frac{1}{20} b^{10} B e^8 x^{20}
\end{aligned}$$

**Problem 1069: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^7 dx$$

Optimal (type 1, 329 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(A b - a B) (b d - a e)^7 (a + b x)^{11}}{11 b^9} + \frac{(b d - a e)^6 (b B d + 7 A b e - 8 a B e) (a + b x)^{12}}{12 b^9} + \\
 & \frac{7 e (b d - a e)^5 (b B d + 3 A b e - 4 a B e) (a + b x)^{13}}{13 b^9} + \\
 & \frac{e^2 (b d - a e)^4 (3 b B d + 5 A b e - 8 a B e) (a + b x)^{14}}{2 b^9} + \\
 & \frac{7 e^3 (b d - a e)^3 (b B d + A b e - 2 a B e) (a + b x)^{15}}{3 b^9} + \\
 & \frac{7 e^4 (b d - a e)^2 (5 b B d + 3 A b e - 8 a B e) (a + b x)^{16}}{16 b^9} + \\
 & \frac{7 e^5 (b d - a e) (3 b B d + A b e - 4 a B e) (a + b x)^{17}}{17 b^9} + \\
 & \frac{e^6 (7 b B d + A b e - 8 a B e) (a + b x)^{18}}{18 b^9} + \frac{B e^7 (a + b x)^{19}}{19 b^9}
 \end{aligned}$$

Result (type 1, 2034 leaves):

$$\begin{aligned}
 & a^{10} A d^7 x + \frac{1}{2} a^9 d^6 (10 A b d + a B d + 7 a A e) x^2 + \\
 & \frac{1}{3} a^8 d^5 (a B d (10 b d + 7 a e) + A (45 b^2 d^2 + 70 a b d e + 21 a^2 e^2)) x^3 + \\
 & \frac{1}{4} a^7 d^4 (a B d (45 b^2 d^2 + 70 a b d e + 21 a^2 e^2) + 5 A (24 b^3 d^3 + 63 a b^2 d^2 e + 42 a^2 b d e^2 + 7 a^3 e^3)) x^4 + \\
 & a^6 d^3 (a B d (24 b^3 d^3 + 63 a b^2 d^2 e + 42 a^2 b d e^2 + 7 a^3 e^3) + \\
 & \quad 7 A (6 b^4 d^4 + 24 a b^3 d^3 e + 27 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4)) x^5 + \\
 & \frac{7}{6} a^5 d^2 (5 a B d (6 b^4 d^4 + 24 a b^3 d^3 e + 27 a^2 b^2 d^2 e^2 + 10 a^3 b d e^3 + a^4 e^4) + \\
 & \quad A (36 b^5 d^5 + 210 a b^4 d^4 e + 360 a^2 b^3 d^3 e^2 + 225 a^3 b^2 d^2 e^3 + 50 a^4 b d e^4 + 3 a^5 e^5)) x^6 + \\
 & a^4 d (a B d (36 b^5 d^5 + 210 a b^4 d^4 e + 360 a^2 b^3 d^3 e^2 + 225 a^3 b^2 d^2 e^3 + 50 a^4 b d e^4 + 3 a^5 e^5) + \\
 & \quad A (30 b^6 d^6 + 252 a b^5 d^5 e + 630 a^2 b^4 d^4 e^2 + 600 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + 30 a^5 b d e^5 + a^6 e^6)) x^7 + \\
 & \frac{1}{8} a^3 (7 a B d (30 b^6 d^6 + 252 a b^5 d^5 e + 630 a^2 b^4 d^4 e^2 + 600 a^3 b^3 d^3 e^3 + 225 a^4 b^2 d^2 e^4 + \\
 & \quad 30 a^5 b d e^5 + a^6 e^6) + A (120 b^7 d^7 + 1470 a b^6 d^6 e + 5292 a^2 b^5 d^5 e^2 + \\
 & \quad 7350 a^3 b^4 d^4 e^3 + 4200 a^4 b^3 d^3 e^4 + 945 a^5 b^2 d^2 e^5 + 70 a^6 b d e^6 + a^7 e^7)) x^8 + \\
 & \frac{1}{9} a^2 (a B (120 b^7 d^7 + 1470 a b^6 d^6 e + 5292 a^2 b^5 d^5 e^2 + 7350 a^3 b^4 d^4 e^3 + 4200 a^4 b^3 d^3 e^4 + \\
 & \quad 945 a^5 b^2 d^2 e^5 + 70 a^6 b d e^6 + a^7 e^7) + 5 A b (9 b^7 d^7 + 168 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + \\
 & \quad 1764 a^3 b^4 d^4 e^3 + 1470 a^4 b^3 d^3 e^4 + 504 a^5 b^2 d^2 e^5 + 63 a^6 b d e^6 + 2 a^7 e^7)) x^9 + \\
 & \frac{1}{2} a b (a B (9 b^7 d^7 + 168 a b^6 d^6 e + 882 a^2 b^5 d^5 e^2 + 1764 a^3 b^4 d^4 e^3 + 1470 a^4 b^3 d^3 e^4 + \\
 & \quad 504 a^5 b^2 d^2 e^5 + 63 a^6 b d e^6 + 2 a^7 e^7) + A b (2 b^7 d^7 + 63 a b^6 d^6 e + 504 a^2 b^5 d^5 e^2 + \\
 & \quad 1470 a^3 b^4 d^4 e^3 + 1764 a^4 b^3 d^3 e^4 + 882 a^5 b^2 d^2 e^5 + 168 a^6 b d e^6 + 9 a^7 e^7)) x^{10} + \\
 & \frac{1}{11} b^2 (5 a B (2 b^7 d^7 + 63 a b^6 d^6 e + 504 a^2 b^5 d^5 e^2 + 1470 a^3 b^4 d^4 e^3 + 1764 a^4 b^3 d^3 e^4 + \\
 & \quad 882 a^5 b^2 d^2 e^5 + 168 a^6 b d e^6 + 9 a^7 e^7) + A b (b^7 d^7 + 70 a b^6 d^6 e + 945 a^2 b^5 d^5 e^2 + \\
 & \quad 4200 a^3 b^4 d^4 e^3 + 7350 a^4 b^3 d^3 e^4 + 5292 a^5 b^2 d^2 e^5 + 1470 a^6 b d e^6 + 120 a^7 e^7)) x^{11} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{12} b^3 (120 a^7 B e^7 + 4200 a^3 b^4 d^3 e^3 (B d + A e) + 1764 a^5 b^2 d e^5 (3 B d + A e) + \\
 & \quad 210 a^6 b e^6 (7 B d + A e) + 70 a b^6 d^5 e (B d + 3 A e) + 1470 a^4 b^3 d^2 e^4 (5 B d + 3 A e) + \\
 & \quad 315 a^2 b^5 d^4 e^2 (3 B d + 5 A e) + b^7 d^6 (B d + 7 A e)) x^{12} + \\
 & \frac{7}{13} b^4 e (30 a^6 B e^6 + 225 a^2 b^4 d^3 e^2 (B d + A e) + 210 a^4 b^2 d e^4 (3 B d + A e) + 36 a^5 b e^5 (7 B d + A e) + \\
 & \quad b^6 d^5 (B d + 3 A e) + 120 a^3 b^3 d^2 e^3 (5 B d + 3 A e) + 10 a b^5 d^4 e (3 B d + 5 A e)) x^{13} + \\
 & \frac{1}{2} b^5 e^2 (36 a^5 B e^5 + 50 a b^4 d^3 e (B d + A e) + 120 a^3 b^2 d e^3 (3 B d + A e) + \\
 & \quad 30 a^4 b e^4 (7 B d + A e) + 45 a^2 b^3 d^2 e^2 (5 B d + 3 A e) + b^5 d^4 (3 B d + 5 A e)) x^{14} + \\
 & \frac{1}{3} b^6 e^3 (42 a^4 B e^4 + 7 b^4 d^3 (B d + A e) + 63 a^2 b^2 d e^2 (3 B d + A e) + \\
 & \quad 24 a^3 b e^3 (7 B d + A e) + 14 a b^3 d^2 e (5 B d + 3 A e)) x^{15} + \\
 & \frac{1}{16} b^7 e^4 (120 a^3 B e^3 + 70 a b^2 d e (3 B d + A e) + 45 a^2 b e^2 (7 B d + A e) + 7 b^3 d^2 (5 B d + 3 A e)) x^{16} + \\
 & \frac{1}{17} b^8 e^5 (45 a^2 B e^2 + 7 b^2 d (3 B d + A e) + 10 a b e (7 B d + A e)) x^{17} + \\
 & \frac{1}{18} b^9 e^6 (7 b B d + A b e + 10 a B e) x^{18} + \\
 & \frac{1}{19} b^{10} B e^7 x^{19}
 \end{aligned}$$

**Problem 1070: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^6 dx$$

Optimal (type 1, 290 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(A b - a B) (b d - a e)^6 (a + b x)^{11}}{11 b^8} + \frac{(b d - a e)^5 (b B d + 6 A b e - 7 a B e) (a + b x)^{12}}{12 b^8} + \\
 & \frac{3 e (b d - a e)^4 (2 b B d + 5 A b e - 7 a B e) (a + b x)^{13}}{13 b^8} + \\
 & \frac{5 e^2 (b d - a e)^3 (3 b B d + 4 A b e - 7 a B e) (a + b x)^{14}}{14 b^8} + \\
 & \frac{e^3 (b d - a e)^2 (4 b B d + 3 A b e - 7 a B e) (a + b x)^{15}}{3 b^8} + \\
 & \frac{3 e^4 (b d - a e) (5 b B d + 2 A b e - 7 a B e) (a + b x)^{16}}{16 b^8} + \\
 & \frac{e^5 (6 b B d + A b e - 7 a B e) (a + b x)^{17}}{17 b^8} + \frac{B e^6 (a + b x)^{18}}{18 b^8}
 \end{aligned}$$

Result (type 1, 1788 leaves):

$$\begin{aligned}
 & a^{10} A d^6 x + \frac{1}{2} a^9 d^5 (10 A b d + a B d + 6 a A e) x^2 + \\
 & \frac{1}{3} a^8 d^4 (2 a B d (5 b d + 3 a e) + 15 A (3 b^2 d^2 + 4 a b d e + a^2 e^2)) x^3 + \\
 & \frac{5}{4} a^7 d^3 (3 a B d (3 b^2 d^2 + 4 a b d e + a^2 e^2) + A (24 b^3 d^3 + 54 a b^2 d^2 e + 30 a^2 b d e^2 + 4 a^3 e^3)) x^4 + \\
 & a^6 d^2 (2 a B d (12 b^3 d^3 + 27 a b^2 d^2 e + 15 a^2 b d e^2 + 2 a^3 e^3) + \\
 & \quad A (42 b^4 d^4 + 144 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 40 a^3 b d e^3 + 3 a^4 e^4)) x^5 + \\
 & \frac{1}{6} a^5 d (5 a B d (42 b^4 d^4 + 144 a b^3 d^3 e + 135 a^2 b^2 d^2 e^2 + 40 a^3 b d e^3 + 3 a^4 e^4) + \\
 & \quad 6 A (42 b^5 d^5 + 210 a b^4 d^4 e + 300 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + a^5 e^5)) x^6 + \frac{1}{7} a^4 \\
 & (6 a B d (42 b^5 d^5 + 210 a b^4 d^4 e + 300 a^2 b^3 d^3 e^2 + 150 a^3 b^2 d^2 e^3 + 25 a^4 b d e^4 + a^5 e^5) + A (210 b^6 d^6 + \\
 & \quad 1512 a b^5 d^5 e + 3150 a^2 b^4 d^4 e^2 + 2400 a^3 b^3 d^3 e^3 + 675 a^4 b^2 d^2 e^4 + 60 a^5 b d e^5 + a^6 e^6)) x^7 + \frac{1}{8} a^3 \\
 & (10 A b (12 b^6 d^6 + 126 a b^5 d^5 e + 378 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 180 a^4 b^2 d^2 e^4 + 27 a^5 b d e^5 + a^6 e^6) + \\
 & \quad a B (210 b^6 d^6 + 1512 a b^5 d^5 e + 3150 a^2 b^4 d^4 e^2 + \\
 & \quad 2400 a^3 b^3 d^3 e^3 + 675 a^4 b^2 d^2 e^4 + 60 a^5 b d e^5 + a^6 e^6)) x^8 + \frac{5}{9} a^2 b \\
 & (9 A b (b^6 d^6 + 16 a b^5 d^5 e + 70 a^2 b^4 d^4 e^2 + 112 a^3 b^3 d^3 e^3 + 70 a^4 b^2 d^2 e^4 + 16 a^5 b d e^5 + a^6 e^6) + 2 a B \\
 & \quad (12 b^6 d^6 + 126 a b^5 d^5 e + 378 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 180 a^4 b^2 d^2 e^4 + 27 a^5 b d e^5 + a^6 e^6)) x^9 + \\
 & \frac{1}{2} a b^2 (9 a B (b^6 d^6 + 16 a b^5 d^5 e + 70 a^2 b^4 d^4 e^2 + 112 a^3 b^3 d^3 e^3 + 70 a^4 b^2 d^2 e^4 + 16 a^5 b d e^5 + a^6 e^6) + \\
 & \quad 2 a B (b^6 d^6 + 27 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 378 a^4 b^2 d^2 e^4 + 126 a^5 b d e^5 + 12 a^6 e^6)) \\
 & x^{10} + \frac{1}{11} b^3 (10 a B (b^6 d^6 + 27 a b^5 d^5 e + 180 a^2 b^4 d^4 e^2 + 420 a^3 b^3 d^3 e^3 + 378 a^4 b^2 d^2 e^4 + \\
 & \quad 126 a^5 b d e^5 + 12 a^6 e^6) + A b (b^6 d^6 + 60 a b^5 d^5 e + 675 a^2 b^4 d^4 e^2 + \\
 & \quad 2400 a^3 b^3 d^3 e^3 + 3150 a^4 b^2 d^2 e^4 + 1512 a^5 b d e^5 + 210 a^6 e^6)) x^{11} + \frac{1}{12} b^4 \\
 & (210 a^6 B e^6 + 252 a^5 b e^5 (6 B d + A e) + 630 a^4 b^2 d e^4 (5 B d + 2 A e) + 600 a^3 b^3 d^2 e^3 (4 B d + 3 A e) + \\
 & \quad 225 a^2 b^4 d^3 e^2 (3 B d + 4 A e) + 30 a b^5 d^4 e (2 B d + 5 A e) + b^6 d^5 (B d + 6 A e)) x^{12} + \\
 & \frac{1}{13} b^5 e (252 a^5 B e^5 + 210 a^4 b e^4 (6 B d + A e) + 360 a^3 b^2 d e^3 (5 B d + 2 A e) + \\
 & \quad 225 a^2 b^3 d^2 e^2 (4 B d + 3 A e) + 50 a b^4 d^3 e (3 B d + 4 A e) + 3 b^5 d^4 (2 B d + 5 A e)) x^{13} + \\
 & \frac{5}{14} b^6 e^2 (42 a^4 B e^4 + 24 a^3 b e^3 (6 B d + A e) + 27 a^2 b^2 d e^2 (5 B d + 2 A e) + \\
 & \quad 10 a b^3 d^2 e (4 B d + 3 A e) + b^4 d^3 (3 B d + 4 A e)) x^{14} + \\
 & \frac{1}{3} b^7 e^3 (24 a^3 B e^3 + 9 a^2 b e^2 (6 B d + A e) + 6 a b^2 d e (5 B d + 2 A e) + b^3 d^2 (4 B d + 3 A e)) x^{15} + \\
 & \frac{1}{16} b^8 e^4 (45 a^2 B e^2 + 10 a b e (6 B d + A e) + 3 b^2 d (5 B d + 2 A e)) x^{16} + \\
 & \frac{1}{17} b^9 e^5 (6 b B d + A b e + 10 a B e) x^{17} + \\
 & \frac{1}{18} b^{10} B e^6 x^{18}
 \end{aligned}$$

Problem 1071: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (A + B x) (d + e x)^5 dx$$

Optimal (type 1, 243 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e)^5 (a + b x)^{11}}{11 b^7} + \frac{(b d - a e)^4 (b B d + 5 A b e - 6 a B e) (a + b x)^{12}}{12 b^7} +$$

$$\frac{5 e (b d - a e)^3 (b B d + 2 A b e - 3 a B e) (a + b x)^{13}}{13 b^7} +$$

$$\frac{5 e^2 (b d - a e)^2 (b B d + A b e - 2 a B e) (a + b x)^{14}}{7 b^7} + \frac{e^3 (b d - a e) (2 b B d + A b e - 3 a B e) (a + b x)^{15}}{3 b^7} +$$

$$\frac{e^4 (5 b B d + A b e - 6 a B e) (a + b x)^{16}}{16 b^7} + \frac{B e^5 (a + b x)^{17}}{17 b^7}$$

Result (type 1, 1509 leaves):

$$\begin{aligned}
 & a^{10} A d^5 x + \frac{1}{2} a^9 d^4 (a B d + 5 A (2 b d + a e)) x^2 + \\
 & \frac{5}{3} a^8 d^3 (a B d (2 b d + a e) + A (9 b^2 d^2 + 10 a b d e + 2 a^2 e^2)) x^3 + \\
 & \frac{5}{4} a^7 d^2 (a B d (9 b^2 d^2 + 10 a b d e + 2 a^2 e^2) + A (24 b^3 d^3 + 45 a b^2 d^2 e + 20 a^2 b d e^2 + 2 a^3 e^3)) x^4 + \\
 & a^6 d (a B d (24 b^3 d^3 + 45 a b^2 d^2 e + 20 a^2 b d e^2 + 2 a^3 e^3) + \\
 & \quad A (42 b^4 d^4 + 120 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 20 a^3 b d e^3 + a^4 e^4)) x^5 + \\
 & \frac{1}{6} a^5 (5 a B d (42 b^4 d^4 + 120 a b^3 d^3 e + 90 a^2 b^2 d^2 e^2 + 20 a^3 b d e^3 + a^4 e^4) + \\
 & \quad A (252 b^5 d^5 + 1050 a b^4 d^4 e + 1200 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 50 a^4 b d e^4 + a^5 e^5)) x^6 + \\
 & \frac{1}{7} a^4 (a B (252 b^5 d^5 + 1050 a b^4 d^4 e + 1200 a^2 b^3 d^3 e^2 + 450 a^3 b^2 d^2 e^3 + 50 a^4 b d e^4 + a^5 e^5) + \\
 & \quad 5 A b (42 b^5 d^5 + 252 a b^4 d^4 e + 420 a^2 b^3 d^3 e^2 + 240 a^3 b^2 d^2 e^3 + 45 a^4 b d e^4 + 2 a^5 e^5)) x^7 + \\
 & \frac{5}{8} a^3 b (a B (42 b^5 d^5 + 252 a b^4 d^4 e + 420 a^2 b^3 d^3 e^2 + 240 a^3 b^2 d^2 e^3 + 45 a^4 b d e^4 + 2 a^5 e^5) + \\
 & \quad 3 A b (8 b^5 d^5 + 70 a b^4 d^4 e + 168 a^2 b^3 d^3 e^2 + 140 a^3 b^2 d^2 e^3 + 40 a^4 b d e^4 + 3 a^5 e^5)) x^8 + \\
 & \frac{5}{3} a^2 b^2 (a B (8 b^5 d^5 + 70 a b^4 d^4 e + 168 a^2 b^3 d^3 e^2 + 140 a^3 b^2 d^2 e^3 + 40 a^4 b d e^4 + 3 a^5 e^5) + \\
 & \quad A b (3 b^5 d^5 + 40 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 168 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 8 a^5 e^5)) x^9 + \\
 & \frac{1}{2} a b^3 (3 a B (3 b^5 d^5 + 40 a b^4 d^4 e + 140 a^2 b^3 d^3 e^2 + 168 a^3 b^2 d^2 e^3 + 70 a^4 b d e^4 + 8 a^5 e^5) + \\
 & \quad A b (2 b^5 d^5 + 45 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 252 a^4 b d e^4 + 42 a^5 e^5)) x^{10} + \\
 & \frac{1}{11} b^4 (5 a B (2 b^5 d^5 + 45 a b^4 d^4 e + 240 a^2 b^3 d^3 e^2 + 420 a^3 b^2 d^2 e^3 + 252 a^4 b d e^4 + 42 a^5 e^5) + \\
 & \quad A b (b^5 d^5 + 50 a b^4 d^4 e + 450 a^2 b^3 d^3 e^2 + 1200 a^3 b^2 d^2 e^3 + 1050 a^4 b d e^4 + 252 a^5 e^5)) x^{11} + \\
 & \frac{1}{12} b^5 (252 a^5 B e^5 + 450 a^2 b^3 d^2 e^2 (B d + A e) + 600 a^3 b^2 d e^3 (2 B d + A e) + \\
 & \quad 210 a^4 b e^4 (5 B d + A e) + 50 a b^4 d^3 e (B d + 2 A e) + b^5 d^4 (B d + 5 A e)) x^{12} + \\
 & \frac{5}{13} b^6 e (42 a^4 B e^4 + 20 a b^3 d^2 e (B d + A e) + 45 a^2 b^2 d e^2 (2 B d + A e) + \\
 & \quad 24 a^3 b e^3 (5 B d + A e) + b^4 d^3 (B d + 2 A e)) x^{13} + \\
 & \frac{5}{14} b^7 e^2 (24 a^3 B e^3 + 2 b^3 d^2 (B d + A e) + 10 a b^2 d e (2 B d + A e) + 9 a^2 b e^2 (5 B d + A e)) x^{14} + \\
 & \frac{1}{3} b^8 e^3 (9 a^2 B e^2 + b^2 d (2 B d + A e) + 2 a b e (5 B d + A e)) x^{15} + \\
 & \frac{1}{16} b^9 e^4 (5 b B d + A b e + 10 a B e) x^{16} + \frac{1}{17} b^{10} B e^5 x^{17}
 \end{aligned}$$

**Problem 1072: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^4 dx$$

Optimal (type 1, 204 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e)^4 (a + b x)^{11}}{11 b^6} + \frac{(b d - a e)^3 (b B d + 4 A b e - 5 a B e) (a + b x)^{12}}{12 b^6} +$$

$$\frac{2 e (b d - a e)^2 (2 b B d + 3 A b e - 5 a B e) (a + b x)^{13}}{13 b^6} +$$

$$\frac{e^2 (b d - a e) (3 b B d + 2 A b e - 5 a B e) (a + b x)^{14}}{7 b^6} +$$

$$\frac{e^3 (4 b B d + A b e - 5 a B e) (a + b x)^{15}}{15 b^6} + \frac{B e^4 (a + b x)^{16}}{16 b^6}$$

Result (type 1, 1098 leaves):

$$\frac{1}{240240} x \left( 8008 a^{10} \left( 6 A \left( 5 d^4 + 10 d^3 e x + 10 d^2 e^2 x^2 + 5 d e^3 x^3 + e^4 x^4 \right) + \right. \right.$$

$$B x \left( 15 d^4 + 40 d^3 e x + 45 d^2 e^2 x^2 + 24 d e^3 x^3 + 5 e^4 x^4 \right) \left. \right) +$$

$$11440 a^9 b x \left( 7 A \left( 15 d^4 + 40 d^3 e x + 45 d^2 e^2 x^2 + 24 d e^3 x^3 + 5 e^4 x^4 \right) + \right.$$

$$2 B x \left( 35 d^4 + 105 d^3 e x + 126 d^2 e^2 x^2 + 70 d e^3 x^3 + 15 e^4 x^4 \right) \left. \right) +$$

$$12870 a^8 b^2 x^2 \left( 8 A \left( 35 d^4 + 105 d^3 e x + 126 d^2 e^2 x^2 + 70 d e^3 x^3 + 15 e^4 x^4 \right) + \right.$$

$$3 B x \left( 70 d^4 + 224 d^3 e x + 280 d^2 e^2 x^2 + 160 d e^3 x^3 + 35 e^4 x^4 \right) \left. \right) +$$

$$11440 a^7 b^3 x^3 \left( 9 A \left( 70 d^4 + 224 d^3 e x + 280 d^2 e^2 x^2 + 160 d e^3 x^3 + 35 e^4 x^4 \right) + \right.$$

$$4 B x \left( 126 d^4 + 420 d^3 e x + 540 d^2 e^2 x^2 + 315 d e^3 x^3 + 70 e^4 x^4 \right) \left. \right) +$$

$$40040 a^6 b^4 x^4 \left( 2 A \left( 126 d^4 + 420 d^3 e x + 540 d^2 e^2 x^2 + 315 d e^3 x^3 + 70 e^4 x^4 \right) + \right.$$

$$B x \left( 210 d^4 + 720 d^3 e x + 945 d^2 e^2 x^2 + 560 d e^3 x^3 + 126 e^4 x^4 \right) \left. \right) +$$

$$4368 a^5 b^5 x^5 \left( 11 A \left( 210 d^4 + 720 d^3 e x + 945 d^2 e^2 x^2 + 560 d e^3 x^3 + 126 e^4 x^4 \right) + \right.$$

$$6 B x \left( 330 d^4 + 1155 d^3 e x + 1540 d^2 e^2 x^2 + 924 d e^3 x^3 + 210 e^4 x^4 \right) \left. \right) +$$

$$1820 a^4 b^6 x^6 \left( 12 A \left( 330 d^4 + 1155 d^3 e x + 1540 d^2 e^2 x^2 + 924 d e^3 x^3 + 210 e^4 x^4 \right) + \right.$$

$$7 B x \left( 495 d^4 + 1760 d^3 e x + 2376 d^2 e^2 x^2 + 1440 d e^3 x^3 + 330 e^4 x^4 \right) \left. \right) +$$

$$560 a^3 b^7 x^7 \left( 13 A \left( 495 d^4 + 1760 d^3 e x + 2376 d^2 e^2 x^2 + 1440 d e^3 x^3 + 330 e^4 x^4 \right) + \right.$$

$$8 B x \left( 715 d^4 + 2574 d^3 e x + 3510 d^2 e^2 x^2 + 2145 d e^3 x^3 + 495 e^4 x^4 \right) \left. \right) +$$

$$120 a^2 b^8 x^8 \left( 14 A \left( 715 d^4 + 2574 d^3 e x + 3510 d^2 e^2 x^2 + 2145 d e^3 x^3 + 495 e^4 x^4 \right) + \right.$$

$$9 B x \left( 1001 d^4 + 3640 d^3 e x + 5005 d^2 e^2 x^2 + 3080 d e^3 x^3 + 715 e^4 x^4 \right) \left. \right) +$$

$$80 a b^9 x^9 \left( 3 A \left( 1001 d^4 + 3640 d^3 e x + 5005 d^2 e^2 x^2 + 3080 d e^3 x^3 + 715 e^4 x^4 \right) + \right.$$

$$2 B x \left( 1365 d^4 + 5005 d^3 e x + 6930 d^2 e^2 x^2 + 4290 d e^3 x^3 + 1001 e^4 x^4 \right) \left. \right) +$$

$$b^{10} x^{10} \left( 16 A \left( 1365 d^4 + 5005 d^3 e x + 6930 d^2 e^2 x^2 + 4290 d e^3 x^3 + 1001 e^4 x^4 \right) + \right.$$

$$\left. \left. 11 B x \left( 1820 d^4 + 6720 d^3 e x + 9360 d^2 e^2 x^2 + 5824 d e^3 x^3 + 1365 e^4 x^4 \right) \right) \right)$$

**Problem 1073: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^3 dx$$

Optimal (type 1, 159 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e)^3 (a + b x)^{11}}{11 b^5} + \frac{(b d - a e)^2 (b B d + 3 A b e - 4 a B e) (a + b x)^{12}}{12 b^5} +$$

$$\frac{3 e (b d - a e) (b B d + A b e - 2 a B e) (a + b x)^{13}}{13 b^5} +$$

$$\frac{e^2 (3 b B d + A b e - 4 a B e) (a + b x)^{14}}{14 b^5} + \frac{B e^3 (a + b x)^{15}}{15 b^5}$$

Result (type 1, 855 leaves):

$$\frac{1}{60060}$$

$$\times \left( 3003 a^{10} \left( 5 A \left( 4 d^3 + 6 d^2 e x + 4 d e^2 x^2 + e^3 x^3 \right) + B x \left( 10 d^3 + 20 d^2 e x + 15 d e^2 x^2 + 4 e^3 x^3 \right) \right) + 10010 a^9 b x \left( 3 A \left( 10 d^3 + 20 d^2 e x + 15 d e^2 x^2 + 4 e^3 x^3 \right) + B x \left( 20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 10 e^3 x^3 \right) \right) + 6435 a^8 b^2 x^2 \left( 7 A \left( 20 d^3 + 45 d^2 e x + 36 d e^2 x^2 + 10 e^3 x^3 \right) + 3 B x \left( 35 d^3 + 84 d^2 e x + 70 d e^2 x^2 + 20 e^3 x^3 \right) \right) + 25740 a^7 b^3 x^3 \left( 2 A \left( 35 d^3 + 84 d^2 e x + 70 d e^2 x^2 + 20 e^3 x^3 \right) + B x \left( 56 d^3 + 140 d^2 e x + 120 d e^2 x^2 + 35 e^3 x^3 \right) \right) + 5005 a^6 b^4 x^4 \left( 9 A \left( 56 d^3 + 140 d^2 e x + 120 d e^2 x^2 + 35 e^3 x^3 \right) + 5 B x \left( 84 d^3 + 216 d^2 e x + 189 d e^2 x^2 + 56 e^3 x^3 \right) \right) + 6006 a^5 b^5 x^5 \left( 5 A \left( 84 d^3 + 216 d^2 e x + 189 d e^2 x^2 + 56 e^3 x^3 \right) + 3 B x \left( 120 d^3 + 315 d^2 e x + 280 d e^2 x^2 + 84 e^3 x^3 \right) \right) + 1365 a^4 b^6 x^6 \left( 11 A \left( 120 d^3 + 315 d^2 e x + 280 d e^2 x^2 + 84 e^3 x^3 \right) + 7 B x \left( 165 d^3 + 440 d^2 e x + 396 d e^2 x^2 + 120 e^3 x^3 \right) \right) + 1820 a^3 b^7 x^7 \left( 3 A \left( 165 d^3 + 440 d^2 e x + 396 d e^2 x^2 + 120 e^3 x^3 \right) + 2 B x \left( 220 d^3 + 594 d^2 e x + 540 d e^2 x^2 + 165 e^3 x^3 \right) \right) + 105 a^2 b^8 x^8 \left( 13 A \left( 220 d^3 + 594 d^2 e x + 540 d e^2 x^2 + 165 e^3 x^3 \right) + 9 B x \left( 286 d^3 + 780 d^2 e x + 715 d e^2 x^2 + 220 e^3 x^3 \right) \right) + 30 a b^9 x^9 \left( 7 A \left( 286 d^3 + 780 d^2 e x + 715 d e^2 x^2 + 220 e^3 x^3 \right) + 5 B x \left( 364 d^3 + 1001 d^2 e x + 924 d e^2 x^2 + 286 e^3 x^3 \right) \right) + b^{10} x^{10} \left( 15 A \left( 364 d^3 + 1001 d^2 e x + 924 d e^2 x^2 + 286 e^3 x^3 \right) + 11 B x \left( 455 d^3 + 1260 d^2 e x + 1170 d e^2 x^2 + 364 e^3 x^3 \right) \right) \right)$$

**Problem 1074: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x)^2 dx$$

Optimal (type 1, 118 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e)^2 (a + b x)^{11}}{11 b^4} + \frac{(b d - a e) (b B d + 2 A b e - 3 a B e) (a + b x)^{12}}{12 b^4} +$$

$$\frac{e (2 b B d + A b e - 3 a B e) (a + b x)^{13}}{13 b^4} + \frac{B e^2 (a + b x)^{14}}{14 b^4}$$

Result (type 1, 614 leaves):

$$\frac{1}{12012} x \left( 1001 a^{10} \left( 4 A \left( 3 d^2 + 3 d e x + e^2 x^2 \right) + B x \left( 6 d^2 + 8 d e x + 3 e^2 x^2 \right) \right) + \right. \\
2002 a^9 b x \left( 5 A \left( 6 d^2 + 8 d e x + 3 e^2 x^2 \right) + 2 B x \left( 10 d^2 + 15 d e x + 6 e^2 x^2 \right) \right) + \\
9009 a^8 b^2 x^2 \left( 2 A \left( 10 d^2 + 15 d e x + 6 e^2 x^2 \right) + B x \left( 15 d^2 + 24 d e x + 10 e^2 x^2 \right) \right) + \\
3432 a^7 b^3 x^3 \left( 7 A \left( 15 d^2 + 24 d e x + 10 e^2 x^2 \right) + 4 B x \left( 21 d^2 + 35 d e x + 15 e^2 x^2 \right) \right) + \\
3003 a^6 b^4 x^4 \left( 8 A \left( 21 d^2 + 35 d e x + 15 e^2 x^2 \right) + 5 B x \left( 28 d^2 + 48 d e x + 21 e^2 x^2 \right) \right) + \\
6006 a^5 b^5 x^5 \left( 3 A \left( 28 d^2 + 48 d e x + 21 e^2 x^2 \right) + 2 B x \left( 36 d^2 + 63 d e x + 28 e^2 x^2 \right) \right) + \\
1001 a^4 b^6 x^6 \left( 10 A \left( 36 d^2 + 63 d e x + 28 e^2 x^2 \right) + 7 B x \left( 45 d^2 + 80 d e x + 36 e^2 x^2 \right) \right) + \\
364 a^3 b^7 x^7 \left( 11 A \left( 45 d^2 + 80 d e x + 36 e^2 x^2 \right) + 8 B x \left( 55 d^2 + 99 d e x + 45 e^2 x^2 \right) \right) + \\
273 a^2 b^8 x^8 \left( 4 A \left( 55 d^2 + 99 d e x + 45 e^2 x^2 \right) + 3 B x \left( 66 d^2 + 120 d e x + 55 e^2 x^2 \right) \right) + \\
14 a b^9 x^9 \left( 13 A \left( 66 d^2 + 120 d e x + 55 e^2 x^2 \right) + 10 B x \left( 78 d^2 + 143 d e x + 66 e^2 x^2 \right) \right) + \\
b^{10} x^{10} \left( 14 A \left( 78 d^2 + 143 d e x + 66 e^2 x^2 \right) + 11 B x \left( 91 d^2 + 168 d e x + 78 e^2 x^2 \right) \right) \Big) )$$

**Problem 1075: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) (d + e x) dx$$

Optimal (type 1, 75 leaves, 2 steps):

$$\frac{(A b - a B) (b d - a e) (a + b x)^{11}}{11 b^3} + \frac{(b B d + A b e - 2 a B e) (a + b x)^{12}}{12 b^3} + \frac{B e (a + b x)^{13}}{13 b^3}$$

Result (type 1, 383 leaves):

$$\frac{1}{66} a b^9 x^{10} (66 A d + 60 B d x + 60 A e x + 55 B e x^2) + \\
\frac{1}{22} a^2 b^8 x^9 (110 A d + 99 B d x + 99 A e x + 90 B e x^2) + \frac{1}{6} a^{10} x (3 A (2 d + e x) + B x (3 d + 2 e x)) + \\
\frac{3}{4} a^8 b^2 x^3 (5 A (4 d + 3 e x) + 3 B x (5 d + 4 e x)) + \frac{5}{6} a^9 b x^2 (B x (4 d + 3 e x) + A (6 d + 4 e x)) + \\
2 a^7 b^3 x^4 (3 A (5 d + 4 e x) + 2 B x (6 d + 5 e x)) + a^6 b^4 x^5 (7 A (6 d + 5 e x) + 5 B x (7 d + 6 e x)) + \\
\frac{3}{2} a^5 b^5 x^6 (4 A (7 d + 6 e x) + 3 B x (8 d + 7 e x)) + \frac{5}{12} a^4 b^6 x^7 (9 A (8 d + 7 e x) + 7 B x (9 d + 8 e x)) + \\
\frac{1}{3} a^3 b^7 x^8 (5 A (9 d + 8 e x) + 4 B x (10 d + 9 e x)) + \frac{b^{10} x^{11} (13 A (12 d + 11 e x) + 11 B x (13 d + 12 e x))}{1716}$$

**Problem 1076: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{10} (A + B x) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(A b - a B) (a + b x)^{11}}{11 b^2} + \frac{B (a + b x)^{12}}{12 b^2}$$

Result (type 1, 198 leaves):

$$\frac{1}{132} x (66 a^{10} (2 A + B x) + 220 a^9 b x (3 A + 2 B x) + 495 a^8 b^2 x^2 (4 A + 3 B x) + 792 a^7 b^3 x^3 (5 A + 4 B x) + 924 a^6 b^4 x^4 (6 A + 5 B x) + 792 a^5 b^5 x^5 (7 A + 6 B x) + 495 a^4 b^6 x^6 (8 A + 7 B x) + 220 a^3 b^7 x^7 (9 A + 8 B x) + 66 a^2 b^8 x^8 (10 A + 9 B x) + 12 a b^9 x^9 (11 A + 10 B x) + b^{10} x^{10} (12 A + 11 B x))$$

Problem 1077: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^{10} (A+Bx)}{d+ex} dx$$

Optimal (type 3, 348 leaves, 2 steps):

$$\begin{aligned} & \frac{b (bd - ae)^9 (Bd - Ae) x}{e^{11}} - \frac{(bd - ae)^8 (Bd - Ae) (a + bx)^2}{2 e^{10}} + \frac{(bd - ae)^7 (Bd - Ae) (a + bx)^3}{3 e^9} - \\ & \frac{(bd - ae)^6 (Bd - Ae) (a + bx)^4}{4 e^8} + \frac{(bd - ae)^5 (Bd - Ae) (a + bx)^5}{5 e^7} - \\ & \frac{(bd - ae)^4 (Bd - Ae) (a + bx)^6}{6 e^6} + \frac{(bd - ae)^3 (Bd - Ae) (a + bx)^7}{7 e^5} - \\ & \frac{(bd - ae)^2 (Bd - Ae) (a + bx)^8}{8 e^4} + \frac{(bd - ae) (Bd - Ae) (a + bx)^9}{9 e^3} - \\ & \frac{(Bd - Ae) (a + bx)^{10}}{10 e^2} + \frac{B (a + bx)^{11}}{11 b e} - \frac{(bd - ae)^{10} (Bd - Ae) \text{Log}[d + ex]}{e^{12}} \end{aligned}$$

Result (type 3, 1252 leaves):

$$\frac{1}{27720 e^{11}} x \left( 27720 a^{10} B e^{10} + 138600 a^9 b e^9 \left( -2 B d + 2 A e + B e x \right) + \right. \\
207900 a^8 b^2 e^8 \left( 3 A e \left( -2 d + e x \right) + B \left( 6 d^2 - 3 d e x + 2 e^2 x^2 \right) \right) + \\
277200 a^7 b^3 e^7 \left( 2 A e \left( 6 d^2 - 3 d e x + 2 e^2 x^2 \right) + B \left( -12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3 \right) \right) + \\
97020 a^6 b^4 e^6 \left( 5 A e \left( -12 d^3 + 6 d^2 e x - 4 d e^2 x^2 + 3 e^3 x^3 \right) + \right. \\
\left. B \left( 60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4 \right) \right) + \\
116424 a^5 b^5 e^5 \left( A e \left( 60 d^4 - 30 d^3 e x + 20 d^2 e^2 x^2 - 15 d e^3 x^3 + 12 e^4 x^4 \right) + \right. \\
\left. B \left( -60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5 \right) \right) + \\
13860 a^4 b^6 e^4 \left( 7 A e \left( -60 d^5 + 30 d^4 e x - 20 d^3 e^2 x^2 + 15 d^2 e^3 x^3 - 12 d e^4 x^4 + 10 e^5 x^5 \right) + \right. \\
\left. B \left( 420 d^6 - 210 d^5 e x + 140 d^4 e^2 x^2 - 105 d^3 e^3 x^3 + 84 d^2 e^4 x^4 - 70 d e^5 x^5 + 60 e^6 x^6 \right) \right) + 3960 a^3 \\
b^7 e^3 \left( 2 A e \left( 420 d^6 - 210 d^5 e x + 140 d^4 e^2 x^2 - 105 d^3 e^3 x^3 + 84 d^2 e^4 x^4 - 70 d e^5 x^5 + 60 e^6 x^6 \right) + \right. \\
\left. B \left( -840 d^7 + 420 d^6 e x - 280 d^5 e^2 x^2 + 210 d^4 e^3 x^3 - \right. \right. \\
\left. \left. 168 d^3 e^4 x^4 + 140 d^2 e^5 x^5 - 120 d e^6 x^6 + 105 e^7 x^7 \right) \right) + \\
495 a^2 b^8 e^2 \left( 3 A e \left( -840 d^7 + 420 d^6 e x - 280 d^5 e^2 x^2 + 210 d^4 e^3 x^3 - 168 d^3 e^4 x^4 + \right. \right. \\
\left. \left. 140 d^2 e^5 x^5 - 120 d e^6 x^6 + 105 e^7 x^7 \right) + B \left( 2520 d^8 - 1260 d^7 e x + 840 d^6 e^2 x^2 - \right. \right. \\
\left. \left. 630 d^5 e^3 x^3 + 504 d^4 e^4 x^4 - 420 d^3 e^5 x^5 + 360 d^2 e^6 x^6 - 315 d e^7 x^7 + 280 e^8 x^8 \right) \right) + \\
110 a b^9 e \left( A e \left( 2520 d^8 - 1260 d^7 e x + 840 d^6 e^2 x^2 - 630 d^5 e^3 x^3 + 504 d^4 e^4 x^4 - 420 d^3 e^5 x^5 + \right. \right. \\
\left. \left. 360 d^2 e^6 x^6 - 315 d e^7 x^7 + 280 e^8 x^8 \right) + B \left( -2520 d^9 + 1260 d^8 e x - 840 d^7 e^2 x^2 + 630 d^6 e^3 \right. \right. \\
\left. \left. x^3 - 504 d^5 e^4 x^4 + 420 d^4 e^5 x^5 - 360 d^3 e^6 x^6 + 315 d^2 e^7 x^7 - 280 d e^8 x^8 + 252 e^9 x^9 \right) \right) + \\
b^{10} \left( 11 A e \left( -2520 d^9 + 1260 d^8 e x - 840 d^7 e^2 x^2 + 630 d^6 e^3 x^3 - 504 d^5 e^4 x^4 + 420 d^4 e^5 x^5 - \right. \right. \\
\left. \left. 360 d^3 e^6 x^6 + 315 d^2 e^7 x^7 - 280 d e^8 x^8 + 252 e^9 x^9 \right) + B \left( 27720 d^{10} - 13860 d^9 e x + \right. \right. \\
\left. \left. 9240 d^8 e^2 x^2 - 6930 d^7 e^3 x^3 + 5544 d^6 e^4 x^4 - 4620 d^5 e^5 x^5 + 3960 d^4 e^6 x^6 - 3465 d^3 e^7 x^7 + \right. \right. \\
\left. \left. 3080 d^2 e^8 x^8 - 2772 d e^9 x^9 + 2520 e^{10} x^{10} \right) \right) \Big) + \frac{(b d - a e)^{10} (-B d + A e) \operatorname{Log}[d + e x]}{e^{12}}$$

**Problem 1078: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^2} dx$$

Optimal (type 3, 445 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) x}{e^{11}} + \\
 & \frac{(b d - a e)^{10} (B d - A e)}{e^{12} (d + e x)} + \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e) (d + e x)^2}{2 e^{12}} - \\
 & \frac{10 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e) (d + e x)^3}{e^{12}} + \\
 & \frac{21 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e) (d + e x)^4}{2 e^{12}} - \\
 & \frac{42 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e) (d + e x)^5}{5 e^{12}} + \\
 & \frac{5 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e) (d + e x)^6}{e^{12}} - \\
 & \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e) (d + e x)^7}{7 e^{12}} + \\
 & \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^8}{8 e^{12}} - \frac{b^9 (11 b B d - A b e - 10 a B e) (d + e x)^9}{9 e^{12}} + \\
 & \frac{b^{10} B (d + e x)^{10}}{10 e^{12}} + \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e) \operatorname{Log}[d + e x]}{e^{12}}
 \end{aligned}$$

Result (type 3, 1486 leaves):

$$\frac{1}{2520 e^{12} (d + e x)} \left( -2520 a^{10} e^{10} (-B d + A e) + 25200 a^9 b e^9 (A d e + B (-d^2 + d e x + e^2 x^2)) + \right. \\
56700 a^8 b^2 e^8 (2 A e (-d^2 + d e x + e^2 x^2) + B (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3)) + 50400 a^7 b^3 e^7 \\
(3 A e (2 d^3 - 4 d^2 e x - 3 d e^2 x^2 + e^3 x^3) + 2 B (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4)) + \\
44100 a^6 b^4 e^6 (4 A e (-3 d^4 + 9 d^3 e x + 6 d^2 e^2 x^2 - 2 d e^3 x^3 + e^4 x^4) + \\
B (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5)) + \\
10584 a^5 b^5 e^5 (5 A e (12 d^5 - 48 d^4 e x - 30 d^3 e^2 x^2 + 10 d^2 e^3 x^3 - 5 d e^4 x^4 + 3 e^5 x^5) - \\
6 B (10 d^6 - 50 d^5 e x - 30 d^4 e^2 x^2 + 10 d^3 e^3 x^3 - 5 d^2 e^4 x^4 + 3 d e^5 x^5 - 2 e^6 x^6)) + \\
8820 a^4 b^6 e^4 (6 A e (-10 d^6 + 50 d^5 e x + 30 d^4 e^2 x^2 - 10 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 3 d e^5 x^5 + 2 e^6 x^6) + \\
B (60 d^7 - 360 d^6 e x - 210 d^5 e^2 x^2 + 70 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 21 d^2 e^5 x^5 - 14 d e^6 x^6 + 10 e^7 x^7)) + \\
720 a^3 b^7 e^3 (7 A e (60 d^7 - 360 d^6 e x - 210 d^5 e^2 x^2 + 70 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + \\
21 d^2 e^5 x^5 - 14 d e^6 x^6 + 10 e^7 x^7) - 4 B (105 d^8 - 735 d^7 e x - 420 d^6 e^2 x^2 + \\
140 d^5 e^3 x^3 - 70 d^4 e^4 x^4 + 42 d^3 e^5 x^5 - 28 d^2 e^6 x^6 + 20 d e^7 x^7 - 15 e^8 x^8)) + \\
135 a^2 b^8 e^2 (8 A e (-105 d^8 + 735 d^7 e x + 420 d^6 e^2 x^2 - 140 d^5 e^3 x^3 + 70 d^4 e^4 x^4 - 42 d^3 e^5 x^5 + \\
28 d^2 e^6 x^6 - 20 d e^7 x^7 + 15 e^8 x^8) + 3 B (280 d^9 - 2240 d^8 e x - 1260 d^7 e^2 x^2 + 420 d^6 e^3 x^3 - \\
210 d^5 e^4 x^4 + 126 d^4 e^5 x^5 - 84 d^3 e^6 x^6 + 60 d^2 e^7 x^7 - 45 d e^8 x^8 + 35 e^9 x^9)) + \\
10 a b^9 e (9 A e (280 d^9 - 2240 d^8 e x - 1260 d^7 e^2 x^2 + 420 d^6 e^3 x^3 - 210 d^5 e^4 x^4 + \\
126 d^4 e^5 x^5 - 84 d^3 e^6 x^6 + 60 d^2 e^7 x^7 - 45 d e^8 x^8 + 35 e^9 x^9) - \\
10 B (252 d^{10} - 2268 d^9 e x - 1260 d^8 e^2 x^2 + 420 d^7 e^3 x^3 - 210 d^6 e^4 x^4 + 126 d^5 e^5 x^5 - \\
84 d^4 e^6 x^6 + 60 d^3 e^7 x^7 - 45 d^2 e^8 x^8 + 35 d e^9 x^9 - 28 e^{10} x^{10})) + \\
b^{10} (10 A e (-252 d^{10} + 2268 d^9 e x + 1260 d^8 e^2 x^2 - 420 d^7 e^3 x^3 + 210 d^6 e^4 x^4 - \\
126 d^5 e^5 x^5 + 84 d^4 e^6 x^6 - 60 d^3 e^7 x^7 + 45 d^2 e^8 x^8 - 35 d e^9 x^9 + 28 e^{10} x^{10})) + \\
B (2520 d^{11} - 25200 d^{10} e x - 13860 d^9 e^2 x^2 + 4620 d^8 e^3 x^3 - 2310 d^7 e^4 x^4 + 1386 d^6 e^5 x^5 - \\
924 d^5 e^6 x^6 + 660 d^4 e^7 x^7 - 495 d^3 e^8 x^8 + 385 d^2 e^9 x^9 - 308 d e^{10} x^{10} + 252 e^{11} x^{11})) + \\
2520 (b d - a e)^9 (11 b B d - 10 A b e - a B e) (d + e x) \text{Log}[d + e x] \Big)$$

**Problem 1079: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^3} dx$$

Optimal (type 3, 445 leaves, 2 steps):

$$\begin{aligned}
 & \frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e) x}{e^{11}} + \frac{(b d - a e)^{10} (B d - A e)}{2 e^{12} (d + e x)^2} - \\
 & \frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{e^{12} (d + e x)} - \frac{15 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e) (d + e x)^2}{e^{12}} + \\
 & \frac{14 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e) (d + e x)^3}{e^{12}} - \\
 & \frac{21 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e) (d + e x)^4}{2 e^{12}} + \\
 & \frac{6 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e) (d + e x)^5}{e^{12}} - \\
 & \frac{5 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e) (d + e x)^6}{2 e^{12}} + \\
 & \frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d + e x)^7}{7 e^{12}} - \frac{b^9 (11 b B d - A b e - 10 a B e) (d + e x)^8}{8 e^{12}} + \\
 & \frac{b^{10} B (d + e x)^9}{9 e^{12}} - \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) \text{Log}[d + e x]}{e^{12}}
 \end{aligned}$$

Result (type 3, 1480 leaves):

$$\begin{aligned}
 & \frac{1}{504 e^{12} (d + e x)^2} \left( -252 a^{10} e^{10} (A e + B (d + 2 e x)) - 2520 a^9 b e^9 (A e (d + 2 e x) - B d (3 d + 4 e x)) + \right. \\
 & 11340 a^8 b^2 e^8 (A d e (3 d + 4 e x) + B (-5 d^3 - 4 d^2 e x + 4 d e^2 x^2 + 2 e^3 x^3)) + 30240 a^7 b^3 e^7 \\
 & (A e (-5 d^3 - 4 d^2 e x + 4 d e^2 x^2 + 2 e^3 x^3) + B (7 d^4 + 2 d^3 e x - 11 d^2 e^2 x^2 - 4 d e^3 x^3 + e^4 x^4)) + \\
 & 17640 a^6 b^4 e^6 (3 A e (7 d^4 + 2 d^3 e x - 11 d^2 e^2 x^2 - 4 d e^3 x^3 + e^4 x^4) + \\
 & B (-27 d^5 + 6 d^4 e x + 63 d^3 e^2 x^2 + 20 d^2 e^3 x^3 - 5 d e^4 x^4 + 2 e^5 x^5)) + \\
 & 10584 a^5 b^5 e^5 (2 A e (-27 d^5 + 6 d^4 e x + 63 d^3 e^2 x^2 + 20 d^2 e^3 x^3 - 5 d e^4 x^4 + 2 e^5 x^5) + \\
 & 3 B (22 d^6 - 16 d^5 e x - 68 d^4 e^2 x^2 - 20 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 2 d e^5 x^5 + e^6 x^6)) + \\
 & 5292 a^4 b^6 e^4 (5 A e (22 d^6 - 16 d^5 e x - 68 d^4 e^2 x^2 - 20 d^3 e^3 x^3 + 5 d^2 e^4 x^4 - 2 d e^5 x^5 + e^6 x^6) + \\
 & B (-130 d^7 + 160 d^6 e x + 500 d^5 e^2 x^2 + 140 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + 14 d^2 e^5 x^5 - 7 d e^6 x^6 + 4 e^7 x^7)) + \\
 & 1008 a^3 b^7 e^3 (3 A e (-130 d^7 + 160 d^6 e x + 500 d^5 e^2 x^2 + 140 d^4 e^3 x^3 - 35 d^3 e^4 x^4 + \\
 & 14 d^2 e^5 x^5 - 7 d e^6 x^6 + 4 e^7 x^7) + 2 B (225 d^8 - 390 d^7 e x - 1035 d^6 e^2 x^2 - \\
 & 280 d^5 e^3 x^3 + 70 d^4 e^4 x^4 - 28 d^3 e^5 x^5 + 14 d^2 e^6 x^6 - 8 d e^7 x^7 + 5 e^8 x^8)) + \\
 & 108 a^2 b^8 e^2 (7 A e (225 d^8 - 390 d^7 e x - 1035 d^6 e^2 x^2 - 280 d^5 e^3 x^3 + 70 d^4 e^4 x^4 - 28 d^3 e^5 x^5 + \\
 & 14 d^2 e^6 x^6 - 8 d e^7 x^7 + 5 e^8 x^8) - 3 B (595 d^9 - 1330 d^8 e x - 3185 d^7 e^2 x^2 - 840 d^6 e^3 x^3 + \\
 & 210 d^5 e^4 x^4 - 84 d^4 e^5 x^5 + 42 d^3 e^6 x^6 - 24 d^2 e^7 x^7 + 15 d e^8 x^8 - 10 e^9 x^9)) + \\
 & 18 a b^9 e (4 A e (-595 d^9 + 1330 d^8 e x + 3185 d^7 e^2 x^2 + 840 d^6 e^3 x^3 - 210 d^5 e^4 x^4 + \\
 & 84 d^4 e^5 x^5 - 42 d^3 e^6 x^6 + 24 d^2 e^7 x^7 - 15 d e^8 x^8 + 10 e^9 x^9) + \\
 & 5 B (532 d^{10} - 1456 d^9 e x - 3248 d^8 e^2 x^2 - 840 d^7 e^3 x^3 + 210 d^6 e^4 x^4 - 84 d^5 e^5 x^5 + \\
 & 42 d^4 e^6 x^6 - 24 d^3 e^7 x^7 + 15 d^2 e^8 x^8 - 10 d e^9 x^9 + 7 e^{10} x^{10})) + \\
 & b^{10} (9 A e (532 d^{10} - 1456 d^9 e x - 3248 d^8 e^2 x^2 - 840 d^7 e^3 x^3 + 210 d^6 e^4 x^4 - 84 d^5 e^5 x^5 + \\
 & 42 d^4 e^6 x^6 - 24 d^3 e^7 x^7 + 15 d^2 e^8 x^8 - 10 d e^9 x^9 + 7 e^{10} x^{10})) + \\
 & B (-5292 d^{11} + 17136 d^{10} e x + 36288 d^9 e^2 x^2 + 9240 d^8 e^3 x^3 - 2310 d^7 e^4 x^4 + 924 d^6 e^5 x^5 - \\
 & 462 d^5 e^6 x^6 + 264 d^4 e^7 x^7 - 165 d^3 e^8 x^8 + 110 d^2 e^9 x^9 - 77 d e^{10} x^{10} + 56 e^{11} x^{11})) - \\
 & 2520 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e) (d + e x)^2 \text{Log}[d + e x]
 \end{aligned}$$

### Problem 1086: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^{10} (A+Bx)}{(d+ex)^{10}} dx$$

Optimal (type 3, 441 leaves, 2 steps):

$$\begin{aligned} & - \frac{b^9 (10bBd - Abe - 10aBe) x}{e^{11}} + \frac{b^{10} B x^2}{2 e^{10}} + \frac{(bd - ae)^{10} (Bd - Ae)}{9 e^{12} (d + ex)^9} - \\ & \frac{(bd - ae)^9 (11bBd - 10Abe - aBe)}{8 e^{12} (d + ex)^8} + \frac{5b (bd - ae)^8 (11bBd - 9Abe - 2aBe)}{7 e^{12} (d + ex)^7} - \\ & \frac{5b^2 (bd - ae)^7 (11bBd - 8Abe - 3aBe)}{2 e^{12} (d + ex)^6} + \frac{6b^3 (bd - ae)^6 (11bBd - 7Abe - 4aBe)}{e^{12} (d + ex)^5} - \\ & \frac{21b^4 (bd - ae)^5 (11bBd - 6Abe - 5aBe)}{2 e^{12} (d + ex)^4} + \frac{14b^5 (bd - ae)^4 (11bBd - 5Abe - 6aBe)}{e^{12} (d + ex)^3} - \\ & \frac{15b^6 (bd - ae)^3 (11bBd - 4Abe - 7aBe)}{e^{12} (d + ex)^2} + \frac{15b^7 (bd - ae)^2 (11bBd - 3Abe - 8aBe)}{e^{12} (d + ex)} + \\ & \frac{5b^8 (bd - ae) (11bBd - 2Abe - 9aBe) \operatorname{Log}[d + ex]}{e^{12}} \end{aligned}$$

Result (type 3, 1460 leaves):

$$\begin{aligned}
 & \frac{1}{504 e^{12} (d+e x)^9} \\
 & \left( 7 a^{10} e^{10} (8 A e + B (d+9 e x)) + 10 a^9 b e^9 (7 A e (d+9 e x) + 2 B (d^2 + 9 d e x + 36 e^2 x^2)) + \right. \\
 & 45 a^8 b^2 e^8 (2 A e (d^2 + 9 d e x + 36 e^2 x^2) + B (d^3 + 9 d^2 e x + 36 d e^2 x^2 + 84 e^3 x^3)) + 24 a^7 b^3 e^7 \\
 & (5 A e (d^3 + 9 d^2 e x + 36 d e^2 x^2 + 84 e^3 x^3) + 4 B (d^4 + 9 d^3 e x + 36 d^2 e^2 x^2 + 84 d e^3 x^3 + 126 e^4 x^4)) + \\
 & 42 a^6 b^4 e^6 (4 A e (d^4 + 9 d^3 e x + 36 d^2 e^2 x^2 + 84 d e^3 x^3 + 126 e^4 x^4) + \\
 & 5 B (d^5 + 9 d^4 e x + 36 d^3 e^2 x^2 + 84 d^2 e^3 x^3 + 126 d e^4 x^4 + 126 e^5 x^5)) + \\
 & 252 a^5 b^5 e^5 (A e (d^5 + 9 d^4 e x + 36 d^3 e^2 x^2 + 84 d^2 e^3 x^3 + 126 d e^4 x^4 + 126 e^5 x^5) + \\
 & 2 B (d^6 + 9 d^5 e x + 36 d^4 e^2 x^2 + 84 d^3 e^3 x^3 + 126 d^2 e^4 x^4 + 126 d e^5 x^5 + 84 e^6 x^6)) + \\
 & 210 a^4 b^6 e^4 (2 A e (d^6 + 9 d^5 e x + 36 d^4 e^2 x^2 + 84 d^3 e^3 x^3 + 126 d^2 e^4 x^4 + 126 d e^5 x^5 + 84 e^6 x^6) + \\
 & 7 B (d^7 + 9 d^6 e x + 36 d^5 e^2 x^2 + 84 d^4 e^3 x^3 + 126 d^3 e^4 x^4 + 126 d^2 e^5 x^5 + 84 d e^6 x^6 + 36 e^7 x^7)) + \\
 & 840 a^3 b^7 e^3 (A e (d^7 + 9 d^6 e x + 36 d^5 e^2 x^2 + 84 d^4 e^3 x^3 + 126 d^3 e^4 x^4 + 126 d^2 e^5 x^5 + \\
 & 84 d e^6 x^6 + 36 e^7 x^7) + 8 B (d^8 + 9 d^7 e x + 36 d^6 e^2 x^2 + 84 d^5 e^3 x^3 + \\
 & 126 d^4 e^4 x^4 + 126 d^3 e^5 x^5 + 84 d^2 e^6 x^6 + 36 d e^7 x^7 + 9 e^8 x^8)) - \\
 & 9 a^2 b^8 e^2 (-280 A e (d^8 + 9 d^7 e x + 36 d^6 e^2 x^2 + 84 d^5 e^3 x^3 + 126 d^4 e^4 x^4 + 126 d^3 e^5 x^5 + 84 d^2 e^6 x^6 + \\
 & 36 d e^7 x^7 + 9 e^8 x^8) + B d (7129 d^8 + 61641 d^7 e x + 235224 d^6 e^2 x^2 + 518616 d^5 e^3 x^3 + \\
 & 725004 d^4 e^4 x^4 + 661500 d^3 e^5 x^5 + 388080 d^2 e^6 x^6 + 136080 d e^7 x^7 + 22680 e^8 x^8)) - \\
 & 2 a b^9 e (A d e (7129 d^8 + 61641 d^7 e x + 235224 d^6 e^2 x^2 + 518616 d^5 e^3 x^3 + 725004 d^4 e^4 x^4 + \\
 & 661500 d^3 e^5 x^5 + 388080 d^2 e^6 x^6 + 136080 d e^7 x^7 + 22680 e^8 x^8)) - \\
 & 10 B (4861 d^{10} + 41229 d^9 e x + 153576 d^8 e^2 x^2 + 328104 d^7 e^3 x^3 + 439236 d^6 e^4 x^4 + 375732 \\
 & d^5 e^5 x^5 + 197568 d^4 e^6 x^6 + 54432 d^3 e^7 x^7 + 2268 d^2 e^8 x^8 - 2268 d e^9 x^9 - 252 e^{10} x^{10})) - \\
 & b^{10} (-2 A e (4861 d^{10} + 41229 d^9 e x + 153576 d^8 e^2 x^2 + 328104 d^7 e^3 x^3 + 439236 d^6 e^4 x^4 + 375732 \\
 & d^5 e^5 x^5 + 197568 d^4 e^6 x^6 + 54432 d^3 e^7 x^7 + 2268 d^2 e^8 x^8 - 2268 d e^9 x^9 - 252 e^{10} x^{10})) + \\
 & B (42131 d^{11} + 351459 d^{10} e x + 1281096 d^9 e^2 x^2 + 2656584 d^8 e^3 x^3 + \\
 & 3402756 d^7 e^4 x^4 + 2704212 d^6 e^5 x^5 + 1220688 d^5 e^6 x^6 + 190512 d^4 e^7 x^7 - \\
 & 77112 d^3 e^8 x^8 - 36288 d^2 e^9 x^9 - 2772 d e^{10} x^{10} + 252 e^{11} x^{11})) - \\
 & 2520 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e) (d+e x)^9 \text{Log}[d+e x]
 \end{aligned}$$

**Problem 1087: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^{10} (A+Bx)}{(d+ex)^{11}} dx$$

Optimal (type 3, 446 leaves, 2 steps):

$$\frac{b^{10} B x}{e^{11}} + \frac{(b d - a e)^{10} (B d - A e)}{10 e^{12} (d + e x)^{10}} -$$

$$\frac{(b d - a e)^9 (11 b B d - 10 A b e - a B e)}{9 e^{12} (d + e x)^9} + \frac{5 b (b d - a e)^8 (11 b B d - 9 A b e - 2 a B e)}{8 e^{12} (d + e x)^8} -$$

$$\frac{15 b^2 (b d - a e)^7 (11 b B d - 8 A b e - 3 a B e)}{7 e^{12} (d + e x)^7} + \frac{5 b^3 (b d - a e)^6 (11 b B d - 7 A b e - 4 a B e)}{e^{12} (d + e x)^6} -$$

$$\frac{42 b^4 (b d - a e)^5 (11 b B d - 6 A b e - 5 a B e)}{5 e^{12} (d + e x)^5} + \frac{21 b^5 (b d - a e)^4 (11 b B d - 5 A b e - 6 a B e)}{2 e^{12} (d + e x)^4} -$$

$$\frac{10 b^6 (b d - a e)^3 (11 b B d - 4 A b e - 7 a B e)}{e^{12} (d + e x)^3} + \frac{15 b^7 (b d - a e)^2 (11 b B d - 3 A b e - 8 a B e)}{2 e^{12} (d + e x)^2} -$$

$$\frac{5 b^8 (b d - a e) (11 b B d - 2 A b e - 9 a B e)}{e^{12} (d + e x)} - \frac{b^9 (11 b B d - A b e - 10 a B e) \text{Log}[d + e x]}{e^{12}}$$

Result (type 3, 1447 leaves):

$$\frac{1}{2520 e^{12} (d + e x)^{10}}$$

$$\left( 28 a^{10} e^{10} (9 A e + B (d + 10 e x)) + 70 a^9 b e^9 (4 A e (d + 10 e x) + B (d^2 + 10 d e x + 45 e^2 x^2)) + \right.$$

$$45 a^8 b^2 e^8 (7 A e (d^2 + 10 d e x + 45 e^2 x^2) + 3 B (d^3 + 10 d^2 e x + 45 d e^2 x^2 + 120 e^3 x^3)) +$$

$$120 a^7 b^3 e^7 (3 A e (d^3 + 10 d^2 e x + 45 d e^2 x^2 + 120 e^3 x^3) +$$

$$2 B (d^4 + 10 d^3 e x + 45 d^2 e^2 x^2 + 120 d e^3 x^3 + 210 e^4 x^4)) +$$

$$420 a^6 b^4 e^6 (A e (d^4 + 10 d^3 e x + 45 d^2 e^2 x^2 + 120 d e^3 x^3 + 210 e^4 x^4) +$$

$$B (d^5 + 10 d^4 e x + 45 d^3 e^2 x^2 + 120 d^2 e^3 x^3 + 210 d e^4 x^4 + 252 e^5 x^5)) +$$

$$252 a^5 b^5 e^5 (2 A e (d^5 + 10 d^4 e x + 45 d^3 e^2 x^2 + 120 d^2 e^3 x^3 + 210 d e^4 x^4 + 252 e^5 x^5) +$$

$$3 B (d^6 + 10 d^5 e x + 45 d^4 e^2 x^2 + 120 d^3 e^3 x^3 + 210 d^2 e^4 x^4 + 252 d e^5 x^5 + 210 e^6 x^6)) + 210 a^4 b^6$$

$$e^4 (3 A e (d^6 + 10 d^5 e x + 45 d^4 e^2 x^2 + 120 d^3 e^3 x^3 + 210 d^2 e^4 x^4 + 252 d e^5 x^5 + 210 e^6 x^6) + 7 B$$

$$(d^7 + 10 d^6 e x + 45 d^5 e^2 x^2 + 120 d^4 e^3 x^3 + 210 d^3 e^4 x^4 + 252 d^2 e^5 x^5 + 210 d e^6 x^6 + 120 e^7 x^7)) +$$

$$840 a^3 b^7 e^3 (A e (d^7 + 10 d^6 e x + 45 d^5 e^2 x^2 + 120 d^4 e^3 x^3 + 210 d^3 e^4 x^4 + 252 d^2 e^5 x^5 +$$

$$210 d e^6 x^6 + 120 e^7 x^7) + 4 B (d^8 + 10 d^7 e x + 45 d^6 e^2 x^2 + 120 d^5 e^3 x^3 +$$

$$210 d^4 e^4 x^4 + 252 d^3 e^5 x^5 + 210 d^2 e^6 x^6 + 120 d e^7 x^7 + 45 e^8 x^8)) +$$

$$1260 a^2 b^8 e^2 (A e (d^8 + 10 d^7 e x + 45 d^6 e^2 x^2 + 120 d^5 e^3 x^3 + 210 d^4 e^4 x^4 + 252 d^3 e^5 x^5 +$$

$$210 d^2 e^6 x^6 + 120 d e^7 x^7 + 45 e^8 x^8) + 9 B (d^9 + 10 d^8 e x + 45 d^7 e^2 x^2 + 120 d^6 e^3 x^3 +$$

$$210 d^5 e^4 x^4 + 252 d^4 e^5 x^5 + 210 d^3 e^6 x^6 + 120 d^2 e^7 x^7 + 45 d e^8 x^8 + 10 e^9 x^9)) -$$

$$10 a b^9 e (-252 A e (d^9 + 10 d^8 e x + 45 d^7 e^2 x^2 + 120 d^6 e^3 x^3 + 210 d^5 e^4 x^4 +$$

$$252 d^4 e^5 x^5 + 210 d^3 e^6 x^6 + 120 d^2 e^7 x^7 + 45 d e^8 x^8 + 10 e^9 x^9) +$$

$$B d (7381 d^9 + 71290 d^8 e x + 308205 d^7 e^2 x^2 + 784080 d^6 e^3 x^3 + 1296540 d^5 e^4 x^4 +$$

$$1450008 d^4 e^5 x^5 + 1102500 d^3 e^6 x^6 + 554400 d^2 e^7 x^7 + 170100 d e^8 x^8 + 25200 e^9 x^9)) -$$

$$b^{10} (A d e (7381 d^9 + 71290 d^8 e x + 308205 d^7 e^2 x^2 + 784080 d^6 e^3 x^3 + 1296540 d^5 e^4 x^4 +$$

$$1450008 d^4 e^5 x^5 + 1102500 d^3 e^6 x^6 + 554400 d^2 e^7 x^7 + 170100 d e^8 x^8 + 25200 e^9 x^9) -$$

$$B (55991 d^{11} + 532190 d^{10} e x + 2256255 d^9 e^2 x^2 + 5600880 d^8 e^3 x^3 +$$

$$8969940 d^7 e^4 x^4 + 9599688 d^6 e^5 x^5 + 6835500 d^5 e^6 x^6 + 3074400 d^4 e^7 x^7 +$$

$$737100 d^3 e^8 x^8 + 25200 d^2 e^9 x^9 - 25200 d e^{10} x^{10} - 2520 e^{11} x^{11})) +$$

$$2520 b^9 (11 b B d - A b e - 10 a B e) (d + e x)^{10} \text{Log}[d + e x]$$

### Problem 1088: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^{10} (A+Bx)}{(d+ex)^{12}} dx$$

Optimal (type 3, 321 leaves, 3 steps):

$$\begin{aligned} & -\frac{(Bd-Ae)(a+bx)^{11}}{11e(bd-ae)(d+ex)^{11}} - \frac{B(bd-ae)^{10}}{10e^{12}(d+ex)^{10}} + \frac{10bB(bd-ae)^9}{9e^{12}(d+ex)^9} - \frac{45b^2B(bd-ae)^8}{8e^{12}(d+ex)^8} + \\ & \frac{120b^3B(bd-ae)^7}{7e^{12}(d+ex)^7} - \frac{35b^4B(bd-ae)^6}{e^{12}(d+ex)^6} + \frac{252b^5B(bd-ae)^5}{5e^{12}(d+ex)^5} - \frac{105b^6B(bd-ae)^4}{2e^{12}(d+ex)^4} + \\ & \frac{40b^7B(bd-ae)^3}{e^{12}(d+ex)^3} - \frac{45b^8B(bd-ae)^2}{2e^{12}(d+ex)^2} + \frac{10b^9B(bd-ae)}{e^{12}(d+ex)} + \frac{b^{10}B \operatorname{Log}[d+ex]}{e^{12}} \end{aligned}$$

Result (type 3, 1443 leaves):

$$\begin{aligned} & -\frac{1}{27720e^{12}(d+ex)^{11}} \\ & \left( 252a^{10}e^{10}(10Ae+B(d+11ex)) + 280a^9be^9(9Ae(d+11ex) + 2B(d^2+11dex+55e^2x^2)) + \right. \\ & 315a^8b^2e^8(8Ae(d^2+11dex+55e^2x^2) + 3B(d^3+11d^2ex+55de^2x^2+165e^3x^3)) + \\ & 360a^7b^3e^7(7Ae(d^3+11d^2ex+55de^2x^2+165e^3x^3) + \\ & 4B(d^4+11d^3ex+55d^2e^2x^2+165de^3x^3+330e^4x^4)) + \\ & 420a^6b^4e^6(6Ae(d^4+11d^3ex+55d^2e^2x^2+165de^3x^3+330e^4x^4) + \\ & 5B(d^5+11d^4ex+55d^3e^2x^2+165d^2e^3x^3+330de^4x^4+462e^5x^5)) + \\ & 504a^5b^5e^5(5Ae(d^5+11d^4ex+55d^3e^2x^2+165d^2e^3x^3+330de^4x^4+462e^5x^5) + \\ & 6B(d^6+11d^5ex+55d^4e^2x^2+165d^3e^3x^3+330d^2e^4x^4+462de^5x^5+462e^6x^6)) + 630a^4b^6 \\ & e^4(4Ae(d^6+11d^5ex+55d^4e^2x^2+165d^3e^3x^3+330d^2e^4x^4+462de^5x^5+462e^6x^6) + 7B \\ & (d^7+11d^6ex+55d^5e^2x^2+165d^4e^3x^3+330d^3e^4x^4+462d^2e^5x^5+462de^6x^6+330e^7x^7)) + \\ & 840a^3b^7e^3(3Ae(d^7+11d^6ex+55d^5e^2x^2+165d^4e^3x^3+330d^3e^4x^4+462d^2e^5x^5+ \\ & 462de^6x^6+330e^7x^7) + 8B(d^8+11d^7ex+55d^6e^2x^2+165d^5e^3x^3+ \\ & 330d^4e^4x^4+462d^3e^5x^5+462d^2e^6x^6+330de^7x^7+165e^8x^8)) + \\ & 1260a^2b^8e^2(2Ae(d^8+11d^7ex+55d^6e^2x^2+165d^5e^3x^3+330d^4e^4x^4+462d^3e^5x^5+ \\ & 462d^2e^6x^6+330de^7x^7+165e^8x^8) + 9B(d^9+11d^8ex+55d^7e^2x^2+165d^6e^3x^3+ \\ & 330d^5e^4x^4+462d^4e^5x^5+462d^3e^6x^6+330d^2e^7x^7+165de^8x^8+55e^9x^9)) + \\ & 2520ab^9e(Ae(d^9+11d^8ex+55d^7e^2x^2+165d^6e^3x^3+330d^5e^4x^4+462d^4e^5x^5+462d^3e^6x^6+ \\ & 330d^2e^7x^7+165de^8x^8+55e^9x^9) + 10B(d^{10}+11d^9ex+55d^8e^2x^2+165d^7e^3x^3+ \\ & 330d^6e^4x^4+462d^5e^5x^5+462d^4e^6x^6+330d^3e^7x^7+165d^2e^8x^8+55de^9x^9+11e^{10}x^{10})) + \\ & b^{10}(2520Ae(d^{10}+11d^9ex+55d^8e^2x^2+165d^7e^3x^3+330d^6e^4x^4+462d^5e^5x^5+ \\ & 462d^4e^6x^6+330d^3e^7x^7+165d^2e^8x^8+55de^9x^9+11e^{10}x^{10}) - \\ & Bd(83711d^{10}+893101d^9ex+4313045d^8e^2x^2+12430935d^7e^3x^3+23718420d^6e^4x^4+ \\ & 31376268d^5e^5x^5+29241828d^4e^6x^6+19057500d^3e^7x^7+8385300d^2e^8x^8+ \\ & 2286900de^9x^9+304920e^{10}x^{10})) - 27720b^{10}B(d+ex)^{11} \operatorname{Log}[d+ex] \end{aligned}$$

### Problem 1089: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^{10} (A+Bx)}{(d+ex)^{13}} dx$$

Optimal (type 1, 86 leaves, 2 steps):

$$-\frac{(Bd-Ae)(a+bx)^{11}}{12e(bd-ae)(d+ex)^{12}} + \frac{(11bBd+Abe-12aBe)(a+bx)^{11}}{132e(bd-ae)^2(d+ex)^{11}}$$

Result (type 1, 1421 leaves):

$$\begin{aligned} & -\frac{1}{132e^{12}(d+ex)^{12}} \\ & (a^{10}e^{10}(11Ae+B(d+12ex)) + 2a^9be^9(5Ae(d+12ex) + B(d^2+12dex+66e^2x^2)) + \\ & 3a^8b^2e^8(3Ae(d^2+12dex+66e^2x^2) + B(d^3+12d^2ex+66de^2x^2+220e^3x^3)) + \\ & 4a^7b^3e^7(2Ae(d^3+12d^2ex+66de^2x^2+220e^3x^3) + \\ & B(d^4+12d^3ex+66d^2e^2x^2+220de^3x^3+495e^4x^4)) + \\ & a^6b^4e^6(7Ae(d^4+12d^3ex+66d^2e^2x^2+220de^3x^3+495e^4x^4) + \\ & 5B(d^5+12d^4ex+66d^3e^2x^2+220d^2e^3x^3+495de^4x^4+792e^5x^5)) + \\ & 6a^5b^5e^5(Ae(d^5+12d^4ex+66d^3e^2x^2+220d^2e^3x^3+495de^4x^4+792e^5x^5) + \\ & B(d^6+12d^5ex+66d^4e^2x^2+220d^3e^3x^3+495d^2e^4x^4+792de^5x^5+924e^6x^6)) + \\ & a^4b^6e^4(5Ae(d^6+12d^5ex+66d^4e^2x^2+220d^3e^3x^3+495d^2e^4x^4+792de^5x^5+924e^6x^6) + 7B \\ & (d^7+12d^6ex+66d^5e^2x^2+220d^4e^3x^3+495d^3e^4x^4+792d^2e^5x^5+924de^6x^6+792e^7x^7)) + \\ & 4a^3b^7e^3(Ae(d^7+12d^6ex+66d^5e^2x^2+220d^4e^3x^3+495d^3e^4x^4+792d^2e^5x^5+ \\ & 924de^6x^6+792e^7x^7) + 2B(d^8+12d^7ex+66d^6e^2x^2+220d^5e^3x^3+ \\ & 495d^4e^4x^4+792d^3e^5x^5+924d^2e^6x^6+792de^7x^7+495e^8x^8)) + \\ & 3a^2b^8e^2(Ae(d^8+12d^7ex+66d^6e^2x^2+220d^5e^3x^3+495d^4e^4x^4+792d^3e^5x^5+ \\ & 924d^2e^6x^6+792de^7x^7+495e^8x^8) + 3B(d^9+12d^8ex+66d^7e^2x^2+220d^6e^3x^3+ \\ & 495d^5e^4x^4+792d^4e^5x^5+924d^3e^6x^6+792d^2e^7x^7+495de^8x^8+220e^9x^9)) + \\ & 2ab^9e(Ae(d^9+12d^8ex+66d^7e^2x^2+220d^6e^3x^3+495d^5e^4x^4+792d^4e^5x^5+924d^3e^6x^6+ \\ & 792d^2e^7x^7+495de^8x^8+220e^9x^9) + 5B(d^{10}+12d^9ex+66d^8e^2x^2+220d^7e^3x^3+495d^6 \\ & e^4x^4+792d^5e^5x^5+924d^4e^6x^6+792d^3e^7x^7+495d^2e^8x^8+220de^9x^9+66e^{10}x^{10})) + \\ & b^{10}(Ae(d^{10}+12d^9ex+66d^8e^2x^2+220d^7e^3x^3+495d^6e^4x^4+792d^5e^5x^5+ \\ & 924d^4e^6x^6+792d^3e^7x^7+495d^2e^8x^8+220de^9x^9+66e^{10}x^{10}) + \\ & 11B(d^{11}+12d^{10}ex+66d^9e^2x^2+220d^8e^3x^3+495d^7e^4x^4+792d^6e^5x^5+ \\ & 924d^5e^6x^6+792d^4e^7x^7+495d^3e^8x^8+220d^2e^9x^9+66de^{10}x^{10}+12e^{11}x^{11})) \end{aligned}$$

### Problem 1090: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^{10} (A+Bx)}{(d+ex)^{14}} dx$$

Optimal (type 1, 135 leaves, 3 steps):

$$\begin{aligned}
 & - \frac{(Bd - Ae) (a + bx)^{11}}{13e (bd - ae) (d + ex)^{13}} + \\
 & \frac{(11bBd + 2Abe - 13aBe) (a + bx)^{11}}{156e (bd - ae)^2 (d + ex)^{12}} + \frac{b (11bBd + 2Abe - 13aBe) (a + bx)^{11}}{1716e (bd - ae)^3 (d + ex)^{11}}
 \end{aligned}$$

Result (type 1, 1433 leaves):

$$\begin{aligned}
 & - \frac{1}{1716e^{12} (d + ex)^{13}} \\
 & \left( 11a^{10}e^{10} (12Ae + B(d + 13ex)) + 10a^9be^9 (11Ae(d + 13ex) + 2B(d^2 + 13dex + 78e^2x^2)) + \right. \\
 & 9a^8b^2e^8 (10Ae(d^2 + 13dex + 78e^2x^2) + 3B(d^3 + 13d^2ex + 78de^2x^2 + 286e^3x^3)) + \\
 & 8a^7b^3e^7 (9Ae(d^3 + 13d^2ex + 78de^2x^2 + 286e^3x^3) + \\
 & \quad 4B(d^4 + 13d^3ex + 78d^2e^2x^2 + 286de^3x^3 + 715e^4x^4)) + \\
 & 7a^6b^4e^6 (8Ae(d^4 + 13d^3ex + 78d^2e^2x^2 + 286de^3x^3 + 715e^4x^4) + \\
 & \quad 5B(d^5 + 13d^4ex + 78d^3e^2x^2 + 286d^2e^3x^3 + 715de^4x^4 + 1287e^5x^5)) + \\
 & 6a^5b^5e^5 (7Ae(d^5 + 13d^4ex + 78d^3e^2x^2 + 286d^2e^3x^3 + 715de^4x^4 + 1287e^5x^5) + 6B \\
 & \quad (d^6 + 13d^5ex + 78d^4e^2x^2 + 286d^3e^3x^3 + 715d^2e^4x^4 + 1287de^5x^5 + 1716e^6x^6)) + 5a^4b^6e^4 \\
 & \quad (6Ae(d^6 + 13d^5ex + 78d^4e^2x^2 + 286d^3e^3x^3 + 715d^2e^4x^4 + 1287de^5x^5 + 1716e^6x^6) + 7B(d^7 + \\
 & \quad 13d^6ex + 78d^5e^2x^2 + 286d^4e^3x^3 + 715d^3e^4x^4 + 1287d^2e^5x^5 + 1716de^6x^6 + 1716e^7x^7)) + \\
 & 4a^3b^7e^3 (5Ae(d^7 + 13d^6ex + 78d^5e^2x^2 + 286d^4e^3x^3 + 715d^3e^4x^4 + 1287d^2e^5x^5 + \\
 & \quad 1716de^6x^6 + 1716e^7x^7) + 8B(d^8 + 13d^7ex + 78d^6e^2x^2 + 286d^5e^3x^3 + \\
 & \quad 715d^4e^4x^4 + 1287d^3e^5x^5 + 1716d^2e^6x^6 + 1716de^7x^7 + 1287e^8x^8)) + \\
 & 3a^2b^8e^2 (4Ae(d^8 + 13d^7ex + 78d^6e^2x^2 + 286d^5e^3x^3 + 715d^4e^4x^4 + 1287d^3e^5x^5 + \\
 & \quad 1716d^2e^6x^6 + 1716de^7x^7 + 1287e^8x^8) + 9B(d^9 + 13d^8ex + 78d^7e^2x^2 + 286d^6e^3x^3 + \\
 & \quad 715d^5e^4x^4 + 1287d^4e^5x^5 + 1716d^3e^6x^6 + 1716d^2e^7x^7 + 1287de^8x^8 + 715e^9x^9)) + \\
 & 2ab^9e (3Ae(d^9 + 13d^8ex + 78d^7e^2x^2 + 286d^6e^3x^3 + 715d^5e^4x^4 + 1287d^4e^5x^5 + \\
 & \quad 1716d^3e^6x^6 + 1716d^2e^7x^7 + 1287de^8x^8 + 715e^9x^9) + \\
 & \quad 10B(d^{10} + 13d^9ex + 78d^8e^2x^2 + 286d^7e^3x^3 + 715d^6e^4x^4 + 1287d^5e^5x^5 + \\
 & \quad 1716d^4e^6x^6 + 1716d^3e^7x^7 + 1287d^2e^8x^8 + 715de^9x^9 + 286e^{10}x^{10})) + \\
 & b^{10} (2Ae(d^{10} + 13d^9ex + 78d^8e^2x^2 + 286d^7e^3x^3 + 715d^6e^4x^4 + 1287d^5e^5x^5 + \\
 & \quad 1716d^4e^6x^6 + 1716d^3e^7x^7 + 1287d^2e^8x^8 + 715de^9x^9 + 286e^{10}x^{10}) + \\
 & \quad 11B(d^{11} + 13d^{10}ex + 78d^9e^2x^2 + 286d^8e^3x^3 + 715d^7e^4x^4 + 1287d^6e^5x^5 + \\
 & \quad 1716d^5e^6x^6 + 1716d^4e^7x^7 + 1287d^3e^8x^8 + 715d^2e^9x^9 + 286de^{10}x^{10} + 78e^{11}x^{11})) )
 \end{aligned}$$

**Problem 1091: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + bx)^{10} (A + Bx)}{(d + ex)^{15}} dx$$

Optimal (type 1, 185 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{(Bd - Ae) (a + bx)^{11}}{14e (bd - ae) (d + ex)^{14}} + \frac{(11bBd + 3Abe - 14aBe) (a + bx)^{11}}{182e (bd - ae)^2 (d + ex)^{13}} + \\
 & \frac{b (11bBd + 3Abe - 14aBe) (a + bx)^{11}}{1092e (bd - ae)^3 (d + ex)^{12}} + \frac{b^2 (11bBd + 3Abe - 14aBe) (a + bx)^{11}}{12012e (bd - ae)^4 (d + ex)^{11}}
 \end{aligned}$$

Result (type 1, 1430 leaves):

$$\begin{aligned}
 & - \frac{1}{12012 e^{12} (d + e x)^{14}} \\
 & \left( 66 a^{10} e^{10} \left( 13 A e + B (d + 14 e x) \right) + 110 a^9 b e^9 \left( 6 A e (d + 14 e x) + B (d^2 + 14 d e x + 91 e^2 x^2) \right) + \right. \\
 & 45 a^8 b^2 e^8 \left( 11 A e (d^2 + 14 d e x + 91 e^2 x^2) + 3 B (d^3 + 14 d^2 e x + 91 d e^2 x^2 + 364 e^3 x^3) \right) + \\
 & 72 a^7 b^3 e^7 \left( 5 A e (d^3 + 14 d^2 e x + 91 d e^2 x^2 + 364 e^3 x^3) + \right. \\
 & \quad \left. 2 B (d^4 + 14 d^3 e x + 91 d^2 e^2 x^2 + 364 d e^3 x^3 + 1001 e^4 x^4) \right) + \\
 & 28 a^6 b^4 e^6 \left( 9 A e (d^4 + 14 d^3 e x + 91 d^2 e^2 x^2 + 364 d e^3 x^3 + 1001 e^4 x^4) + \right. \\
 & \quad \left. 5 B (d^5 + 14 d^4 e x + 91 d^3 e^2 x^2 + 364 d^2 e^3 x^3 + 1001 d e^4 x^4 + 2002 e^5 x^5) \right) + \\
 & 42 a^5 b^5 e^5 \left( 4 A e (d^5 + 14 d^4 e x + 91 d^3 e^2 x^2 + 364 d^2 e^3 x^3 + 1001 d e^4 x^4 + 2002 e^5 x^5) + \right. \\
 & \quad \left. 3 B (d^6 + 14 d^5 e x + 91 d^4 e^2 x^2 + 364 d^3 e^3 x^3 + 1001 d^2 e^4 x^4 + 2002 d e^5 x^5 + 3003 e^6 x^6) \right) + \\
 & 105 a^4 b^6 e^4 \left( A e (d^6 + 14 d^5 e x + 91 d^4 e^2 x^2 + 364 d^3 e^3 x^3 + 1001 d^2 e^4 x^4 + 2002 d e^5 x^5 + 3003 e^6 x^6) + \right. \\
 & \quad \left. B (d^7 + 14 d^6 e x + 91 d^5 e^2 x^2 + 364 d^4 e^3 x^3 + \right. \\
 & \quad \quad \left. 1001 d^3 e^4 x^4 + 2002 d^2 e^5 x^5 + 3003 d e^6 x^6 + 3432 e^7 x^7) \right) + \\
 & 20 a^3 b^7 e^3 \left( 3 A e (d^7 + 14 d^6 e x + 91 d^5 e^2 x^2 + 364 d^4 e^3 x^3 + 1001 d^3 e^4 x^4 + 2002 d^2 e^5 x^5 + \right. \\
 & \quad \left. 3003 d e^6 x^6 + 3432 e^7 x^7) + 4 B (d^8 + 14 d^7 e x + 91 d^6 e^2 x^2 + 364 d^5 e^3 x^3 + \right. \\
 & \quad \left. 1001 d^4 e^4 x^4 + 2002 d^3 e^5 x^5 + 3003 d^2 e^6 x^6 + 3432 d e^7 x^7 + 3003 e^8 x^8) \right) + \\
 & 6 a^2 b^8 e^2 \left( 5 A e (d^8 + 14 d^7 e x + 91 d^6 e^2 x^2 + 364 d^5 e^3 x^3 + 1001 d^4 e^4 x^4 + 2002 d^3 e^5 x^5 + \right. \\
 & \quad \left. 3003 d^2 e^6 x^6 + 3432 d e^7 x^7 + 3003 e^8 x^8) + 9 B (d^9 + 14 d^8 e x + 91 d^7 e^2 x^2 + 364 d^6 e^3 x^3 + \right. \\
 & \quad \left. 1001 d^5 e^4 x^4 + 2002 d^4 e^5 x^5 + 3003 d^3 e^6 x^6 + 3432 d^2 e^7 x^7 + 3003 d e^8 x^8 + 2002 e^9 x^9) \right) + \\
 & 6 a b^9 e \left( 2 A e (d^9 + 14 d^8 e x + 91 d^7 e^2 x^2 + 364 d^6 e^3 x^3 + 1001 d^5 e^4 x^4 + 2002 d^4 e^5 x^5 + \right. \\
 & \quad \left. 3003 d^3 e^6 x^6 + 3432 d^2 e^7 x^7 + 3003 d e^8 x^8 + 2002 e^9 x^9) + \right. \\
 & \quad \left. 5 B (d^{10} + 14 d^9 e x + 91 d^8 e^2 x^2 + 364 d^7 e^3 x^3 + 1001 d^6 e^4 x^4 + 2002 d^5 e^5 x^5 + \right. \\
 & \quad \quad \left. 3003 d^4 e^6 x^6 + 3432 d^3 e^7 x^7 + 3003 d^2 e^8 x^8 + 2002 d e^9 x^9 + 1001 e^{10} x^{10}) \right) + \\
 & b^{10} \left( 3 A e (d^{10} + 14 d^9 e x + 91 d^8 e^2 x^2 + 364 d^7 e^3 x^3 + 1001 d^6 e^4 x^4 + 2002 d^5 e^5 x^5 + \right. \\
 & \quad \left. 3003 d^4 e^6 x^6 + 3432 d^3 e^7 x^7 + 3003 d^2 e^8 x^8 + 2002 d e^9 x^9 + 1001 e^{10} x^{10}) + \right. \\
 & \quad \left. 11 B (d^{11} + 14 d^{10} e x + 91 d^9 e^2 x^2 + 364 d^8 e^3 x^3 + 1001 d^7 e^4 x^4 + 2002 d^6 e^5 x^5 + \right. \\
 & \quad \quad \left. 3003 d^5 e^6 x^6 + 3432 d^4 e^7 x^7 + 3003 d^3 e^8 x^8 + 2002 d^2 e^9 x^9 + 1001 d e^{10} x^{10} + 364 e^{11} x^{11}) \right)
 \end{aligned}$$

### Problem 1092: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{16}} dx$$

Optimal (type 1, 235 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{(B d - A e) (a + b x)^{11}}{15 e (b d - a e) (d + e x)^{15}} + \\
 & \frac{(11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{210 e (b d - a e)^2 (d + e x)^{14}} + \frac{b (11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{910 e (b d - a e)^3 (d + e x)^{13}} + \\
 & \frac{b^2 (11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{5460 e (b d - a e)^4 (d + e x)^{12}} + \frac{b^3 (11 b B d + 4 A b e - 15 a B e) (a + b x)^{11}}{60060 e (b d - a e)^5 (d + e x)^{11}}
 \end{aligned}$$

Result (type 1, 1430 leaves):

1

$$\begin{aligned}
 & 60060 e^{12} (d + e x)^{15} \\
 & (286 a^{10} e^{10} (14 A e + B (d + 15 e x)) + 220 a^9 b e^9 (13 A e (d + 15 e x) + 2 B (d^2 + 15 d e x + 105 e^2 x^2)) + \\
 & 495 a^8 b^2 e^8 (4 A e (d^2 + 15 d e x + 105 e^2 x^2) + B (d^3 + 15 d^2 e x + 105 d e^2 x^2 + 455 e^3 x^3)) + \\
 & 120 a^7 b^3 e^7 (11 A e (d^3 + 15 d^2 e x + 105 d e^2 x^2 + 455 e^3 x^3) + \\
 & 4 B (d^4 + 15 d^3 e x + 105 d^2 e^2 x^2 + 455 d e^3 x^3 + 1365 e^4 x^4)) + \\
 & 420 a^6 b^4 e^6 (2 A e (d^4 + 15 d^3 e x + 105 d^2 e^2 x^2 + 455 d e^3 x^3 + 1365 e^4 x^4) + \\
 & B (d^5 + 15 d^4 e x + 105 d^3 e^2 x^2 + 455 d^2 e^3 x^3 + 1365 d e^4 x^4 + 3003 e^5 x^5)) + \\
 & 168 a^5 b^5 e^5 (3 A e (d^5 + 15 d^4 e x + 105 d^3 e^2 x^2 + 455 d^2 e^3 x^3 + 1365 d e^4 x^4 + 3003 e^5 x^5) + \\
 & 2 B (d^6 + 15 d^5 e x + 105 d^4 e^2 x^2 + 455 d^3 e^3 x^3 + 1365 d^2 e^4 x^4 + 3003 d e^5 x^5 + 5005 e^6 x^6)) + 35 a^4 \\
 & b^6 e^4 (8 A e (d^6 + 15 d^5 e x + 105 d^4 e^2 x^2 + 455 d^3 e^3 x^3 + 1365 d^2 e^4 x^4 + 3003 d e^5 x^5 + 5005 e^6 x^6) + \\
 & 7 B (d^7 + 15 d^6 e x + 105 d^5 e^2 x^2 + 455 d^4 e^3 x^3 + \\
 & 1365 d^3 e^4 x^4 + 3003 d^2 e^5 x^5 + 5005 d e^6 x^6 + 6435 e^7 x^7)) + \\
 & 20 a^3 b^7 e^3 (7 A e (d^7 + 15 d^6 e x + 105 d^5 e^2 x^2 + 455 d^4 e^3 x^3 + 1365 d^3 e^4 x^4 + 3003 d^2 e^5 x^5 + \\
 & 5005 d e^6 x^6 + 6435 e^7 x^7) + 8 B (d^8 + 15 d^7 e x + 105 d^6 e^2 x^2 + 455 d^5 e^3 x^3 + \\
 & 1365 d^4 e^4 x^4 + 3003 d^3 e^5 x^5 + 5005 d^2 e^6 x^6 + 6435 d e^7 x^7 + 6435 e^8 x^8)) + \\
 & 30 a^2 b^8 e^2 (2 A e (d^8 + 15 d^7 e x + 105 d^6 e^2 x^2 + 455 d^5 e^3 x^3 + 1365 d^4 e^4 x^4 + 3003 d^3 e^5 x^5 + \\
 & 5005 d^2 e^6 x^6 + 6435 d e^7 x^7 + 6435 e^8 x^8) + 3 B (d^9 + 15 d^8 e x + 105 d^7 e^2 x^2 + 455 d^6 e^3 x^3 + \\
 & 1365 d^5 e^4 x^4 + 3003 d^4 e^5 x^5 + 5005 d^3 e^6 x^6 + 6435 d^2 e^7 x^7 + 6435 d e^8 x^8 + 5005 e^9 x^9)) + \\
 & 20 a b^9 e (A e (d^9 + 15 d^8 e x + 105 d^7 e^2 x^2 + 455 d^6 e^3 x^3 + 1365 d^5 e^4 x^4 + 3003 d^4 e^5 x^5 + \\
 & 5005 d^3 e^6 x^6 + 6435 d^2 e^7 x^7 + 6435 d e^8 x^8 + 5005 e^9 x^9) + \\
 & 2 B (d^{10} + 15 d^9 e x + 105 d^8 e^2 x^2 + 455 d^7 e^3 x^3 + 1365 d^6 e^4 x^4 + 3003 d^5 e^5 x^5 + \\
 & 5005 d^4 e^6 x^6 + 6435 d^3 e^7 x^7 + 6435 d^2 e^8 x^8 + 5005 d e^9 x^9 + 3003 e^{10} x^{10})) + \\
 & b^{10} (4 A e (d^{10} + 15 d^9 e x + 105 d^8 e^2 x^2 + 455 d^7 e^3 x^3 + 1365 d^6 e^4 x^4 + 3003 d^5 e^5 x^5 + \\
 & 5005 d^4 e^6 x^6 + 6435 d^3 e^7 x^7 + 6435 d^2 e^8 x^8 + 5005 d e^9 x^9 + 3003 e^{10} x^{10}) + \\
 & 11 B (d^{11} + 15 d^{10} e x + 105 d^9 e^2 x^2 + 455 d^8 e^3 x^3 + 1365 d^7 e^4 x^4 + 3003 d^6 e^5 x^5 + \\
 & 5005 d^5 e^6 x^6 + 6435 d^4 e^7 x^7 + 6435 d^3 e^8 x^8 + 5005 d^2 e^9 x^9 + 3003 d e^{10} x^{10} + 1365 e^{11} x^{11})) )
 \end{aligned}$$

**Problem 1093: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{17}} dx$$

Optimal (type 1, 285 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{(B d - A e) (a + b x)^{11}}{16 e (b d - a e) (d + e x)^{16}} + \frac{(11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{240 e (b d - a e)^2 (d + e x)^{15}} + \\
 & \frac{b (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{840 e (b d - a e)^3 (d + e x)^{14}} + \frac{b^2 (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{3640 e (b d - a e)^4 (d + e x)^{13}} + \\
 & \frac{b^3 (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{21840 e (b d - a e)^5 (d + e x)^{12}} + \frac{b^4 (11 b B d + 5 A b e - 16 a B e) (a + b x)^{11}}{240240 e (b d - a e)^6 (d + e x)^{11}}
 \end{aligned}$$

Result (type 1, 1429 leaves):

1

$$\begin{aligned}
 & \frac{240240 e^{12} (d + e x)^{16}}{(1001 a^{10} e^{10} (15 A e + B (d + 16 e x)) + 1430 a^9 b e^9 (7 A e (d + 16 e x) + B (d^2 + 16 d e x + 120 e^2 x^2)) + \\
 & 495 a^8 b^2 e^8 (13 A e (d^2 + 16 d e x + 120 e^2 x^2) + 3 B (d^3 + 16 d^2 e x + 120 d e^2 x^2 + 560 e^3 x^3)) + \\
 & 1320 a^7 b^3 e^7 (3 A e (d^3 + 16 d^2 e x + 120 d e^2 x^2 + 560 e^3 x^3) + \\
 & B (d^4 + 16 d^3 e x + 120 d^2 e^2 x^2 + 560 d e^3 x^3 + 1820 e^4 x^4)) + \\
 & 210 a^6 b^4 e^6 (11 A e (d^4 + 16 d^3 e x + 120 d^2 e^2 x^2 + 560 d e^3 x^3 + 1820 e^4 x^4) + \\
 & 5 B (d^5 + 16 d^4 e x + 120 d^3 e^2 x^2 + 560 d^2 e^3 x^3 + 1820 d e^4 x^4 + 4368 e^5 x^5)) + \\
 & 252 a^5 b^5 e^5 (5 A e (d^5 + 16 d^4 e x + 120 d^3 e^2 x^2 + 560 d^2 e^3 x^3 + 1820 d e^4 x^4 + 4368 e^5 x^5) + \\
 & 3 B (d^6 + 16 d^5 e x + 120 d^4 e^2 x^2 + 560 d^3 e^3 x^3 + 1820 d^2 e^4 x^4 + 4368 d e^5 x^5 + 8008 e^6 x^6)) + 70 a^4 \\
 & b^6 e^4 (9 A e (d^6 + 16 d^5 e x + 120 d^4 e^2 x^2 + 560 d^3 e^3 x^3 + 1820 d^2 e^4 x^4 + 4368 d e^5 x^5 + 8008 e^6 x^6) + \\
 & 7 B (d^7 + 16 d^6 e x + 120 d^5 e^2 x^2 + 560 d^4 e^3 x^3 + \\
 & 1820 d^3 e^4 x^4 + 4368 d^2 e^5 x^5 + 8008 d e^6 x^6 + 11440 e^7 x^7)) + \\
 & 280 a^3 b^7 e^3 (A e (d^7 + 16 d^6 e x + 120 d^5 e^2 x^2 + 560 d^4 e^3 x^3 + 1820 d^3 e^4 x^4 + 4368 d^2 e^5 x^5 + \\
 & 8008 d e^6 x^6 + 11440 e^7 x^7) + B (d^8 + 16 d^7 e x + 120 d^6 e^2 x^2 + 560 d^5 e^3 x^3 + \\
 & 1820 d^4 e^4 x^4 + 4368 d^3 e^5 x^5 + 8008 d^2 e^6 x^6 + 11440 d e^7 x^7 + 12870 e^8 x^8)) + \\
 & 15 a^2 b^8 e^2 (7 A e (d^8 + 16 d^7 e x + 120 d^6 e^2 x^2 + 560 d^5 e^3 x^3 + 1820 d^4 e^4 x^4 + 4368 d^3 e^5 x^5 + \\
 & 8008 d^2 e^6 x^6 + 11440 d e^7 x^7 + 12870 e^8 x^8) + 9 B (d^9 + 16 d^8 e x + 120 d^7 e^2 x^2 + 560 d^6 e^3 x^3 + \\
 & 1820 d^5 e^4 x^4 + 4368 d^4 e^5 x^5 + 8008 d^3 e^6 x^6 + 11440 d^2 e^7 x^7 + 12870 d e^8 x^8 + 11440 e^9 x^9)) + \\
 & 10 a b^9 e (3 A e (d^9 + 16 d^8 e x + 120 d^7 e^2 x^2 + 560 d^6 e^3 x^3 + 1820 d^5 e^4 x^4 + 4368 d^4 e^5 x^5 + \\
 & 8008 d^3 e^6 x^6 + 11440 d^2 e^7 x^7 + 12870 d e^8 x^8 + 11440 e^9 x^9) + \\
 & 5 B (d^{10} + 16 d^9 e x + 120 d^8 e^2 x^2 + 560 d^7 e^3 x^3 + 1820 d^6 e^4 x^4 + 4368 d^5 e^5 x^5 + \\
 & 8008 d^4 e^6 x^6 + 11440 d^3 e^7 x^7 + 12870 d^2 e^8 x^8 + 11440 d e^9 x^9 + 8008 e^{10} x^{10})) + \\
 & b^{10} (5 A e (d^{10} + 16 d^9 e x + 120 d^8 e^2 x^2 + 560 d^7 e^3 x^3 + 1820 d^6 e^4 x^4 + 4368 d^5 e^5 x^5 + \\
 & 8008 d^4 e^6 x^6 + 11440 d^3 e^7 x^7 + 12870 d^2 e^8 x^8 + 11440 d e^9 x^9 + 8008 e^{10} x^{10}) + \\
 & 11 B (d^{11} + 16 d^{10} e x + 120 d^9 e^2 x^2 + 560 d^8 e^3 x^3 + 1820 d^7 e^4 x^4 + 4368 d^6 e^5 x^5 + 8008 d^5 e^6 x^6 + \\
 & 11440 d^4 e^7 x^7 + 12870 d^3 e^8 x^8 + 11440 d^2 e^9 x^9 + 8008 d e^{10} x^{10} + 4368 e^{11} x^{11}))
 \end{aligned}$$

**Problem 1094: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{18}} dx$$

Optimal (type 1, 335 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{(B d - A e) (a + b x)^{11}}{17 e (b d - a e) (d + e x)^{17}} + \frac{(11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{272 e (b d - a e)^2 (d + e x)^{16}} + \\
 & \frac{b (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{816 e (b d - a e)^3 (d + e x)^{15}} + \frac{b^2 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{2856 e (b d - a e)^4 (d + e x)^{14}} + \\
 & \frac{b^3 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{12376 e (b d - a e)^5 (d + e x)^{13}} + \\
 & \frac{b^4 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{74256 e (b d - a e)^6 (d + e x)^{12}} + \frac{b^5 (11 b B d + 6 A b e - 17 a B e) (a + b x)^{11}}{816816 e (b d - a e)^7 (d + e x)^{11}}
 \end{aligned}$$

Result (type 1, 1433 leaves):

$$\begin{aligned}
 & - \frac{1}{816816 e^{12} (d+ex)^{17}} \left( 3003 a^{10} e^{10} (16 A e + B (d+17 ex)) + \right. \\
 & \quad 2002 a^9 b e^9 (15 A e (d+17 ex) + 2 B (d^2 + 17 d ex + 136 e^2 x^2)) + \\
 & \quad 1287 a^8 b^2 e^8 (14 A e (d^2 + 17 d ex + 136 e^2 x^2) + 3 B (d^3 + 17 d^2 ex + 136 d e^2 x^2 + 680 e^3 x^3)) + \\
 & \quad 792 a^7 b^3 e^7 (13 A e (d^3 + 17 d^2 ex + 136 d e^2 x^2 + 680 e^3 x^3) + \\
 & \quad \quad 4 B (d^4 + 17 d^3 ex + 136 d^2 e^2 x^2 + 680 d e^3 x^3 + 2380 e^4 x^4)) + \\
 & \quad 462 a^6 b^4 e^6 (12 A e (d^4 + 17 d^3 ex + 136 d^2 e^2 x^2 + 680 d e^3 x^3 + 2380 e^4 x^4) + \\
 & \quad \quad 5 B (d^5 + 17 d^4 ex + 136 d^3 e^2 x^2 + 680 d^2 e^3 x^3 + 2380 d e^4 x^4 + 6188 e^5 x^5)) + \\
 & \quad 252 a^5 b^5 e^5 (11 A e (d^5 + 17 d^4 ex + 136 d^3 e^2 x^2 + 680 d^2 e^3 x^3 + 2380 d e^4 x^4 + 6188 e^5 x^5) + 6 B \\
 & \quad \quad (d^6 + 17 d^5 ex + 136 d^4 e^2 x^2 + 680 d^3 e^3 x^3 + 2380 d^2 e^4 x^4 + 6188 d e^5 x^5 + 12376 e^6 x^6)) + 126 a^4 \\
 & \quad b^6 e^4 (10 A e (d^6 + 17 d^5 ex + 136 d^4 e^2 x^2 + 680 d^3 e^3 x^3 + 2380 d^2 e^4 x^4 + 6188 d e^5 x^5 + 12376 e^6 x^6) + \\
 & \quad \quad 7 B (d^7 + 17 d^6 ex + 136 d^5 e^2 x^2 + 680 d^4 e^3 x^3 + \\
 & \quad \quad \quad 2380 d^3 e^4 x^4 + 6188 d^2 e^5 x^5 + 12376 d e^6 x^6 + 19448 e^7 x^7)) + \\
 & \quad 56 a^3 b^7 e^3 (9 A e (d^7 + 17 d^6 ex + 136 d^5 e^2 x^2 + 680 d^4 e^3 x^3 + 2380 d^3 e^4 x^4 + 6188 d^2 e^5 x^5 + \\
 & \quad \quad 12376 d e^6 x^6 + 19448 e^7 x^7) + 8 B (d^8 + 17 d^7 ex + 136 d^6 e^2 x^2 + 680 d^5 e^3 x^3 + \\
 & \quad \quad \quad 2380 d^4 e^4 x^4 + 6188 d^3 e^5 x^5 + 12376 d^2 e^6 x^6 + 19448 d e^7 x^7 + 24310 e^8 x^8)) + \\
 & \quad 21 a^2 b^8 e^2 (8 A e (d^8 + 17 d^7 ex + 136 d^6 e^2 x^2 + 680 d^5 e^3 x^3 + 2380 d^4 e^4 x^4 + \\
 & \quad \quad 6188 d^3 e^5 x^5 + 12376 d^2 e^6 x^6 + 19448 d e^7 x^7 + 24310 e^8 x^8) + \\
 & \quad \quad 9 B (d^9 + 17 d^8 ex + 136 d^7 e^2 x^2 + 680 d^6 e^3 x^3 + 2380 d^5 e^4 x^4 + 6188 d^4 e^5 x^5 + \\
 & \quad \quad \quad 12376 d^3 e^6 x^6 + 19448 d^2 e^7 x^7 + 24310 d e^8 x^8 + 24310 e^9 x^9)) + \\
 & \quad 6 a b^9 e (7 A e (d^9 + 17 d^8 ex + 136 d^7 e^2 x^2 + 680 d^6 e^3 x^3 + 2380 d^5 e^4 x^4 + 6188 d^4 e^5 x^5 + \\
 & \quad \quad 12376 d^3 e^6 x^6 + 19448 d^2 e^7 x^7 + 24310 d e^8 x^8 + 24310 e^9 x^9) + \\
 & \quad \quad 10 B (d^{10} + 17 d^9 ex + 136 d^8 e^2 x^2 + 680 d^7 e^3 x^3 + 2380 d^6 e^4 x^4 + 6188 d^5 e^5 x^5 + \\
 & \quad \quad \quad 12376 d^4 e^6 x^6 + 19448 d^3 e^7 x^7 + 24310 d^2 e^8 x^8 + 24310 d e^9 x^9 + 19448 e^{10} x^{10})) + \\
 & \quad b^{10} (6 A e (d^{10} + 17 d^9 ex + 136 d^8 e^2 x^2 + 680 d^7 e^3 x^3 + 2380 d^6 e^4 x^4 + 6188 d^5 e^5 x^5 + \\
 & \quad \quad 12376 d^4 e^6 x^6 + 19448 d^3 e^7 x^7 + 24310 d^2 e^8 x^8 + 24310 d e^9 x^9 + 19448 e^{10} x^{10}) + \\
 & \quad \quad 11 B (d^{11} + 17 d^{10} ex + 136 d^9 e^2 x^2 + 680 d^8 e^3 x^3 + 2380 d^7 e^4 x^4 + 6188 d^6 e^5 x^5 + 12376 d^5 e^6 x^6 + \\
 & \quad \quad \quad 19448 d^4 e^7 x^7 + 24310 d^3 e^8 x^8 + 24310 d^2 e^9 x^9 + 19448 d e^{10} x^{10} + 12376 e^{11} x^{11})) )
 \end{aligned}$$

**Problem 1095: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^{10} (A+Bx)}{(d+ex)^{19}} dx$$

Optimal (type 1, 385 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{(Bd - Ae) (a + bx)^{11}}{18e (bd - ae) (d + ex)^{18}} + \frac{(11bBd + 7Abe - 18aBe) (a + bx)^{11}}{306e (bd - ae)^2 (d + ex)^{17}} + \\
 & \frac{b (11bBd + 7Abe - 18aBe) (a + bx)^{11}}{816e (bd - ae)^3 (d + ex)^{16}} + \frac{b^2 (11bBd + 7Abe - 18aBe) (a + bx)^{11}}{2448e (bd - ae)^4 (d + ex)^{15}} + \\
 & \frac{b^3 (11bBd + 7Abe - 18aBe) (a + bx)^{11}}{8568e (bd - ae)^5 (d + ex)^{14}} + \frac{b^4 (11bBd + 7Abe - 18aBe) (a + bx)^{11}}{37128e (bd - ae)^6 (d + ex)^{13}} + \\
 & \frac{b^5 (11bBd + 7Abe - 18aBe) (a + bx)^{11}}{222768e (bd - ae)^7 (d + ex)^{12}} + \frac{b^6 (11bBd + 7Abe - 18aBe) (a + bx)^{11}}{2450448e (bd - ae)^8 (d + ex)^{11}}
 \end{aligned}$$

Result (type 1, 1428 leaves):

$$\begin{aligned}
 & - \frac{1}{2450448e^{12} (d + ex)^{18}} (8008a^{10}e^{10} (17Ae + B (d + 18ex)) + \\
 & 10010a^9b^9e^9 (8Ae (d + 18ex) + B (d^2 + 18dex + 153e^2x^2)) + \\
 & 9009a^8b^2e^8 (5Ae (d^2 + 18dex + 153e^2x^2) + B (d^3 + 18d^2ex + 153de^2x^2 + 816e^3x^3)) + \\
 & 3432a^7b^3e^7 (7Ae (d^3 + 18d^2ex + 153de^2x^2 + 816e^3x^3) + \\
 & 2B (d^4 + 18d^3ex + 153d^2e^2x^2 + 816de^3x^3 + 3060e^4x^4)) + \\
 & 924a^6b^4e^6 (13Ae (d^4 + 18d^3ex + 153d^2e^2x^2 + 816de^3x^3 + 3060e^4x^4) + \\
 & 5B (d^5 + 18d^4ex + 153d^3e^2x^2 + 816d^2e^3x^3 + 3060de^4x^4 + 8568e^5x^5)) + \\
 & 2772a^5b^5e^5 (2Ae (d^5 + 18d^4ex + 153d^3e^2x^2 + 816d^2e^3x^3 + 3060de^4x^4 + 8568e^5x^5) + \\
 & B (d^6 + 18d^5ex + 153d^4e^2x^2 + 816d^3e^3x^3 + 3060d^2e^4x^4 + 8568de^5x^5 + 18564e^6x^6)) + 210a^4 \\
 & b^6e^4 (11Ae (d^6 + 18d^5ex + 153d^4e^2x^2 + 816d^3e^3x^3 + 3060d^2e^4x^4 + 8568de^5x^5 + 18564e^6x^6) + \\
 & 7B (d^7 + 18d^6ex + 153d^5e^2x^2 + 816d^4e^3x^3 + \\
 & 3060d^3e^4x^4 + 8568d^2e^5x^5 + 18564de^6x^6 + 31824e^7x^7)) + \\
 & 168a^3b^7e^3 (5Ae (d^7 + 18d^6ex + 153d^5e^2x^2 + 816d^4e^3x^3 + 3060d^3e^4x^4 + 8568d^2e^5x^5 + \\
 & 18564de^6x^6 + 31824e^7x^7) + 4B (d^8 + 18d^7ex + 153d^6e^2x^2 + 816d^5e^3x^3 + \\
 & 3060d^4e^4x^4 + 8568d^3e^5x^5 + 18564d^2e^6x^6 + 31824de^7x^7 + 43758e^8x^8)) + 252a^2b^8e^2 \\
 & (Ae (d^8 + 18d^7ex + 153d^6e^2x^2 + 816d^5e^3x^3 + 3060d^4e^4x^4 + 8568d^3e^5x^5 + 18564d^2e^6x^6 + \\
 & 31824de^7x^7 + 43758e^8x^8) + B (d^9 + 18d^8ex + 153d^7e^2x^2 + 816d^6e^3x^3 + 3060d^5e^4x^4 + \\
 & 8568d^4e^5x^5 + 18564d^3e^6x^6 + 31824d^2e^7x^7 + 43758de^8x^8 + 48620e^9x^9)) + \\
 & 14ab^9e (4Ae (d^9 + 18d^8ex + 153d^7e^2x^2 + 816d^6e^3x^3 + 3060d^5e^4x^4 + 8568d^4e^5x^5 + \\
 & 18564d^3e^6x^6 + 31824d^2e^7x^7 + 43758de^8x^8 + 48620e^9x^9) + \\
 & 5B (d^{10} + 18d^9ex + 153d^8e^2x^2 + 816d^7e^3x^3 + 3060d^6e^4x^4 + 8568d^5e^5x^5 + \\
 & 18564d^4e^6x^6 + 31824d^3e^7x^7 + 43758d^2e^8x^8 + 48620de^9x^9 + 43758e^{10}x^{10})) + \\
 & b^{10} (7Ae (d^{10} + 18d^9ex + 153d^8e^2x^2 + 816d^7e^3x^3 + 3060d^6e^4x^4 + 8568d^5e^5x^5 + \\
 & 18564d^4e^6x^6 + 31824d^3e^7x^7 + 43758d^2e^8x^8 + 48620de^9x^9 + 43758e^{10}x^{10}) + \\
 & 11B (d^{11} + 18d^{10}ex + 153d^9e^2x^2 + 816d^8e^3x^3 + 3060d^7e^4x^4 + 8568d^6e^5x^5 + 18564d^5e^6x^6 + \\
 & 31824d^4e^7x^7 + 43758d^3e^8x^8 + 48620d^2e^9x^9 + 43758de^{10}x^{10} + 31824e^{11}x^{11}))
 \end{aligned}$$

Problem 1096: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^{10} (A + Bx)}{(d + ex)^{20}} dx$$

Optimal (type 1, 460 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(bd-ae)^{10} (Bd-Ae)}{19 e^{12} (d+ex)^{19}} - \frac{(bd-ae)^9 (11bBd-10Abe-aBe)}{18 e^{12} (d+ex)^{18}} + \\
 & \frac{5b (bd-ae)^8 (11bBd-9Abe-2aBe)}{17 e^{12} (d+ex)^{17}} - \frac{15b^2 (bd-ae)^7 (11bBd-8Abe-3aBe)}{16 e^{12} (d+ex)^{16}} + \\
 & \frac{2b^3 (bd-ae)^6 (11bBd-7Abe-4aBe)}{e^{12} (d+ex)^{15}} - \frac{3b^4 (bd-ae)^5 (11bBd-6Abe-5aBe)}{e^{12} (d+ex)^{14}} + \\
 & \frac{42b^5 (bd-ae)^4 (11bBd-5Abe-6aBe)}{13 e^{12} (d+ex)^{13}} - \frac{5b^6 (bd-ae)^3 (11bBd-4Abe-7aBe)}{2 e^{12} (d+ex)^{12}} + \\
 & \frac{15b^7 (bd-ae)^2 (11bBd-3Abe-8aBe)}{11 e^{12} (d+ex)^{11}} - \frac{b^8 (bd-ae) (11bBd-2Abe-9aBe)}{2 e^{12} (d+ex)^{10}} + \\
 & \frac{b^9 (11bBd-Abe-10aBe)}{9 e^{12} (d+ex)^9} - \frac{b^{10} B}{8 e^{12} (d+ex)^8}
 \end{aligned}$$

Result (type 1, 1433 leaves):

$$\begin{aligned}
 & - \frac{1}{6651216 e^{12} (d + e x)^{19}} \left( 19448 a^{10} e^{10} (18 A e + B (d + 19 e x)) + \right. \\
 & 11440 a^9 b e^9 (17 A e (d + 19 e x) + 2 B (d^2 + 19 d e x + 171 e^2 x^2)) + \\
 & 6435 a^8 b^2 e^8 (16 A e (d^2 + 19 d e x + 171 e^2 x^2) + 3 B (d^3 + 19 d^2 e x + 171 d e^2 x^2 + 969 e^3 x^3)) + \\
 & 3432 a^7 b^3 e^7 (15 A e (d^3 + 19 d^2 e x + 171 d e^2 x^2 + 969 e^3 x^3) + \\
 & 4 B (d^4 + 19 d^3 e x + 171 d^2 e^2 x^2 + 969 d e^3 x^3 + 3876 e^4 x^4)) + \\
 & 1716 a^6 b^4 e^6 (14 A e (d^4 + 19 d^3 e x + 171 d^2 e^2 x^2 + 969 d e^3 x^3 + 3876 e^4 x^4) + \\
 & 5 B (d^5 + 19 d^4 e x + 171 d^3 e^2 x^2 + 969 d^2 e^3 x^3 + 3876 d e^4 x^4 + 11628 e^5 x^5)) + \\
 & 792 a^5 b^5 e^5 (13 A e (d^5 + 19 d^4 e x + 171 d^3 e^2 x^2 + 969 d^2 e^3 x^3 + 3876 d e^4 x^4 + 11628 e^5 x^5) + \\
 & 6 B (d^6 + 19 d^5 e x + 171 d^4 e^2 x^2 + 969 d^3 e^3 x^3 + 3876 d^2 e^4 x^4 + 11628 d e^5 x^5 + 27132 e^6 x^6)) + \\
 & 330 a^4 b^6 e^4 (12 A e (d^6 + 19 d^5 e x + 171 d^4 e^2 x^2 + 969 d^3 e^3 x^3 + 3876 d^2 e^4 x^4 + \\
 & 11628 d e^5 x^5 + 27132 e^6 x^6) + 7 B (d^7 + 19 d^6 e x + 171 d^5 e^2 x^2 + \\
 & 969 d^4 e^3 x^3 + 3876 d^3 e^4 x^4 + 11628 d^2 e^5 x^5 + 27132 d e^6 x^6 + 50388 e^7 x^7)) + \\
 & 120 a^3 b^7 e^3 (11 A e (d^7 + 19 d^6 e x + 171 d^5 e^2 x^2 + 969 d^4 e^3 x^3 + 3876 d^3 e^4 x^4 + 11628 d^2 e^5 x^5 + \\
 & 27132 d e^6 x^6 + 50388 e^7 x^7) + 8 B (d^8 + 19 d^7 e x + 171 d^6 e^2 x^2 + 969 d^5 e^3 x^3 + \\
 & 3876 d^4 e^4 x^4 + 11628 d^3 e^5 x^5 + 27132 d^2 e^6 x^6 + 50388 d e^7 x^7 + 75582 e^8 x^8)) + \\
 & 36 a^2 b^8 e^2 (10 A e (d^8 + 19 d^7 e x + 171 d^6 e^2 x^2 + 969 d^5 e^3 x^3 + 3876 d^4 e^4 x^4 + \\
 & 11628 d^3 e^5 x^5 + 27132 d^2 e^6 x^6 + 50388 d e^7 x^7 + 75582 e^8 x^8) + \\
 & 9 B (d^9 + 19 d^8 e x + 171 d^7 e^2 x^2 + 969 d^6 e^3 x^3 + 3876 d^5 e^4 x^4 + 11628 d^4 e^5 x^5 + \\
 & 27132 d^3 e^6 x^6 + 50388 d^2 e^7 x^7 + 75582 d e^8 x^8 + 92378 e^9 x^9)) + \\
 & 8 a b^9 e (9 A e (d^9 + 19 d^8 e x + 171 d^7 e^2 x^2 + 969 d^6 e^3 x^3 + 3876 d^5 e^4 x^4 + 11628 d^4 e^5 x^5 + \\
 & 27132 d^3 e^6 x^6 + 50388 d^2 e^7 x^7 + 75582 d e^8 x^8 + 92378 e^9 x^9) + \\
 & 10 B (d^{10} + 19 d^9 e x + 171 d^8 e^2 x^2 + 969 d^7 e^3 x^3 + 3876 d^6 e^4 x^4 + 11628 d^5 e^5 x^5 + \\
 & 27132 d^4 e^6 x^6 + 50388 d^3 e^7 x^7 + 75582 d^2 e^8 x^8 + 92378 d e^9 x^9 + 92378 e^{10} x^{10})) + \\
 & b^{10} (8 A e (d^{10} + 19 d^9 e x + 171 d^8 e^2 x^2 + 969 d^7 e^3 x^3 + 3876 d^6 e^4 x^4 + 11628 d^5 e^5 x^5 + \\
 & 27132 d^4 e^6 x^6 + 50388 d^3 e^7 x^7 + 75582 d^2 e^8 x^8 + 92378 d e^9 x^9 + 92378 e^{10} x^{10}) + \\
 & 11 B (d^{11} + 19 d^{10} e x + 171 d^9 e^2 x^2 + 969 d^8 e^3 x^3 + 3876 d^7 e^4 x^4 + 11628 d^6 e^5 x^5 + 27132 d^5 e^6 x^6 + \\
 & 50388 d^4 e^7 x^7 + 75582 d^3 e^8 x^8 + 92378 d^2 e^9 x^9 + 92378 d e^{10} x^{10} + 75582 e^{11} x^{11})) \Big)
 \end{aligned}$$

**Problem 1097: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10} (A + B x)}{(d + e x)^{21}} dx$$

Optimal (type 1, 462 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(bd - ae)^{10} (Bd - Ae)}{20 e^{12} (d + ex)^{20}} - \frac{(bd - ae)^9 (11 b B d - 10 A b e - a B e)}{19 e^{12} (d + ex)^{19}} + \\
 & \frac{5 b (bd - ae)^8 (11 b B d - 9 A b e - 2 a B e)}{18 e^{12} (d + ex)^{18}} - \frac{15 b^2 (bd - ae)^7 (11 b B d - 8 A b e - 3 a B e)}{17 e^{12} (d + ex)^{17}} + \\
 & \frac{15 b^3 (bd - ae)^6 (11 b B d - 7 A b e - 4 a B e)}{8 e^{12} (d + ex)^{16}} - \frac{14 b^4 (bd - ae)^5 (11 b B d - 6 A b e - 5 a B e)}{5 e^{12} (d + ex)^{15}} + \\
 & \frac{3 b^5 (bd - ae)^4 (11 b B d - 5 A b e - 6 a B e)}{e^{12} (d + ex)^{14}} - \frac{30 b^6 (bd - ae)^3 (11 b B d - 4 A b e - 7 a B e)}{13 e^{12} (d + ex)^{13}} + \\
 & \frac{5 b^7 (bd - ae)^2 (11 b B d - 3 A b e - 8 a B e)}{4 e^{12} (d + ex)^{12}} - \frac{5 b^8 (bd - ae) (11 b B d - 2 A b e - 9 a B e)}{11 e^{12} (d + ex)^{11}} + \\
 & \frac{b^9 (11 b B d - A b e - 10 a B e)}{10 e^{12} (d + ex)^{10}} - \frac{b^{10} B}{9 e^{12} (d + ex)^9}
 \end{aligned}$$

Result (type 1, 1428 leaves):

$$\begin{aligned}
 & - \frac{1}{16628040 e^{12} (d + ex)^{20}} (43758 a^{10} e^{10} (19 A e + B (d + 20 e x)) + \\
 & 48620 a^9 b e^9 (9 A e (d + 20 e x) + B (d^2 + 20 d e x + 190 e^2 x^2)) + \\
 & 12870 a^8 b^2 e^8 (17 A e (d^2 + 20 d e x + 190 e^2 x^2) + 3 B (d^3 + 20 d^2 e x + 190 d e^2 x^2 + 1140 e^3 x^3)) + \\
 & 25740 a^7 b^3 e^7 (4 A e (d^3 + 20 d^2 e x + 190 d e^2 x^2 + 1140 e^3 x^3) + \\
 & B (d^4 + 20 d^3 e x + 190 d^2 e^2 x^2 + 1140 d e^3 x^3 + 4845 e^4 x^4)) + \\
 & 15015 a^6 b^4 e^6 (3 A e (d^4 + 20 d^3 e x + 190 d^2 e^2 x^2 + 1140 d e^3 x^3 + 4845 e^4 x^4) + \\
 & B (d^5 + 20 d^4 e x + 190 d^3 e^2 x^2 + 1140 d^2 e^3 x^3 + 4845 d e^4 x^4 + 15504 e^5 x^5)) + \\
 & 2574 a^5 b^5 e^5 (7 A e (d^5 + 20 d^4 e x + 190 d^3 e^2 x^2 + 1140 d^2 e^3 x^3 + 4845 d e^4 x^4 + 15504 e^5 x^5) + \\
 & 3 B (d^6 + 20 d^5 e x + 190 d^4 e^2 x^2 + 1140 d^3 e^3 x^3 + 4845 d^2 e^4 x^4 + 15504 d e^5 x^5 + 38760 e^6 x^6)) + \\
 & 495 a^4 b^6 e^4 (13 A e (d^6 + 20 d^5 e x + 190 d^4 e^2 x^2 + 1140 d^3 e^3 x^3 + 4845 d^2 e^4 x^4 + \\
 & 15504 d e^5 x^5 + 38760 e^6 x^6) + 7 B (d^7 + 20 d^6 e x + 190 d^5 e^2 x^2 + \\
 & 1140 d^4 e^3 x^3 + 4845 d^3 e^4 x^4 + 15504 d^2 e^5 x^5 + 38760 d e^6 x^6 + 77520 e^7 x^7)) + \\
 & 660 a^3 b^7 e^3 (3 A e (d^7 + 20 d^6 e x + 190 d^5 e^2 x^2 + 1140 d^4 e^3 x^3 + 4845 d^3 e^4 x^4 + 15504 d^2 e^5 x^5 + \\
 & 38760 d e^6 x^6 + 77520 e^7 x^7) + 2 B (d^8 + 20 d^7 e x + 190 d^6 e^2 x^2 + 1140 d^5 e^3 x^3 + \\
 & 4845 d^4 e^4 x^4 + 15504 d^3 e^5 x^5 + 38760 d^2 e^6 x^6 + 77520 d e^7 x^7 + 125970 e^8 x^8)) + \\
 & 45 a^2 b^8 e^2 (11 A e (d^8 + 20 d^7 e x + 190 d^6 e^2 x^2 + 1140 d^5 e^3 x^3 + 4845 d^4 e^4 x^4 + \\
 & 15504 d^3 e^5 x^5 + 38760 d^2 e^6 x^6 + 77520 d e^7 x^7 + 125970 e^8 x^8) + \\
 & 9 B (d^9 + 20 d^8 e x + 190 d^7 e^2 x^2 + 1140 d^6 e^3 x^3 + 4845 d^5 e^4 x^4 + 15504 d^4 e^5 x^5 + \\
 & 38760 d^3 e^6 x^6 + 77520 d^2 e^7 x^7 + 125970 d e^8 x^8 + 167960 e^9 x^9)) + \\
 & 90 a b^9 e (A e (d^9 + 20 d^8 e x + 190 d^7 e^2 x^2 + 1140 d^6 e^3 x^3 + 4845 d^5 e^4 x^4 + 15504 d^4 e^5 x^5 + \\
 & 38760 d^3 e^6 x^6 + 77520 d^2 e^7 x^7 + 125970 d e^8 x^8 + 167960 e^9 x^9) + \\
 & B (d^{10} + 20 d^9 e x + 190 d^8 e^2 x^2 + 1140 d^7 e^3 x^3 + 4845 d^6 e^4 x^4 + 15504 d^5 e^5 x^5 + \\
 & 38760 d^4 e^6 x^6 + 77520 d^3 e^7 x^7 + 125970 d^2 e^8 x^8 + 167960 d e^9 x^9 + 184756 e^{10} x^{10})) + \\
 & b^{10} (9 A e (d^{10} + 20 d^9 e x + 190 d^8 e^2 x^2 + 1140 d^7 e^3 x^3 + 4845 d^6 e^4 x^4 + 15504 d^5 e^5 x^5 + \\
 & 38760 d^4 e^6 x^6 + 77520 d^3 e^7 x^7 + 125970 d^2 e^8 x^8 + 167960 d e^9 x^9 + 184756 e^{10} x^{10}) + 11 B \\
 & (d^{11} + 20 d^{10} e x + 190 d^9 e^2 x^2 + 1140 d^8 e^3 x^3 + 4845 d^7 e^4 x^4 + 15504 d^6 e^5 x^5 + 38760 d^5 e^6 x^6 + \\
 & 77520 d^4 e^7 x^7 + 125970 d^3 e^8 x^8 + 167960 d^2 e^9 x^9 + 184756 d e^{10} x^{10} + 167960 e^{11} x^{11}))
 \end{aligned}$$

Problem 1098: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^{10} (A + Bx)}{(d + ex)^{22}} dx$$

Optimal (type 1, 464 leaves, 2 steps):

$$\begin{aligned} & \frac{(bd - ae)^{10} (Bd - Ae)}{21 e^{12} (d + ex)^{21}} - \frac{(bd - ae)^9 (11 b B d - 10 A b e - a B e)}{20 e^{12} (d + ex)^{20}} + \\ & \frac{5 b (bd - ae)^8 (11 b B d - 9 A b e - 2 a B e)}{19 e^{12} (d + ex)^{19}} - \frac{5 b^2 (bd - ae)^7 (11 b B d - 8 A b e - 3 a B e)}{6 e^{12} (d + ex)^{18}} + \\ & \frac{30 b^3 (bd - ae)^6 (11 b B d - 7 A b e - 4 a B e)}{17 e^{12} (d + ex)^{17}} - \frac{21 b^4 (bd - ae)^5 (11 b B d - 6 A b e - 5 a B e)}{8 e^{12} (d + ex)^{16}} + \\ & \frac{14 b^5 (bd - ae)^4 (11 b B d - 5 A b e - 6 a B e)}{5 e^{12} (d + ex)^{15}} - \frac{15 b^6 (bd - ae)^3 (11 b B d - 4 A b e - 7 a B e)}{7 e^{12} (d + ex)^{14}} + \\ & \frac{15 b^7 (bd - ae)^2 (11 b B d - 3 A b e - 8 a B e)}{13 e^{12} (d + ex)^{13}} - \frac{5 b^8 (bd - ae) (11 b B d - 2 A b e - 9 a B e)}{12 e^{12} (d + ex)^{12}} + \\ & \frac{b^9 (11 b B d - A b e - 10 a B e)}{11 e^{12} (d + ex)^{11}} - \frac{b^{10} B}{10 e^{12} (d + ex)^{10}} \end{aligned}$$

Result (type 1, 1431 leaves):

$$\begin{aligned}
 & - \frac{1}{38798760 e^{12} (d+ex)^{21}} \left( 92378 a^{10} e^{10} (20Ae + B(d+21ex)) + \right. \\
 & 48620 a^9 b e^9 (19Ae(d+21ex) + 2B(d^2 + 21dex + 210e^2x^2)) + \\
 & 72930 a^8 b^2 e^8 (6Ae(d^2 + 21dex + 210e^2x^2) + B(d^3 + 21d^2ex + 210de^2x^2 + 1330e^3x^3)) + \\
 & 11440 a^7 b^3 e^7 (17Ae(d^3 + 21d^2ex + 210de^2x^2 + 1330e^3x^3) + \\
 & 4B(d^4 + 21d^3ex + 210d^2e^2x^2 + 1330de^3x^3 + 5985e^4x^4)) + \\
 & 5005 a^6 b^4 e^6 (16Ae(d^4 + 21d^3ex + 210d^2e^2x^2 + 1330de^3x^3 + 5985e^4x^4) + \\
 & 5B(d^5 + 21d^4ex + 210d^3e^2x^2 + 1330d^2e^3x^3 + 5985de^4x^4 + 20349e^5x^5)) + \\
 & 6006 a^5 b^5 e^5 (5Ae(d^5 + 21d^4ex + 210d^3e^2x^2 + 1330d^2e^3x^3 + 5985de^4x^4 + 20349e^5x^5) + \\
 & 2B(d^6 + 21d^5ex + 210d^4e^2x^2 + 1330d^3e^3x^3 + 5985d^2e^4x^4 + 20349de^5x^5 + 54264e^6x^6)) + \\
 & 5005 a^4 b^6 e^4 (2Ae(d^6 + 21d^5ex + 210d^4e^2x^2 + 1330d^3e^3x^3 + 5985d^2e^4x^4 + \\
 & 20349de^5x^5 + 54264e^6x^6) + B(d^7 + 21d^6ex + 210d^5e^2x^2 + 1330d^4e^3x^3 + \\
 & 5985d^3e^4x^4 + 20349d^2e^5x^5 + 54264de^6x^6 + 116280e^7x^7)) + \\
 & 220 a^3 b^7 e^3 (13Ae(d^7 + 21d^6ex + 210d^5e^2x^2 + 1330d^4e^3x^3 + 5985d^3e^4x^4 + 20349d^2e^5x^5 + \\
 & 54264de^6x^6 + 116280e^7x^7) + 8B(d^8 + 21d^7ex + 210d^6e^2x^2 + 1330d^5e^3x^3 + \\
 & 5985d^4e^4x^4 + 20349d^3e^5x^5 + 54264d^2e^6x^6 + 116280de^7x^7 + 203490e^8x^8)) + \\
 & 165 a^2 b^8 e^2 (4Ae(d^8 + 21d^7ex + 210d^6e^2x^2 + 1330d^5e^3x^3 + 5985d^4e^4x^4 + \\
 & 20349d^3e^5x^5 + 54264d^2e^6x^6 + 116280de^7x^7 + 203490e^8x^8) + \\
 & 3B(d^9 + 21d^8ex + 210d^7e^2x^2 + 1330d^6e^3x^3 + 5985d^5e^4x^4 + 20349d^4e^5x^5 + \\
 & 54264d^3e^6x^6 + 116280d^2e^7x^7 + 203490de^8x^8 + 293930e^9x^9)) + \\
 & 10 a b^9 e (11Ae(d^9 + 21d^8ex + 210d^7e^2x^2 + 1330d^6e^3x^3 + 5985d^5e^4x^4 + 20349d^4e^5x^5 + \\
 & 54264d^3e^6x^6 + 116280d^2e^7x^7 + 203490de^8x^8 + 293930e^9x^9) + \\
 & 10B(d^{10} + 21d^9ex + 210d^8e^2x^2 + 1330d^7e^3x^3 + 5985d^6e^4x^4 + 20349d^5e^5x^5 + \\
 & 54264d^4e^6x^6 + 116280d^3e^7x^7 + 203490d^2e^8x^8 + 293930de^9x^9 + 352716e^{10}x^{10})) + \\
 & b^{10} (10Ae(d^{10} + 21d^9ex + 210d^8e^2x^2 + 1330d^7e^3x^3 + 5985d^6e^4x^4 + 20349d^5e^5x^5 + \\
 & 54264d^4e^6x^6 + 116280d^3e^7x^7 + 203490d^2e^8x^8 + 293930de^9x^9 + 352716e^{10}x^{10}) + 11B \\
 & (d^{11} + 21d^{10}ex + 210d^9e^2x^2 + 1330d^8e^3x^3 + 5985d^7e^4x^4 + 20349d^6e^5x^5 + 54264d^5e^6x^6 + \\
 & 116280d^4e^7x^7 + 203490d^3e^8x^8 + 293930d^2e^9x^9 + 352716de^{10}x^{10} + 352716e^{11}x^{11})) \left. \right)
 \end{aligned}$$

Problem 1110: Result more than twice size of optimal antiderivative.

$$\int \frac{(A+Bx)(d+ex)^5}{(a+bx)^2} dx$$

Optimal (type 3, 227 leaves, 2 steps):

$$\begin{aligned}
 & \frac{5e(bd-ae)^3(bBd+2Abe-3aBe)x}{b^6} - \\
 & \frac{(Ab-aB)(bd-ae)^5}{b^7(a+bx)} + \frac{5e^2(bd-ae)^2(bBd+Abe-2aBe)(a+bx)^2}{b^7} + \\
 & \frac{5e^3(bd-ae)(2bBd+Abe-3aBe)(a+bx)^3}{3b^7} + \frac{e^4(5bBd+Abe-6aBe)(a+bx)^4}{4b^7} + \\
 & \frac{Be^5(a+bx)^5}{5b^7} + \frac{(bd-ae)^4(bBd+5Abe-6aBe)\text{Log}[a+bx]}{b^7}
 \end{aligned}$$

Result (type 3, 500 leaves):

$$\frac{1}{60 b^7 (a + b x)} \left( B (-60 a^6 e^5 + 300 a^5 b e^4 (d + e x) + 60 a^4 b^2 e^3 (-10 d^2 - 20 d e x + 3 e^2 x^2) + 30 a^3 b^3 e^2 (20 d^3 + 60 d^2 e x - 25 d e^2 x^2 - 2 e^3 x^3) + 10 a^2 b^4 e (-30 d^4 - 120 d^3 e x + 120 d^2 e^2 x^2 + 25 d e^3 x^3 + 3 e^4 x^4) + b^6 e x^2 (300 d^4 + 300 d^3 e x + 200 d^2 e^2 x^2 + 75 d e^3 x^3 + 12 e^4 x^4) + a b^5 (60 d^5 + 300 d^4 e x - 900 d^3 e^2 x^2 - 400 d^2 e^3 x^3 - 125 d e^4 x^4 - 18 e^5 x^5) \right) - 5 A b (-12 a^5 e^5 + 12 a^4 b e^4 (5 d + 4 e x) + 30 a^3 b^2 e^3 (-4 d^2 - 6 d e x + e^2 x^2) - 10 a^2 b^3 e^2 (-12 d^3 - 24 d^2 e x + 12 d e^2 x^2 + e^3 x^3) + 5 a b^4 e (-12 d^4 - 24 d^3 e x + 36 d^2 e^2 x^2 + 8 d e^3 x^3 + e^4 x^4) + b^5 (12 d^5 - 120 d^3 e^2 x^2 - 60 d^2 e^3 x^3 - 20 d e^4 x^4 - 3 e^5 x^5)) + 60 (b d - a e)^4 (b B d + 5 A b e - 6 a B e) (a + b x) \text{Log}[a + b x]$$

Problem 1180: Result more than twice size of optimal antiderivative.

$$\int (5 - 2 x)^6 (2 + 3 x)^3 (-16 + 33 x) dx$$

Optimal (type 1, 18 leaves, 1 step):

$$-\frac{1}{2} (5 - 2 x)^7 (2 + 3 x)^4$$

Result (type 1, 56 leaves):

$$-2000000 x - 37500 x^2 + 3987500 x^3 - \frac{98125 x^4}{2} - 3816225 x^5 + 1497230 x^6 + 1235404 x^7 - 1256376 x^8 + 452304 x^9 - 76896 x^{10} + 5184 x^{11}$$

Problem 2505: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a + b x} (e + f x) \sqrt{2 b e - a f + b f x}} dx$$

Optimal (type 3, 59 leaves, 2 steps):

$$\frac{\text{ArcTan}\left[\frac{\sqrt{f} \sqrt{a + b x} \sqrt{2 b e - a f + b f x}}{b e - a f}\right]}{\sqrt{f} (b e - a f)}$$

Result (type 3, 81 leaves):

$$\frac{i \text{Log}\left[-\frac{2 i \sqrt{f} (-b e + a f)}{e + f x} + \frac{2 f \sqrt{a + b x} \sqrt{2 b e - a f + b f x}}{e + f x}\right]}{\sqrt{f} (b e - a f)}$$

Problem 2625: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a + b x} \sqrt{c + \frac{b(-1+c)x}{a}} \sqrt{e + \frac{b(-1+e)x}{a}}} dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{2\sqrt{a} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b\sqrt{1-c}}$$

Result (type 4, 129 leaves):

$$- \left( \left( 2(a+bx) \sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+c}} \sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{a}{-1+c}}}}{\sqrt{a+bx}}\right], \frac{-1+c}{-1+e}\right] \right) \right. \\ \left. \left( b \sqrt{-\frac{a}{-1+c}} \sqrt{c+\frac{b(-1+c)x}{a}} \sqrt{e+\frac{b(-1+e)x}{a}} \right) \right) /$$

**Problem 2628: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{e+\frac{b(-1+e)x}{a}}}{\sqrt{a+bx} \sqrt{c+\frac{b(-1+c)x}{a}}} dx$$

Optimal (type 4, 58 leaves, 1 step):

$$\frac{2\sqrt{a} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c}\sqrt{a+bx}}{\sqrt{a}}\right], \frac{1-e}{1-c}\right]}{b\sqrt{1-c}}$$

Result (type 4, 191 leaves):

$$\begin{aligned}
 & - \left( \left( 2 (a+bx)^{3/2} \left( - \frac{\sqrt{-\frac{a}{-1+e}} \left(-1+c+\frac{a}{a+bx}\right) \left(-1+e+\frac{a}{a+bx}\right)}{-1+c} + \right. \right. \right. \\
 & \left. \left. \left. \frac{a \sqrt{\frac{-1+c+\frac{a}{a+bx}}{-1+c}} \sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-\frac{a}{-1+e}}}{\sqrt{a+bx}}\right], \frac{-1+e}{-1+c}\right]}{\sqrt{a+bx}} \right) \right) \right) \\
 & \left( a b \sqrt{-\frac{a}{-1+e}} \sqrt{c+\frac{b(-1+c)x}{a}} \sqrt{e+\frac{b(-1+e)x}{a}} \right)
 \end{aligned}$$

Problem 2629: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx} \sqrt{e+\frac{b(-1+e)x}{a}}} dx$$

Optimal (type 4, 96 leaves, 2 steps):

$$\frac{2 \sqrt{a} \sqrt{c+dx} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-e} \sqrt{a+bx}}{\sqrt{a}}\right], -\frac{ad}{(bc-ad)(1-e)}\right]}{b \sqrt{1-e} \sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Result (type 4, 200 leaves):

$$\begin{aligned}
 & \left( 2 \sqrt{\frac{-1+e+\frac{a}{a+bx}}{-1+e}} \left( b \sqrt{a-\frac{bc}{d}} \sqrt{a+bx} (c+dx) \sqrt{\frac{ae+b(-1+e)x}{(-1+e)(a+bx)}} - \right. \right. \\
 & \left. \left. (bc-ad)(a+bx) \sqrt{\frac{b(c+dx)}{d(a+bx)}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{a-\frac{bc}{d}}}{\sqrt{a+bx}}\right], \frac{ad}{(bc-ad)(-1+e)}\right] \right) \right) \\
 & \left( b^2 \sqrt{a-\frac{bc}{d}} \sqrt{c+dx} \sqrt{e+\frac{b(-1+e)x}{a}} \right)
 \end{aligned}$$

Problem 2630: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+\frac{b(-1+c)x}{a}} \sqrt{e+\frac{b(-1+e)x}{a}}} dx$$

Optimal (type 4, 162 leaves, 2 steps):

$$- \left( \left( 2a \sqrt{c-e} \sqrt{a+bx} \sqrt{-\frac{(1-c)(ae-b(1-e)x)}{a(c-e)}} \right. \right. \\ \left. \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{\sqrt{1-e} \sqrt{c-\frac{b(1-c)x}{a}}}{\sqrt{c-e}} \right], \frac{c-e}{1-e} \right] \right) / \right. \\ \left. \left( b(1-c) \sqrt{1-e} \sqrt{\frac{(1-c)(a+bx)}{a}} \sqrt{e-\frac{b(1-e)x}{a}} \right) \right)$$

Result (type 4, 103 leaves):

$$- \left( \left( 2i a \sqrt{a+bx} \left( \text{EllipticE} \left[ i \text{ArcSinh} \left[ \sqrt{\frac{(-1+c)(a+bx)}{a}} \right], \frac{-1+e}{-1+c} \right] - \text{EllipticF} \left[ \right. \right. \right. \right. \\ \left. \left. \left. \left. i \text{ArcSinh} \left[ \sqrt{\frac{(-1+c)(a+bx)}{a}} \right], \frac{-1+e}{-1+c} \right] \right] \right) / \left( b(-1+e) \sqrt{\frac{(-1+c)(a+bx)}{a}} \right) \right)$$

Problem 2662: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-2x}}{\sqrt{-3-5x} \sqrt{2+3x}} dx$$

Optimal (type 4, 31 leaves, 1 step):

$$\frac{2}{3} \sqrt{\frac{7}{5}} \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{5} \sqrt{2+3x} \right], \frac{2}{35} \right]$$

Result (type 4, 109 leaves):

$$- \left( \left( 2 \left( \frac{3(-3+x+10x^2)}{\sqrt{2+3x}} + \sqrt{35} \sqrt{\frac{-1+2x}{2+3x}} (2+3x) \sqrt{\frac{3+5x}{2+3x}} \right. \right. \right. \\ \left. \left. \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{7}{2}}}{\sqrt{2+3x}}\right], \frac{2}{35}\right] \right) \right) / \left( 15 \sqrt{-3-5x} \sqrt{1-2x} \right) \right)$$

Problem 2666: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{1-2x}}{\sqrt{2+3x} \sqrt{3+5x}} dx$$

Optimal (type 4, 49 leaves, 2 steps):

$$\frac{2 \sqrt{\frac{7}{5}} \sqrt{-3-5x} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{5} \sqrt{2+3x}\right], \frac{2}{35}\right]}{3 \sqrt{3+5x}}$$

Result (type 4, 121 leaves):

$$\left( 2 \sqrt{1-2x} \left( 5 \sqrt{3+5x} (-2+x+6x^2) + \sqrt{33} \sqrt{\frac{-1+2x}{3+5x}} \sqrt{\frac{2+3x}{3+5x}} (3+5x)^2 \right. \right. \\ \left. \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{\frac{11}{2}}}{\sqrt{3+5x}}\right], -\frac{2}{33}\right] \right) \right) / \left( 15 \sqrt{2+3x} (-3+x+10x^2) \right)$$

Problem 2807: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{3+5x}}{\sqrt{1-2x} (2+3x)^{3/2}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$-\frac{2 \sqrt{1-2x} \sqrt{3+5x}}{7 \sqrt{2+3x}} + \frac{2 \sqrt{\frac{5}{7}} \sqrt{-3-5x} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{5} \sqrt{2+3x}\right], \frac{2}{35}\right]}{3 \sqrt{3+5x}}$$

Result (type 4, 70 leaves):

$$\frac{1}{42 + 63x} \left( -6\sqrt{1-2x}\sqrt{2+3x}\sqrt{3+5x} - 2i\sqrt{33}(2+3x)\text{EllipticE}\left[i\text{ArcSinh}\left[\sqrt{9+15x}\right], -\frac{2}{33}\right] \right)$$

**Problem 2829:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1+x}\sqrt{2+x}\sqrt{3+x}} dx$$

Optimal (type 4, 12 leaves, 1 step):

$$-2\text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{3+x}}\right], 2\right]$$

Result (type 4, 55 leaves):

$$\frac{2i\sqrt{1+\frac{1}{1+x}}\text{EllipticF}\left[i\text{ArcSinh}\left[\frac{1}{\sqrt{1+x}}\right], 2\right]}{\sqrt{\frac{2+x}{3+x}}\sqrt{\frac{3+x}{1+x}}}$$

**Problem 2830:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{3-x}\sqrt{1+x}\sqrt{2+x}} dx$$

Optimal (type 4, 16 leaves, 1 step):

$$2\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1+x}}{2}\right], -4\right]$$

Result (type 4, 74 leaves):

$$\frac{i\sqrt{1+\frac{4}{-3+x}}\sqrt{1+\frac{5}{-3+x}}(-3+x)^{3/2}\text{EllipticF}\left[i\text{ArcSinh}\left[\frac{2}{\sqrt{-3+x}}\right], \frac{5}{4}\right]}{\sqrt{-(-3+x)(1+x)}\sqrt{2+x}}$$

**Problem 2831:** Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{2-x}\sqrt{1+x}\sqrt{3+x}} dx$$

Optimal (type 4, 24 leaves, 1 step):

$$\sqrt{2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{3}}\right], -\frac{3}{2}\right]$$

Result (type 4, 67 leaves):

$$\frac{2(3+x) \sqrt{1-\frac{5}{3+x}} \sqrt{1-\frac{2}{3+x}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{5}}{\sqrt{3+x}}\right], \frac{2}{5}\right]}{\sqrt{-50+35(3+x)-5(3+x)^2}}$$

**Problem 2832: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{2-x} \sqrt{3-x} \sqrt{1+x}} dx$$

Optimal (type 4, 18 leaves, 1 step):

$$\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1+x}}{\sqrt{3}}\right], \frac{3}{4}\right]$$

Result (type 4, 65 leaves):

$$\frac{2i \sqrt{1-\frac{3}{2-x}} \sqrt{1+\frac{1}{2-x}} (2-x) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{1}{\sqrt{2-x}}\right], -3\right]}{\sqrt{-(-3+x)(1+x)}}$$

**Problem 2833: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-x} \sqrt{2+x} \sqrt{3+x}} dx$$

Optimal (type 4, 18 leaves, 1 step):

$$2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{2+x}}{\sqrt{3}}\right], -3\right]$$

Result (type 4, 78 leaves):

$$\frac{2i \sqrt{-(-1+x)(2+x)} \sqrt{3+x} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-1+x}}\right], \frac{4}{3}\right]}{\sqrt{3+\frac{9}{-1+x}} (-1+x)^{3/2} \sqrt{\frac{3+x}{-1+x}}}$$

**Problem 2834: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-x} \sqrt{3-x} \sqrt{2+x}} dx$$

Optimal (type 4, 25 leaves, 1 step):

$$\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{2+x}}{\sqrt{3}}\right], \frac{3}{5}\right]}{\sqrt{5}}$$

Result (type 4, 68 leaves):

$$\frac{2 \sqrt{\frac{-3+x}{-1+x}} (-1+x) \sqrt{\frac{2+x}{-1+x}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{-\sqrt{3}}{\sqrt{1-x}}\right], -\frac{2}{3}\right]}{\sqrt{3} \sqrt{6+x-x^2}}$$

**Problem 2835: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-x} \sqrt{2-x} \sqrt{3+x}} dx$$

Optimal (type 4, 23 leaves, 1 step):

$$\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{3+x}}{2}\right], \frac{4}{5}\right]}{\sqrt{5}}$$

Result (type 4, 65 leaves):

$$\frac{2 i \sqrt{1-\frac{4}{1-x}} \sqrt{1+\frac{1}{1-x}} (1-x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{1-x}}\right], -4\right]}{\sqrt{-(-2+x)(3+x)}}$$

**Problem 2836: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-x} \sqrt{2-x} \sqrt{3-x}} dx$$

Optimal (type 4, 14 leaves, 1 step):

$$2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{3-x}}\right], 2\right]$$

Result (type 4, 67 leaves):

$$\frac{2 i \sqrt{\frac{-3+x}{-1+x}} \sqrt{\frac{-2+x}{-1+x}} (-1+x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{1-x}}\right], 2\right]}{\sqrt{2-x} \sqrt{3-x}}$$

**Problem 2837:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-3+x} \sqrt{-2+x} \sqrt{-1+x}} dx$$

Optimal (type 4, 12 leaves, 1 step):

$$-2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{-1+x}}\right], 2\right]$$

Result (type 4, 59 leaves):

$$\frac{2 i \sqrt{1 + \frac{1}{-3+x}} \sqrt{1 + \frac{2}{-3+x}} (-3+x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{1}{\sqrt{-3+x}}\right], 2\right]}{\sqrt{-2+x} \sqrt{-1+x}}$$

**Problem 2839:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-2-x} \sqrt{-3+x} \sqrt{-1+x}} dx$$

Optimal (type 4, 41 leaves, 2 steps):

$$\frac{2 \sqrt{2+x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\frac{2+x}{3}}}\right], \frac{5}{3}\right]}{\sqrt{3} \sqrt{-2-x}}$$

Result (type 4, 72 leaves):

$$\frac{2 i \sqrt{\frac{-3+x}{-1+x}} \sqrt{\frac{-1+x}{2+x}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-2-x}}\right], \frac{5}{3}\right]}{\sqrt{3} \sqrt{\frac{-3+x}{2+x}}}$$

**Problem 2841:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{-1-x} \sqrt{-3+x} \sqrt{-2+x}} dx$$

Optimal (type 4, 41 leaves, 2 steps):

$$\frac{2 \sqrt{1+x} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{\frac{1+x}{3}}}\right], \frac{4}{3}\right]}{\sqrt{3} \sqrt{-1-x}}$$

Result (type 4, 72 leaves):

$$\frac{2 i \sqrt{\frac{-3+x}{-2+x}} \sqrt{\frac{-2+x}{1+x}} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-1-x}}\right], \frac{4}{3}\right]}{\sqrt{3} \sqrt{\frac{-3+x}{1+x}}}$$

**Problem 2842: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{-3-x} \sqrt{-1-x} \sqrt{-2+x}} dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$\frac{2 \sqrt{1+x} \sqrt{3+x} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\frac{3+x}{5} + \frac{x}{5}}}\right], \frac{2}{5}\right]}{\sqrt{5} \sqrt{-3-x} \sqrt{-1-x}}$$

Result (type 4, 75 leaves):

$$\frac{2 i \sqrt{1 + \frac{3}{-2+x}} \sqrt{1 + \frac{5}{-2+x}} (-2+x) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{\sqrt{3}}{\sqrt{-2+x}}\right], \frac{5}{3}\right]}{\sqrt{-15-3(-2+x)} \sqrt{-1-x}}$$

**Problem 2843: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{\sqrt{-2-x} \sqrt{-1-x} \sqrt{-3+x}} dx$$

Optimal (type 4, 57 leaves, 3 steps):

$$\frac{2 \sqrt{1+x} \sqrt{2+x} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{\frac{2+x}{5} + \frac{x}{5}}}\right], \frac{1}{5}\right]}{\sqrt{5} \sqrt{-2-x} \sqrt{-1-x}}$$

Result (type 4, 69 leaves):

$$\frac{i \sqrt{1 + \frac{4}{-3+x}} \sqrt{1 + \frac{5}{-3+x}} (-3+x) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{2}{\sqrt{-3+x}}\right], \frac{5}{4}\right]}{\sqrt{-2-x} \sqrt{-1-x}}$$

**Problem 2844: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-3-x} \sqrt{-2-x} \sqrt{-1-x}} dx$$

Optimal (type 4, 14 leaves, 1 step):

$$2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{-1-x}}\right], 2\right]$$

Result (type 4, 67 leaves):

$$\frac{2 i \sqrt{\frac{1+x}{3+x}} \sqrt{\frac{2+x}{3+x}} (3+x) \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{1}{\sqrt{-3-x}}\right], 2\right]}{\sqrt{-2-x} \sqrt{-1-x}}$$

**Problem 2845: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{1}{(a+bx)^{3/2} \sqrt{c+dx} \sqrt{e+fx}} dx$$

Optimal (type 4, 204 leaves, 4 steps):

$$-\frac{2b\sqrt{c+dx}\sqrt{e+fx}}{(bc-ad)(be-af)\sqrt{a+bx}} + \left( 2\sqrt{f}\sqrt{-de+cf}\sqrt{a+bx}\sqrt{\frac{d(e+fx)}{de-ef}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{f}\sqrt{c+dx}}{\sqrt{-de+cf}}\right], -\frac{b(de-ef)}{(bc-ad)f}\right] \right) / \left( (bc-ad)(be-af)\sqrt{-\frac{d(a+bx)}{bc-ad}}\sqrt{e+fx} \right)$$

Result (type 4, 201 leaves):

$$\left( 2b\sqrt{c+dx}\sqrt{e+fx} \left( -1 - \frac{1}{\sqrt{\frac{b(e+fx)}{be-af}}} i \sqrt{\frac{d(a+bx)}{b(c+dx)}} \left( \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{\frac{d(a+bx)}{bc-ad}}\right], \frac{bcf-adf}{bde-adf}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{\frac{d(a+bx)}{bc-ad}}\right], \frac{bcf-adf}{bde-adf}\right] \right) \right) / \left( (bc-ad)(be-af)\sqrt{a+bx} \right)$$

Problem 2846: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(a+bx)^{5/2} \sqrt{c+dx} \sqrt{e+fx}} dx$$

Optimal (type 4, 437 leaves, 8 steps):

$$\begin{aligned} & -\frac{2b\sqrt{c+dx}\sqrt{e+fx}}{3(bc-ad)(be-af)(a+bx)^{3/2}} + \\ & \frac{4b(bde+bcf-2adf)\sqrt{c+dx}\sqrt{e+fx}}{3(bc-ad)^2(be-af)^2\sqrt{a+bx}} - \left( 4\sqrt{d}(bde+bcf-2adf) \right. \\ & \left. \sqrt{\frac{b(c+dx)}{bc-ad}} \sqrt{e+fx} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right], \frac{(bc-ad)f}{d(be-af)}\right] \right) / \\ & \left( 3(-bc+ad)^{3/2}(be-af)^2\sqrt{c+dx} \sqrt{\frac{b(e+fx)}{be-af}} \right) + \left( 2\sqrt{d}(2bde+bcf-3adf) \right. \\ & \left. \sqrt{\frac{b(c+dx)}{bc-ad}} \sqrt{\frac{b(e+fx)}{be-af}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-bc+ad}}\right], \frac{(bc-ad)f}{d(be-af)}\right] \right) / \\ & (3b(-bc+ad)^{3/2}(be-af)\sqrt{c+dx}\sqrt{e+fx}) \end{aligned}$$

Result (type 4, 449 leaves):

$$\begin{aligned}
 & \frac{1}{3 b \sqrt{-a + \frac{bc}{d}} (bc - ad)^2 (be - af)^2 (a + bx)^{3/2} \sqrt{c + dx} \sqrt{e + fx}} \\
 & 2 \left( b^2 \sqrt{-a + \frac{bc}{d}} (c + dx) (e + fx) ((bc - ad)(be - af) - 2(bde + bcf - 2adf)(a + bx)) + \right. \\
 & (a + bx) \left( 2 b^2 \sqrt{-a + \frac{bc}{d}} (bde + bcf - 2adf)(c + dx)(e + fx) + \right. \\
 & 2 i (bc - ad) f (bde + bcf - 2adf) (a + bx)^{3/2} \sqrt{\frac{b(c + dx)}{d(a + bx)}} \sqrt{\frac{b(e + fx)}{f(a + bx)}} \text{EllipticE} \left[ \right. \\
 & \left. i \text{ArcSinh} \left[ \frac{\sqrt{-a + \frac{bc}{d}}}{\sqrt{a + bx}} \right], \frac{bde - adf}{bcf - adf} \right] - i (bc - ad) f (bde + 2bcf - 3adf) (a + bx)^{3/2} \\
 & \left. \left. \sqrt{\frac{b(c + dx)}{d(a + bx)}} \sqrt{\frac{b(e + fx)}{f(a + bx)}} \text{EllipticF} \left[ i \text{ArcSinh} \left[ \frac{\sqrt{-a + \frac{bc}{d}}}{\sqrt{a + bx}} \right], \frac{bde - adf}{bcf - adf} \right] \right) \right)
 \end{aligned}$$

Problem 2850: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{2 + 3x}}{\sqrt{1 - 2x} \sqrt{3 + 5x}} dx$$

Optimal (type 4, 31 leaves, 1 step):

$$-\sqrt{\frac{7}{5}} \text{EllipticE} \left[ \text{ArcSin} \left[ \sqrt{\frac{5}{11}} \sqrt{1 - 2x} \right], \frac{33}{35} \right]$$

Result (type 4, 129 leaves):

$$\left( \sqrt{2+3x} \sqrt{\frac{-1+2x}{3+5x}} \right. \\ \left. \left( 5 \sqrt{\frac{-1+2x}{3+5x}} \sqrt{\frac{2+3x}{3+5x}} \sqrt{3+5x} + i \sqrt{2} \operatorname{EllipticE} \left[ i \operatorname{ArcSinh} \left[ \frac{1}{\sqrt{9+15x}} \right], -\frac{33}{2} \right] \right) \right) / \left( 5 \sqrt{1-2x} \sqrt{\frac{2+3x}{3+5x}} \right)$$

**Problem 2851:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1-2x} \sqrt{2+3x} \sqrt{3+5x}} dx$$

Optimal (type 4, 29 leaves, 1 step):

$$\frac{2 \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \sqrt{\frac{3}{7}} \sqrt{1-2x} \right], \frac{35}{33} \right]}{\sqrt{33}}$$

Result (type 4, 74 leaves):

$$\frac{i \sqrt{2+3x} \sqrt{\frac{-2+4x}{3+5x}} \operatorname{EllipticF} \left[ i \operatorname{ArcSinh} \left[ \frac{1}{\sqrt{9+15x}} \right], -\frac{33}{2} \right]}{\sqrt{1-2x} \sqrt{\frac{2+3x}{3+5x}}}$$

**Problem 2858:** Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{2+3x}}{\sqrt{1-2x} (3+5x)^{3/2}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$-\frac{2 \sqrt{1-2x} \sqrt{2+3x}}{11 \sqrt{3+5x}} + \frac{2 \sqrt{\frac{7}{5}} \sqrt{-3-5x} \operatorname{EllipticE} \left[ \operatorname{ArcSin} \left[ \sqrt{5} \sqrt{2+3x} \right], \frac{2}{35} \right]}{11 \sqrt{3+5x}}$$

Result (type 4, 61 leaves):

$$\frac{2}{55} \left( -\frac{5 \sqrt{1-2x} \sqrt{2+3x}}{\sqrt{3+5x}} - i \sqrt{33} \operatorname{EllipticE} \left[ i \operatorname{ArcSinh} \left[ \sqrt{9+15x} \right], -\frac{2}{33} \right] \right)$$

**Problem 2872: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\sqrt{x}}{\sqrt{a+2x} \sqrt{c+2x}} dx$$

Optimal (type 4, 86 leaves, 3 steps):

$$\frac{\sqrt{a-c} \sqrt{x} \sqrt{-\frac{c+2x}{a-c}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+2x}}{\sqrt{a-c}}\right], 1 - \frac{c}{a}\right]}{\sqrt{2} \sqrt{-\frac{x}{a}} \sqrt{c+2x}}$$

Result (type 4, 120 leaves):

$$-\left(\left(\left(i c \sqrt{1 + \frac{2x}{a}} \sqrt{1 + \frac{2x}{c}} \left(\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{1}{a}} \sqrt{x}\right], \frac{a}{c}\right] - \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{1}{a}} \sqrt{x}\right], \frac{a}{c}\right]\right)\right)\right) / \left(\sqrt{2} \sqrt{\frac{1}{a}} \sqrt{a+2x} \sqrt{c+2x}\right)$$

**Problem 2873: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{4-x} \sqrt{5-x} \sqrt{-3+x}} dx$$

Optimal (type 4, 18 leaves, 1 step):

$$\sqrt{2} \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{-3+x}\right], \frac{1}{2}\right]$$

Result (type 4, 46 leaves):

$$\frac{2 \sqrt{-15+8x-x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{1}{\sqrt{4-x}}\right], -1\right]}{\sqrt{1 - \frac{1}{(-4+x)^2}} (-4+x)}$$

**Problem 2874: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{4-x} \sqrt{(5-x)(-3+x)}} dx$$

Optimal (type 4, 14 leaves, 3 steps):

$$-2 \text{EllipticF}\left[\text{ArcSin}\left[\sqrt{4-x}\right], -1\right]$$

Result (type 4, 46 leaves):

$$\frac{2 \sqrt{-15 + 8x - x^2} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{4-x}}\right], -1\right]}{\sqrt{1 - \frac{1}{(-4+x)^2}} (-4+x)}$$

Problem 2875: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4-x} \sqrt{-15+8x-x^2}} dx$$

Optimal (type 4, 14 leaves, 2 steps):

$$-2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{4-x}\right], -1\right]$$

Result (type 4, 44 leaves):

$$-\frac{2 \sqrt{1 - \frac{1}{(-4+x)^2}} (-4+x) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{1}{\sqrt{4-x}}\right], -1\right]}{\sqrt{-15+8x-x^2}}$$

Problem 2876: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{6-x} \sqrt{-2+x} \sqrt{-1+x}} dx$$

Optimal (type 4, 16 leaves, 1 step):

$$2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{-2+x}}{2}\right], -4\right]$$

Result (type 4, 74 leaves):

$$\frac{i \sqrt{1 + \frac{4}{-6+x}} \sqrt{1 + \frac{5}{-6+x}} (-6+x)^{3/2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{2}{\sqrt{-6+x}}\right], \frac{5}{4}\right]}{\sqrt{-(-6+x)} (-2+x) \sqrt{-1+x}}$$

Problem 2877: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{(6-x)(-2+x)} \sqrt{-1+x}} dx$$

Optimal (type 4, 25 leaves, 3 steps):

$$-\frac{2 \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{6-x}}{2}\right], \frac{4}{5}\right]}{\sqrt{5}}$$

Result (type 4, 74 leaves):

$$\frac{i \sqrt{1 + \frac{4}{-6+x}} \sqrt{1 + \frac{5}{-6+x}} (-6+x)^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[\frac{2}{\sqrt{-6+x}}\right], \frac{5}{4}\right]}{\sqrt{-(-6+x)} \sqrt{-2+x} \sqrt{-1+x}}$$

Problem 2878: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1+x} \sqrt{-12+8x-x^2}} dx$$

Optimal (type 4, 25 leaves, 2 steps):

$$\frac{2 \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{6-x}}{2}\right], \frac{4}{5}\right]}{\sqrt{5}}$$

Result (type 4, 68 leaves):

$$\frac{2 \sqrt{\frac{-6+x}{-1+x}} \sqrt{\frac{-2+x}{-1+x}} (-1+x) \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{5}}{\sqrt{-1+x}}\right], \frac{1}{5}\right]}{\sqrt{5} \sqrt{-12+8x-x^2}}$$

Problem 2910: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(1-2x)^{3/2} \sqrt{2+3x} \sqrt{3+5x}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$\frac{4 \sqrt{2+3x} \sqrt{3+5x}}{77 \sqrt{1-2x}} + \frac{2 \sqrt{\frac{5}{7}} \sqrt{-3-5x} \text{EllipticE}\left[\text{ArcSin}\left[\sqrt{5} \sqrt{2+3x}\right], \frac{2}{35}\right]}{11 \sqrt{3+5x}}$$

Result (type 4, 61 leaves):

$$\frac{2}{77} \left( \frac{2 \sqrt{2+3x} \sqrt{3+5x}}{\sqrt{1-2x}} - i \sqrt{33} \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{9+15x}\right], -\frac{2}{33}\right] \right)$$

Problem 2989: Result unnecessarily involves higher level functions.

$$\int (a+bx)^{1/3} (c+dx)^{2/3} (e+fx)^2 dx$$

Optimal (type 3, 571 leaves, 5 steps):

$$\frac{1}{81 b^3 d^3} (bc - ad) (10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2))$$

$$(a + b x)^{1/3} (c + d x)^{2/3} + \frac{1}{54 b^3 d^2}$$

$$(10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) (a + b x)^{4/3} (c + d x)^{2/3} +$$

$$\frac{f (15 b d e - 7 b c f - 8 a d f) (a + b x)^{4/3} (c + d x)^{5/3} + f (a + b x)^{4/3} (c + d x)^{5/3} (e + f x)}{36 b^2 d^2 + 4 b d}$$

$$\frac{1}{81 \sqrt{3} b^{11/3} d^{10/3}} (bc - ad)^2 (10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2))$$

$$\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}}\right] + \frac{1}{486 b^{11/3} d^{10/3}}$$

$$(bc - ad)^2 (10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) \text{Log}[a + b x] +$$

$$\frac{1}{162 b^{11/3} d^{10/3}} (bc - ad)^2$$

$$(10 a^2 d^2 f^2 - 10 a b d f (3 d e - c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2)) \text{Log}\left[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}\right]$$

Result (type 5, 311 leaves):

$$\frac{1}{324 b^3 d^4 (a + b x)^{2/3}}$$

$$(c + d x)^{2/3} \left( d (a + b x) (20 a^3 d^3 f^2 - 12 a^2 b d^2 f (5 d e + c f + d f x) + 3 a b^2 d (-3 c^2 f^2 + \right.$$

$$2 c d f (8 e + f x) + 3 d^2 (6 e^2 + 4 e f x + f^2 x^2)) + b^3 (28 c^3 f^2 - 3 c^2 d f (32 e + 7 f x) +$$

$$18 c d^2 (6 e^2 + 4 e f x + f^2 x^2) + 27 d^3 x (6 e^2 + 8 e f x + 3 f^2 x^2)) \left. \right) -$$

$$2 (bc - ad)^2 (10 a^2 d^2 f^2 + 10 a b d f (-3 d e + c f) + b^2 (27 d^2 e^2 - 24 c d e f + 7 c^2 f^2))$$

$$\left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right]$$

**Problem 2990: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{1/3} (c + d x)^{2/3} (e + f x) dx$$

Optimal (type 3, 331 leaves, 4 steps):

$$\frac{(bc - ad) (9bde - 4bcf - 5adf) (a + bx)^{1/3} (c + dx)^{2/3}}{27 b^2 d^2} +$$

$$\frac{(9bde - 4bcf - 5adf) (a + bx)^{4/3} (c + dx)^{2/3}}{18 b^2 d} + \frac{f (a + bx)^{4/3} (c + dx)^{5/3}}{3 b d} +$$

$$\frac{(bc - ad)^2 (9bde - 4bcf - 5adf) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}}\right]}{27 \sqrt{3} b^{8/3} d^{7/3}} +$$

$$\frac{(bc - ad)^2 (9bde - 4bcf - 5adf) \operatorname{Log}[a + bx]}{162 b^{8/3} d^{7/3}} +$$

$$\frac{(bc - ad)^2 (9bde - 4bcf - 5adf) \operatorname{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{54 b^{8/3} d^{7/3}}$$

Result (type 5, 175 leaves):

$$\left( (c + dx)^{2/3} \left( d (a + bx) \right. \right.$$

$$\left. \left. (-5 a^2 d^2 f + a b d (9 d e + 4 c f + 3 d f x) + b^2 (-8 c^2 f + 6 c d (3 e + f x) + 9 d^2 x (3 e + 2 f x))) \right) + \right.$$

$$\left. (bc - ad)^2 (-9 b d e + 4 b c f + 5 a d f) \left( \frac{d (a + bx)}{-bc + ad} \right)^{2/3} \right.$$

$$\left. \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + dx)}{bc - ad}\right] \right) / (54 b^2 d^3 (a + bx)^{2/3})$$

**Problem 2991: Result unnecessarily involves higher level functions.**

$$\int (a + bx)^{1/3} (c + dx)^{2/3} dx$$

Optimal (type 3, 219 leaves, 3 steps):

$$\frac{(bc - ad) (a + bx)^{1/3} (c + dx)^{2/3}}{3 b d} +$$

$$\frac{(a + bx)^{4/3} (c + dx)^{2/3}}{2 b} + \frac{(bc - ad)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}}\right]}{3 \sqrt{3} b^{5/3} d^{4/3}} +$$

$$\frac{(bc - ad)^2 \operatorname{Log}[a + bx]}{18 b^{5/3} d^{4/3}} + \frac{(bc - ad)^2 \operatorname{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{6 b^{5/3} d^{4/3}}$$

Result (type 5, 109 leaves):

$$\frac{1}{6 b d^2 (a + bx)^{2/3}} (c + dx)^{2/3} \left( d (a + bx) (2 b c + a d + 3 b d x) - \right.$$

$$\left. (bc - ad)^2 \left( \frac{d (a + bx)}{-bc + ad} \right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + dx)}{bc - ad}\right] \right)$$

**Problem 2992: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3} (c+dx)^{2/3}}{e+fx} dx$$

Optimal (type 3, 409 leaves, 4 steps):

$$\begin{aligned} & \frac{(a+bx)^{1/3} (c+dx)^{2/3}}{f} + \frac{(3bde - 2bcf - adf) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}}\right]}{\sqrt{3}b^{2/3}d^{1/3}f^2} - \\ & \frac{\sqrt{3}(be-af)^{1/3}(de-cf)^{2/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3}(de-cf)^{1/3}(a+bx)^{1/3}}\right]}{f^2} + \\ & \frac{(3bde - 2bcf - adf) \operatorname{Log}[a+bx]}{6b^{2/3}d^{1/3}f^2} + \frac{(be-af)^{1/3}(de-cf)^{2/3} \operatorname{Log}[e+fx]}{2f^2} - \\ & \frac{3(be-af)^{1/3}(de-cf)^{2/3} \operatorname{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}}\right]}{2f^2} + \\ & \frac{(3bde - 2bcf - adf) \operatorname{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{2b^{2/3}d^{1/3}f^2} \end{aligned}$$

Result (type 6, 541 leaves):

$$\begin{aligned} & \frac{1}{5f(a+bx)^{2/3}}(c+dx)^{2/3} \left( 5(a+bx) - \frac{1}{d^2(e+fx)} \right. \\ & 4(bc-ad) \left( - \left( \left( 5bf(-de+cf)(c+dx) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)}\right] \right) / \right. \right. \\ & \left( 6bf(c+dx) \operatorname{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)}\right] + \right. \\ & \left. b(-3de+3cf) \operatorname{AppellF1}\left[2, \frac{2}{3}, 2, 3, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)}\right] + \right. \\ & \left. \left. 2(bc-ad) f \operatorname{AppellF1}\left[2, \frac{5}{3}, 1, 3, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)}\right] \right) \right) - \\ & \left( 2(de-cf)(3bde-2bcf-adf) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b(c+dx)}{bc-ad}, \frac{f(c+dx)}{-de+cf}\right] \right) / \\ & \left( \frac{1}{c+dx} 8(bc-ad)(-de+cf) \operatorname{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b(c+dx)}{bc-ad}, \frac{f(c+dx)}{-de+cf}\right] + \right. \\ & 3(bc-ad) f \operatorname{AppellF1}\left[\frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, \frac{b(c+dx)}{bc-ad}, \frac{f(c+dx)}{-de+cf}\right] + \\ & \left. \left. 2b(-de+cf) \operatorname{AppellF1}\left[\frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, \frac{b(c+dx)}{bc-ad}, \frac{f(c+dx)}{-de+cf}\right] \right) \right) \end{aligned}$$

Problem 2993: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{1/3} (c+dx)^{2/3}}{(e+fx)^2} dx$$

Optimal (type 3, 417 leaves, 4 steps):

$$\begin{aligned} & - \frac{(a+bx)^{1/3} (c+dx)^{2/3}}{f(e+fx)} - \frac{\sqrt{3} b^{1/3} d^{2/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}}\right]}{f^2} + \\ & \frac{(3bde - bcf - 2adf) \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3}(de-cf)^{1/3}(a+bx)^{1/3}}\right]}{\sqrt{3} f^2 (be-af)^{2/3} (de-cf)^{1/3}} - \\ & \frac{b^{1/3} d^{2/3} \text{Log}[a+bx]}{2 f^2} - \frac{(3bde - bcf - 2adf) \text{Log}[e+fx]}{6 f^2 (be-af)^{2/3} (de-cf)^{1/3}} + \\ & \frac{(3bde - bcf - 2adf) \text{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}}\right]}{2 f^2 (be-af)^{2/3} (de-cf)^{1/3}} - \frac{3 b^{1/3} d^{2/3} \text{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{2 f^2} \end{aligned}$$

Result (type 6, 743 leaves):

$$\begin{aligned}
 & \frac{1}{5 f (a+b x)^{2/3} (e+f x)} (c+d x)^{2/3} \\
 & \left( -5 (a+b x) - \frac{1}{d} 4 b \left( - \left( \left( 5 b c f (c+d x) \operatorname{AppellF1} \left[ 1, \frac{2}{3}, 1, 2, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] \right) / \right. \right. \right. \\
 & \quad \left( 6 b f (c+d x) \operatorname{AppellF1} \left[ 1, \frac{2}{3}, 1, 2, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] + \right. \\
 & \quad \left. b (-3 d e+3 c f) \operatorname{AppellF1} \left[ 2, \frac{2}{3}, 2, 3, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] + \right. \\
 & \quad \left. \left. \left. 2 (b c-a d) f \operatorname{AppellF1} \left[ 2, \frac{5}{3}, 1, 3, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] \right) \right) - \right. \\
 & \quad \left( 5 a d f (c+d x) \operatorname{AppellF1} \left[ 1, \frac{2}{3}, 1, 2, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] \right) / \\
 & \quad \left( -6 b f (c+d x) \operatorname{AppellF1} \left[ 1, \frac{2}{3}, 1, 2, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] + \right. \\
 & \quad \left. 3 b (d e-c f) \operatorname{AppellF1} \left[ 2, \frac{2}{3}, 2, 3, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] + \right. \\
 & \quad \left. \left. \left. 2 (-b c+a d) f \operatorname{AppellF1} \left[ 2, \frac{5}{3}, 1, 3, \frac{b c-a d}{b c+b d x}, \frac{-d e+c f}{f (c+d x)} \right] \right) - \right. \\
 & \quad \left( 6 (b c-a d) (-d e+c f) \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c+d x)}{b c-a d}, \frac{f (c+d x)}{-d e+c f} \right] \right) / \\
 & \quad \left( \frac{1}{c+d x} 8 (b c-a d) (-d e+c f) \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c+d x)}{b c-a d}, \frac{f (c+d x)}{-d e+c f} \right] + \right. \\
 & \quad \left. 3 (b c-a d) f \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, \frac{b (c+d x)}{b c-a d}, \frac{f (c+d x)}{-d e+c f} \right] + \right. \\
 & \quad \left. \left. \left. 2 b (-d e+c f) \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, \frac{b (c+d x)}{b c-a d}, \frac{f (c+d x)}{-d e+c f} \right] \right) \right) \right)
 \end{aligned}$$

**Problem 2994: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x)^{1/3} (c+d x)^{2/3}}{(e+f x)^3} dx$$

Optimal (type 3, 325 leaves, 3 steps):

$$\frac{(a+bx)^{1/3} (c+dx)^{5/3}}{2 (de-cf) (e+fx)^2} - \frac{(bc-ad) (a+bx)^{1/3} (c+dx)^{2/3}}{6 (be-af) (de-cf) (e+fx)} +$$

$$\frac{(bc-ad)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (be-af)^{1/3} (c+dx)^{1/3}}{\sqrt{3} (de-cf)^{1/3} (a+bx)^{1/3}}\right]}{3 \sqrt{3} (be-af)^{5/3} (de-cf)^{4/3}} - \frac{(bc-ad)^2 \operatorname{Log}[e+fx]}{18 (be-af)^{5/3} (de-cf)^{4/3}} +$$

$$\frac{(bc-ad)^2 \operatorname{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3} (c+dx)^{1/3}}{(de-cf)^{1/3}}\right]}{6 (be-af)^{5/3} (de-cf)^{4/3}}$$

Result (type 5, 196 leaves):

$$\left( (a+bx)^{1/3} \left( f (be-af) (c+dx) (-3acf+ad(e-2fx)) + b(2ce+3dex-cfx) - 2(bc-ad)^2 f \right. \right. \\ \left. \left. \left( \frac{(be-af)(c+dx)}{(bc-ad)(e+fx)} \right)^{1/3} (e+fx)^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right] \right) \right) / \\ (6f (be-af)^2 (de-cf) (c+dx)^{1/3} (e+fx)^2)$$

**Problem 2995: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3} (c+dx)^{2/3}}{(e+fx)^4} dx$$

Optimal (type 3, 465 leaves, 4 steps):

$$-\frac{f (a+bx)^{4/3} (c+dx)^{5/3}}{3 (be-af) (de-cf) (e+fx)^3} + \frac{(9bde-5bcf-4adf) (a+bx)^{1/3} (c+dx)^{5/3}}{18 (be-af) (de-cf)^2 (e+fx)^2} -$$

$$\frac{(bc-ad) (9bde-5bcf-4adf) (a+bx)^{1/3} (c+dx)^{2/3}}{54 (be-af)^2 (de-cf)^2 (e+fx)} +$$

$$\left( (bc-ad)^2 (9bde-5bcf-4adf) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (be-af)^{1/3} (c+dx)^{1/3}}{\sqrt{3} (de-cf)^{1/3} (a+bx)^{1/3}}\right] \right) /$$

$$(27 \sqrt{3} (be-af)^{8/3} (de-cf)^{7/3}) - \frac{(bc-ad)^2 (9bde-5bcf-4adf) \operatorname{Log}[e+fx]}{162 (be-af)^{8/3} (de-cf)^{7/3}} +$$

$$\left( (bc-ad)^2 (9bde-5bcf-4adf) \operatorname{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3} (c+dx)^{1/3}}{(de-cf)^{1/3}}\right] \right) /$$

$$(54 (be-af)^{8/3} (de-cf)^{7/3})$$

Result (type 5, 304 leaves):

$$\frac{1}{54 f (b e - a f)^3 (d e - c f)^2 (c + d x)^{1/3} (e + f x)^3} (a + b x)^{1/3} \left( - (b e - a f) (c + d x) \left( 18 (b e - a f)^2 (d e - c f)^2 - 3 (b e - a f) (d e - c f) (3 b d e - b c f - 2 a d f) \right. \right. \\ \left. \left. (e + f x) - (8 a^2 d^2 f^2 - 4 a b d f (3 d e + c f) + b^2 (9 d^2 e^2 - 6 c d e f + 5 c^2 f^2)) (e + f x)^2 \right) + \right. \\ \left. 2 (b c - a d)^2 f (-9 b d e + 5 b c f + 4 a d f) \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{1/3} (e + f x)^3 \right) \\ \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)} \right]$$

**Problem 2996: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{1/3} (e + f x)^2}{(c + d x)^{1/3}} dx$$

Optimal (type 3, 475 leaves, 4 steps):

$$\frac{1}{27 b^2 d^3} (5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) (a + b x)^{1/3} (c + d x)^{2/3} + \\ \frac{f (12 b d e - 7 b c f - 5 a d f) (a + b x)^{4/3} (c + d x)^{2/3} + f (a + b x)^{4/3} (c + d x)^{2/3} (e + f x)}{18 b^2 d^2 + 3 b d} + \\ \frac{1}{27 \sqrt{3} b^{8/3} d^{10/3}} (b c - a d) (5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \\ \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}} \right] + \frac{1}{162 b^{8/3} d^{10/3}} \\ (b c - a d) (5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \text{Log}[a + b x] + \\ \frac{1}{54 b^{8/3} d^{10/3}} (b c - a d) \\ (5 a^2 d^2 f^2 - 2 a b d f (9 d e - 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \text{Log} \left[ -1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}} \right]$$

Result (type 5, 229 leaves):

$$\left( (c + d x)^{2/3} \left( d (a + b x) (-5 a^2 d^2 f^2 + a b d f (-5 c f + 3 d (6 e + f x)) + \right. \right. \\ \left. \left. b^2 (28 c^2 f^2 - 3 c d f (24 e + 7 f x) + 18 d^2 (3 e^2 + 3 e f x + f^2 x^2))) \right) - \right. \\ \left. (b c - a d) (5 a^2 d^2 f^2 + 2 a b d f (-9 d e + 4 c f) + b^2 (27 d^2 e^2 - 36 c d e f + 14 c^2 f^2)) \right. \\ \left. \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right) / (54 b^2 d^4 (a + b x)^{2/3})$$

**Problem 2997: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3} (e+fx)}{(c+dx)^{1/3}} dx$$

Optimal (type 3, 273 leaves, 3 steps):

$$\frac{(3bde - 2bcf - adf)(a+bx)^{1/3}(c+dx)^{2/3}}{3bd^2} + \frac{f(a+bx)^{4/3}(c+dx)^{2/3}}{2bd} +$$

$$\frac{(bc-ad)(3bde - 2bcf - adf) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}}\right]}{3\sqrt{3}b^{5/3}d^{7/3}} +$$

$$\frac{(bc-ad)(3bde - 2bcf - adf) \operatorname{Log}[a+bx]}{18b^{5/3}d^{7/3}} +$$

$$\frac{(bc-ad)(3bde - 2bcf - adf) \operatorname{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{6b^{5/3}d^{7/3}}$$

Result (type 5, 129 leaves):

$$\frac{1}{6bd^3(a+bx)^{2/3}}(c+dx)^{2/3} \left( d(a+bx)(adf + b(6de - 4cf + 3dfx)) + \right.$$

$$\left. (bc-ad)(-3bde + 2bcf + adf) \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} \operatorname{Hypergeometric2F1}\left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 2998: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3}}{(c+dx)^{1/3}} dx$$

Optimal (type 3, 171 leaves, 2 steps):

$$\frac{(a+bx)^{1/3}(c+dx)^{2/3}}{d} + \frac{(bc-ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}}\right]}{\sqrt{3}b^{2/3}d^{4/3}} +$$

$$\frac{(bc-ad) \operatorname{Log}[a+bx]}{6b^{2/3}d^{4/3}} + \frac{(bc-ad) \operatorname{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{2b^{2/3}d^{4/3}}$$

Result (type 5, 76 leaves):

$$\frac{(a+bx)^{1/3}(c+dx)^{2/3} \left( 2 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right]}{\left(\frac{d(a+bx)}{-bc+ad}\right)^{1/3}} \right)}{2d}$$

**Problem 2999: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3}}{(c+dx)^{1/3} (e+fx)} dx$$

Optimal (type 3, 339 leaves, 4 steps):

$$\begin{aligned} & -\frac{\sqrt{3} b^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3} d^{1/3} (a+bx)^{1/3}}\right]}{d^{1/3} f} + \frac{\sqrt{3} (be-af)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3} (de-cf)^{1/3} (a+bx)^{1/3}}\right]}{f (de-cf)^{1/3}} \\ & -\frac{b^{1/3} \operatorname{Log}[a+bx]}{2 d^{1/3} f} - \frac{(be-af)^{1/3} \operatorname{Log}[e+fx]}{2 f (de-cf)^{1/3}} + \\ & \frac{3 (be-af)^{1/3} \operatorname{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}}\right]}{2 f (de-cf)^{1/3}} - \frac{3 b^{1/3} \operatorname{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3} (a+bx)^{1/3}}\right]}{2 d^{1/3} f} \end{aligned}$$

Result (type 6, 290 leaves):

$$\begin{aligned} & -\left(\left(21 (bc-ad) (be-af)^2 (a+bx)^{4/3} \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{3}, 1, \frac{7}{3}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right)\right) / \\ & \left(4 b (-be+af) (c+dx)^{1/3} (e+fx)\right. \\ & \left. \left(7 (bc-ad) (be-af) \operatorname{AppellF1}\left[\frac{4}{3}, \frac{1}{3}, 1, \frac{7}{3}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right.\right. \\ & \left. (a+bx) \left((-3bcf+3adf) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{1}{3}, 2, \frac{10}{3}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right.\right. \\ & \left. \left. d (-be+af) \operatorname{AppellF1}\left[\frac{7}{3}, \frac{4}{3}, 1, \frac{10}{3}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right)\right)\right) \end{aligned}$$

**Problem 3000: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3}}{(c+dx)^{1/3} (e+fx)^2} dx$$

Optimal (type 3, 256 leaves, 2 steps):

$$\begin{aligned} & \frac{(a+bx)^{1/3} (c+dx)^{2/3}}{(de-cf) (e+fx)} + \frac{(bc-ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3} (de-cf)^{1/3} (a+bx)^{1/3}}\right]}{\sqrt{3} (be-af)^{2/3} (de-cf)^{4/3}} - \\ & \frac{(bc-ad) \operatorname{Log}[e+fx]}{6 (be-af)^{2/3} (de-cf)^{4/3}} + \frac{(bc-ad) \operatorname{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}}\right]}{2 (be-af)^{2/3} (de-cf)^{4/3}} \end{aligned}$$

Result (type 5, 124 leaves):

$$\frac{(a+bx)^{1/3} (c+dx)^{2/3} \left( \frac{1}{de-cf} + \frac{\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-de-cf)(a+bx)}{(bc-ad)(e+fx)}\right]}{(-de+cf) \left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{2/3}} \right)}{e+fx}$$

**Problem 3001: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3}}{(c+dx)^{1/3} (e+fx)^3} dx$$

Optimal (type 3, 386 leaves, 3 steps):

$$\begin{aligned} & -\frac{f(a+bx)^{4/3}(c+dx)^{2/3}}{2(be-af)(de-cf)(e+fx)^2} + \frac{(3bde-bcf-2adf)(a+bx)^{1/3}(c+dx)^{2/3}}{3(be-af)(de-cf)^2(e+fx)} + \\ & \left( (bc-ad)(3bde-bcf-2adf) \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3}(de-cf)^{1/3}(a+bx)^{1/3}}\right] \right) / \\ & (3\sqrt{3}(be-af)^{5/3}(de-cf)^{7/3}) - \frac{(bc-ad)(3bde-bcf-2adf) \text{Log}[e+fx]}{18(be-af)^{5/3}(de-cf)^{7/3}} + \\ & \left( (bc-ad)(3bde-bcf-2adf) \text{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}}\right] \right) / \\ & (6(be-af)^{5/3}(de-cf)^{7/3}) \end{aligned}$$

Result (type 5, 212 leaves):

$$\begin{aligned} & \left( (a+bx)^{1/3} \left( (be-af)(c+dx)(3(be-af)(de-cf) + (3bde+bcf-4adf)(e+fx)) + \right. \right. \\ & \quad \left. \left. 2(bc-ad)(-3bde+bcf+2adf) \left( \frac{(be-af)(c+dx)}{(bc-ad)(e+fx)} \right)^{1/3} \right. \right. \\ & \quad \left. \left. (e+fx)^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right] \right) \right) / \\ & (6(be-af)^2(de-cf)^2(c+dx)^{1/3}(e+fx)^2) \end{aligned}$$

**Problem 3002: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/3}}{(c+dx)^{1/3} (e+fx)^4} dx$$

Optimal (type 3, 591 leaves, 5 steps):

$$\begin{aligned}
 & \frac{(a+bx)^{1/3} (c+dx)^{2/3}}{3 (de-cf) (e+fx)^3} + \frac{(6bde+bcf-7adf) (a+bx)^{1/3} (c+dx)^{2/3}}{18 (be-af) (de-cf)^2 (e+fx)^2} + \\
 & \left( (28a^2d^2f^2 - abdf(51de+5cf) + b^2(18d^2e^2 + 15cdef - 5c^2f^2)) (a+bx)^{1/3} (c+dx)^{2/3} \right) / \\
 & \left( 54 (be-af)^2 (de-cf)^3 (e+fx) \right) + \\
 & \left( (bc-ad) (14a^2d^2f^2 - 4abdf(9de-2cf) + b^2(27d^2e^2 - 18cdef + 5c^2f^2)) \right. \\
 & \left. \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 (be-af)^{1/3} (c+dx)^{1/3}}{\sqrt{3} (de-cf)^{1/3} (a+bx)^{1/3}} \right] \right) / \left( 27\sqrt{3} (be-af)^{8/3} (de-cf)^{10/3} \right) - \\
 & \left( (bc-ad) (14a^2d^2f^2 - 4abdf(9de-2cf) + b^2(27d^2e^2 - 18cdef + 5c^2f^2)) \text{Log}[e+fx] \right) / \\
 & \left( 162 (be-af)^{8/3} (de-cf)^{10/3} \right) + \\
 & \left( (bc-ad) (14a^2d^2f^2 - 4abdf(9de-2cf) + b^2(27d^2e^2 - 18cdef + 5c^2f^2)) \right. \\
 & \left. \text{Log} \left[ - (a+bx)^{1/3} + \frac{(be-af)^{1/3} (c+dx)^{1/3}}{(de-cf)^{1/3}} \right] \right) / \left( 54 (be-af)^{8/3} (de-cf)^{10/3} \right)
 \end{aligned}$$

Result (type 5, 334 leaves):

$$\begin{aligned}
 & \frac{1}{54 (be-af)^3 (de-cf)^3 (c+dx)^{1/3} (e+fx)^3} (a+bx)^{1/3} \left( (be-af) (c+dx) \right. \\
 & \left( 18 (be-af)^2 (de-cf)^2 + 3 (be-af) (de-cf) (6bde+bcf-7adf) (e+fx) + \right. \\
 & \left. (28a^2d^2f^2 - abdf(51de+5cf) + b^2(18d^2e^2 + 15cdef - 5c^2f^2)) (e+fx)^2 \right) - \\
 & 2 (bc-ad) (14a^2d^2f^2 + 4abdf(-9de+2cf) + b^2(27d^2e^2 - 18cdef + 5c^2f^2)) \\
 & \left. \left( \frac{(be-af) (c+dx)}{(bc-ad) (e+fx)} \right)^{1/3} (e+fx)^3 \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-de+cf) (a+bx)}{(bc-ad) (e+fx)} \right] \right)
 \end{aligned}$$

**Problem 3003: Result unnecessarily involves higher level functions.**

$$\int \frac{(e+fx)^3}{(a+bx)^{1/3} (c+dx)^{2/3}} dx$$

Optimal (type 3, 587 leaves, 3 steps):

$$\frac{f (a + b x)^{2/3} (c + d x)^{1/3} (e + f x)^2}{3 b d} + \frac{1}{54 b^3 d^3}$$

$$f (a + b x)^{2/3} (c + d x)^{1/3} (28 a^2 d^2 f^2 - a b d f (108 d e - 31 c f) +$$

$$b^2 (144 d^2 e^2 - 135 c d e f + 40 c^2 f^2) + 3 b d f (15 b d e - 8 b c f - 7 a d f) x) + \frac{1}{27 \sqrt{3} b^{10/3} d^{11/3}}$$

$$(14 a^3 d^3 f^3 - 6 a^2 b d^2 f^2 (9 d e - 2 c f) + 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) -$$

$$b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3)) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a + b x)^{1/3}}{\sqrt{3} b^{1/3} (c + d x)^{1/3}}\right] +$$

$$\frac{1}{162 b^{10/3} d^{11/3}} (14 a^3 d^3 f^3 - 6 a^2 b d^2 f^2 (9 d e - 2 c f) + 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) -$$

$$b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3)) \operatorname{Log}[c + d x] + \frac{1}{54 b^{10/3} d^{11/3}}$$

$$(14 a^3 d^3 f^3 - 6 a^2 b d^2 f^2 (9 d e - 2 c f) + 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) -$$

$$b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3)) \operatorname{Log}\left[-1 + \frac{d^{1/3} (a + b x)^{1/3}}{b^{1/3} (c + d x)^{1/3}}\right]$$

Result (type 5, 275 leaves):

$$\frac{1}{54 b^3 d^4 (a + b x)^{1/3}} (c + d x)^{1/3} \left( d f (a + b x) (28 a^2 d^2 f^2 + a b d f (31 c f - 3 d (36 e + 7 f x)) +$$

$$b^2 (40 c^2 f^2 - 3 c d f (45 e + 8 f x) + 9 d^2 (18 e^2 + 9 e f x + 2 f^2 x^2))) +$$

$$2 (-14 a^3 d^3 f^3 + 6 a^2 b d^2 f^2 (9 d e - 2 c f) - 3 a b^2 d f (27 d^2 e^2 - 18 c d e f + 5 c^2 f^2) +$$

$$b^3 (81 d^3 e^3 - 162 c d^2 e^2 f + 135 c^2 d e f^2 - 40 c^3 f^3))$$

$$\left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 3004: Result unnecessarily involves higher level functions.**

$$\int \frac{(e + f x)^2}{(a + b x)^{1/3} (c + d x)^{2/3}} dx$$

Optimal (type 3, 369 leaves, 3 steps):

$$\frac{f (9 b d e - 5 b c f - 4 a d f) (a + b x)^{2/3} (c + d x)^{1/3}}{6 b^2 d^2} + \frac{f (a + b x)^{2/3} (c + d x)^{1/3} (e + f x)}{2 b d}$$

$$\frac{1}{3 \sqrt{3} b^{7/3} d^{8/3}} (2 a^2 d^2 f^2 - 2 a b d f (3 d e - c f) + b^2 (9 d^2 e^2 - 12 c d e f + 5 c^2 f^2))$$

$$\operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a + b x)^{1/3}}{\sqrt{3} b^{1/3} (c + d x)^{1/3}}\right] - \frac{1}{18 b^{7/3} d^{8/3}}$$

$$(2 a^2 d^2 f^2 - 2 a b d f (3 d e - c f) + b^2 (9 d^2 e^2 - 12 c d e f + 5 c^2 f^2)) \operatorname{Log}[c + d x] - \frac{1}{6 b^{7/3} d^{8/3}}$$

$$(2 a^2 d^2 f^2 - 2 a b d f (3 d e - c f) + b^2 (9 d^2 e^2 - 12 c d e f + 5 c^2 f^2)) \operatorname{Log}\left[-1 + \frac{d^{1/3} (a + b x)^{1/3}}{b^{1/3} (c + d x)^{1/3}}\right]$$

Result (type 5, 162 leaves):

$$\left( (c+dx)^{1/3} \left( df(a+bx) (-5bcf - 4adf + 3bd(4e+fx)) + \right. \right. \\ \left. \left. 2(2a^2d^2f^2 + 2abd f(-3de+cf) + b^2(9d^2e^2 - 12cdef + 5c^2f^2)) \right) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} \right. \\ \left. \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) / (6b^2d^3(a+bx)^{1/3})$$

**Problem 3005: Result unnecessarily involves higher level functions.**

$$\int \frac{e+fx}{(a+bx)^{1/3}(c+dx)^{2/3}} dx$$

Optimal (type 3, 200 leaves, 2 steps):

$$\frac{f(a+bx)^{2/3}(c+dx)^{1/3}}{bd} - \frac{(3bde - 2bcf - adf) \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2d^{1/3}(a+bx)^{1/3}}{\sqrt{3}b^{1/3}(c+dx)^{1/3}} \right]}{\sqrt{3}b^{4/3}d^{5/3}} - \\ \frac{(3bde - 2bcf - adf) \text{Log}[c+dx]}{6b^{4/3}d^{5/3}} - \frac{(3bde - 2bcf - adf) \text{Log} \left[ -1 + \frac{d^{1/3}(a+bx)^{1/3}}{b^{1/3}(c+dx)^{1/3}} \right]}{2b^{4/3}d^{5/3}}$$

Result (type 5, 99 leaves):

$$\frac{1}{bd^2(a+bx)^{1/3}}(c+dx)^{1/3} \\ \left( df(a+bx) + (3bde - 2bcf - adf) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 3006: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{1/3}(c+dx)^{2/3}} dx$$

Optimal (type 3, 126 leaves, 1 step):

$$-\frac{\sqrt{3} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2d^{1/3}(a+bx)^{1/3}}{\sqrt{3}b^{1/3}(c+dx)^{1/3}} \right]}{b^{1/3}d^{2/3}} - \frac{\text{Log}[c+dx]}{2b^{1/3}d^{2/3}} - \frac{3 \text{Log} \left[ -1 + \frac{d^{1/3}(a+bx)^{1/3}}{b^{1/3}(c+dx)^{1/3}} \right]}{2b^{1/3}d^{2/3}}$$

Result (type 5, 71 leaves):

$$\frac{3 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} (c+dx)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right]}{d(a+bx)^{1/3}}$$

### Problem 3007: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{1/3} (c+dx)^{2/3} (e+fx)} dx$$

Optimal (type 3, 197 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(de-cf)^{1/3}(a+bx)^{1/3}}{\sqrt{3}(be-af)^{1/3}(c+dx)^{1/3}}\right]}{(be-af)^{1/3}(de-cf)^{2/3}} + \frac{\operatorname{Log}[e+fx]}{2(be-af)^{1/3}(de-cf)^{2/3}} - \frac{3 \operatorname{Log}\left[\frac{(de-cf)^{1/3}(a+bx)^{1/3}}{(be-af)^{1/3}} - (c+dx)^{1/3}\right]}{2(be-af)^{1/3}(de-cf)^{2/3}}$$

Result (type 5, 108 leaves):

$$\left(3(a+bx)^{2/3} \left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right]\right) / \left(2(be-af)(c+dx)^{2/3}\right)$$

### Problem 3008: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{1/3} (c+dx)^{2/3} (e+fx)^2} dx$$

Optimal (type 3, 293 leaves, 2 steps):

$$-\frac{f(a+bx)^{2/3}(c+dx)^{1/3}}{(be-af)(de-cf)(e+fx)} - \frac{(3bde-bcf-2adf) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(de-cf)^{1/3}(a+bx)^{1/3}}{\sqrt{3}(be-af)^{1/3}(c+dx)^{1/3}}\right]}{\sqrt{3}(be-af)^{4/3}(de-cf)^{5/3}} + \frac{(3bde-bcf-2adf) \operatorname{Log}[e+fx]}{6(be-af)^{4/3}(de-cf)^{5/3}} - \frac{(3bde-bcf-2adf) \operatorname{Log}\left[\frac{(de-cf)^{1/3}(a+bx)^{1/3}}{(be-af)^{1/3}} - (c+dx)^{1/3}\right]}{2(be-af)^{4/3}(de-cf)^{5/3}}$$

Result (type 5, 171 leaves):

$$\left((a+bx)^{2/3} \left(\frac{2f(c+dx)}{(-de+cf)(e+fx)} + \left((3bde-bcf-2adf) \left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right]\right) / ((be-af)(de-cf))\right) / \left(2(be-af)(c+dx)^{2/3}\right)$$

### Problem 3009: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{1/3} (c+dx)^{2/3} (e+fx)^3} dx$$

Optimal (type 3, 477 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{f (a+bx)^{2/3} (c+dx)^{1/3}}{2 (be-af) (de-cf) (e+fx)^2} - \frac{f (9bde-4bcf-5adf) (a+bx)^{2/3} (c+dx)^{1/3}}{6 (be-af)^2 (de-cf)^2 (e+fx)} \\
 & \left( (5a^2d^2f^2 - 2abd f (6de-cf) + b^2 (9d^2e^2 - 6cdef + 2c^2f^2)) \right. \\
 & \quad \left. \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 (de-cf)^{1/3} (a+bx)^{1/3}}{\sqrt{3} (be-af)^{1/3} (c+dx)^{1/3}} \right] \right) / \left( 3\sqrt{3} (be-af)^{7/3} (de-cf)^{8/3} \right) + \\
 & \left( (5a^2d^2f^2 - 2abd f (6de-cf) + b^2 (9d^2e^2 - 6cdef + 2c^2f^2)) \text{Log}[e+fx] \right) / \\
 & \left( 18 (be-af)^{7/3} (de-cf)^{8/3} \right) - \\
 & \left( (5a^2d^2f^2 - 2abd f (6de-cf) + b^2 (9d^2e^2 - 6cdef + 2c^2f^2)) \right. \\
 & \quad \left. \text{Log} \left[ \frac{(de-cf)^{1/3} (a+bx)^{1/3}}{(be-af)^{1/3}} - (c+dx)^{1/3} \right] \right) / \left( 6 (be-af)^{7/3} (de-cf)^{8/3} \right)
 \end{aligned}$$

Result (type 5, 244 leaves):

$$\begin{aligned}
 & \left( (a+bx)^{2/3} \left( -f (be-af) (c+dx) (3 (be-af) (de-cf) + (9bde-4bcf-5adf) (e+fx)) + \right. \right. \\
 & \quad \left. \left. (5a^2d^2f^2 + 2abd f (-6de+cf) + b^2 (9d^2e^2 - 6cdef + 2c^2f^2)) \left( \frac{(be-af) (c+dx)}{(bc-ad) (e+fx)} \right)^{2/3} \right. \right. \\
 & \quad \left. \left. (e+fx)^2 \text{Hypergeometric2F1} \left[ \frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{(-de+cf) (a+bx)}{(bc-ad) (e+fx)} \right] \right) \right) / \\
 & \left( 6 (be-af)^3 (de-cf)^2 (c+dx)^{2/3} (e+fx)^2 \right)
 \end{aligned}$$

**Problem 3010: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^3}{(c+dx)^{1/3} (bc+ad+2bdx)^{1/3}} dx$$

Optimal (type 4, 1389 leaves, 7 steps):

$$\begin{aligned}
 & \frac{3(a+bx)^2(c+dx)^{2/3}(bc+ad+2bdx)^{2/3}}{20d^2} + \frac{1}{560d^4} \\
 & 9(bc-ad)(c+dx)^{2/3}(23bc-39ad-16bdx)(bc+ad+2bdx)^{2/3} - \\
 & \left( 81(bc-ad)^3((c+dx)(bc+ad+2bdx))^{1/3} \sqrt{d^2(3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{(d(3bc+ad)+4bd^2x)^2} \right) / \left( 112b^{2/3}d^6(c+dx)^{1/3}(bc+ad+2bdx)^{1/3} \right. \\
 & \quad \left. (3bc+ad+4bdx) \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right) + \\
 & \left( 81 \times 3^{1/4} \sqrt{2-\sqrt{3}} (bc-ad)^{11/3} ((c+dx)(bc+ad+2bdx))^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3}(bc-ad)^{2/3} \right. \right. \\
 & \quad \left. \left. ((c+dx)(ad+b(c+2dx)))^{1/3} + 4b^{2/3}((c+dx)(ad+b(c+2dx)))^{2/3} \right) \right) / \\
 & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \Big) / \\
 & \text{EllipticE}\left[\text{ArcSin}\left[\left( (1-\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( 224b^{2/3}d^4(c+dx)^{1/3}(bc+ad+2bdx)^{1/3}(3bc+ad+4bdx) \sqrt{d^2(3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right) / \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \right) - \\
 & \left( 27 \times 3^{3/4} (bc-ad)^{11/3} ((c+dx)(bc+ad+2bdx))^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3}(bc-ad)^{2/3}((c+dx)(ad+b(c+2dx)))^{1/3} + \right. \right. \\
 & \quad \left. \left. 4b^{2/3}((c+dx)(ad+b(c+2dx)))^{2/3} \right) \right) / \\
 & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \Big) \\
 & \text{EllipticF}\left[\text{ArcSin}\left[\left( (1-\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( 56\sqrt{2} b^{2/3}d^4(c+dx)^{1/3}(bc+ad+2bdx)^{1/3}(3bc+ad+4bdx) \sqrt{d^2(3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right) / \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \right) \Big)
 \end{aligned}$$

Result(type 5, 160 leaves):

$$\begin{aligned}
 & - \left( \left( 3 (a d + b (c + 2 d x)) \right)^{2/3} \right. \\
 & \quad \left( -2 b (c + d x) (145 a^2 d^2 + 2 a b d (-93 c + 52 d x) + b^2 (69 c^2 - 48 c d x + 28 d^2 x^2)) + \right. \\
 & \quad \left. 135 \times 2^{1/3} (b c - a d)^3 \left( \frac{b (c + d x)}{b c - a d} \right)^{1/3} \right. \\
 & \quad \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d + b (c + 2 d x)}{-b c + a d} \right] \right) \right) / \left( 1120 b d^4 (c + d x)^{1/3} \right)
 \end{aligned}$$

**Problem 3011: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^2}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1373 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{45 (bc - ad) (c + dx)^{2/3} (bc + ad + 2bdx)^{2/3}}{112 d^3} + \frac{3 (a + bx) (c + dx)^{2/3} (bc + ad + 2bdx)^{2/3}}{14 d^2} + \\
 & \left( 99 (bc - ad)^2 ((c + dx) (bc + ad + 2bdx))^{1/3} \sqrt{d^2 (3bc + ad + 4bdx)^2} \right. \\
 & \quad \left. \sqrt{(d(3bc + ad) + 4bd^2x)^2} \right) / \left( 112 b^{2/3} d^5 (c + dx)^{1/3} (bc + ad + 2bdx)^{1/3} \right. \\
 & \quad \left. (3bc + ad + 4bdx) \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) \right) - \\
 & \left( 99 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (bc - ad)^{8/3} ((c + dx) (bc + ad + 2bdx))^{1/3} \sqrt{(d(3bc + ad) + 4bd^2x)^2} \right. \\
 & \quad \left( (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) \sqrt{\left( (bc - ad)^{4/3} - 2b^{1/3} (bc - ad)^{2/3} \right.} \\
 & \quad \left. ((c + dx) (ad + b(c + 2dx)))^{1/3} + 4b^{2/3} ((c + dx) (ad + b(c + 2dx)))^{2/3} \right) / \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right)^2 \right) \\
 & \text{EllipticE} \left[ \text{ArcSin} \left[ \left( (1 - \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \left( 224 b^{2/3} d^3 (c + dx)^{1/3} (bc + ad + 2bdx)^{1/3} (3bc + ad + 4bdx) \sqrt{d^2 (3bc + ad + 4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) \right) / \right.} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right)^2 \right) + \\
 & \left( 33 \times 3^{3/4} (bc - ad)^{8/3} ((c + dx) (bc + ad + 2bdx))^{1/3} \sqrt{(d(3bc + ad) + 4bd^2x)^2} \right. \\
 & \quad \left( (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) \\
 & \quad \left. \sqrt{\left( (bc - ad)^{4/3} - 2b^{1/3} (bc - ad)^{2/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} + \right.} \right. \\
 & \quad \left. \left. 4b^{2/3} ((c + dx) (ad + b(c + 2dx)))^{2/3} \right) / \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right)^2 \right) \\
 & \text{EllipticF} \left[ \text{ArcSin} \left[ \left( (1 - \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \left( 56 \sqrt{2} b^{2/3} d^3 (c + dx)^{1/3} (bc + ad + 2bdx)^{1/3} (3bc + ad + 4bdx) \sqrt{d^2 (3bc + ad + 4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right) \right) / \right.} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx) (ad + b(c + 2dx)))^{1/3} \right)^2 \right)
 \end{aligned}$$

Result (type 5, 129 leaves):

$$\left( 3 (a d + b (c + 2 d x))^{2/3} \right. \\ \left. \left( -2 b (c + d x) (15 b c - 23 a d - 8 b d x) + 33 \times 2^{1/3} (b c - a d)^2 \left( \frac{b (c + d x)}{b c - a d} \right)^{1/3} \right. \right. \\ \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{a d + b (c + 2 d x)}{-b c + a d} \right] \right) \right) / (224 b d^3 (c + d x)^{1/3})$$

**Problem 3012: Result unnecessarily involves higher level functions.**

$$\int \frac{a + b x}{(c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3}} dx$$

Optimal (type 4, 1326 leaves, 6 steps):

$$\begin{aligned}
 & \frac{3 (c+dx)^{2/3} (bc+ad+2bdx)^{2/3}}{8d^2} - \\
 & \left( 9 (bc-ad) ((c+dx) (bc+ad+2bdx))^{1/3} \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{(d(3bc+ad)+4bd^2x)^2} \right) / \left( 8b^{2/3} d^4 (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} \right. \\
 & \quad \left. (3bc+ad+4bdx) \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) \right) + \\
 & \left( 9 \times 3^{1/4} \sqrt{2-\sqrt{3}} (bc-ad)^{5/3} ((c+dx) (bc+ad+2bdx))^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3} (bc-ad)^{2/3} \right.} \right. \\
 & \quad \left. \left. ((c+dx) (ad+b(c+2dx)))^{1/3} + 4b^{2/3} ((c+dx) (ad+b(c+2dx)))^{2/3} \right) \right) / \\
 & \quad \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right)^2 \Big) \\
 & \text{EllipticE} \left[ \text{ArcSin} \left[ \left( (1-\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( 16b^{2/3} d^2 (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right)^2 \right) - \\
 & \left( 3 \times 3^{3/4} (bc-ad)^{5/3} ((c+dx) (bc+ad+2bdx))^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3} (bc-ad)^{2/3} ((c+dx) (ad+b(c+2dx)))^{1/3} + \right.} \right. \\
 & \quad \left. \left. 4b^{2/3} ((c+dx) (ad+b(c+2dx)))^{2/3} \right) \right) / \\
 & \quad \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right)^2 \Big) \\
 & \text{EllipticF} \left[ \text{ArcSin} \left[ \left( (1-\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( 4\sqrt{2} b^{2/3} d^2 (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} ((c+dx) (ad+b(c+2dx)))^{1/3} \right)^2 \right) \Big)
 \end{aligned}$$

Result (type 5, 95 leaves):

$$-\frac{1}{16d^2} 3 (c+dx)^{2/3} (ad+b(c+2dx))^{2/3} \left( -2 + \frac{3 \times 2^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{ad+b(c+2dx)}{-bc+ad}\right]}{\left(\frac{b(c+dx)}{b-c-d}\right)^{2/3}} \right)$$

**Problem 3013: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(c+dx)^{1/3} (bc+ad+2bdx)^{1/3}} dx$$

Optimal (type 4, 1283 leaves, 5 steps):

$$\begin{aligned}
 & \left( 3 \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{d^2 (3bc+ad+4bdx)^2} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right) / \\
 & \left( 2b^{2/3} d^3 (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \right. \\
 & \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right) - \\
 & \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} (bc-ad)^{2/3} \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3} (bc-ad)^{2/3} \right.} \\
 & \left. \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} + 4b^{2/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{2/3} \right) / \\
 & \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \Big) \\
 & \text{EllipticE} \left[ \text{ArcSin} \left[ \left( (1-\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) / \right. \right. \\
 & \left. \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( 4b^{2/3} d (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right) /} \right. \\
 & \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) + \\
 & \left( 3^{3/4} (bc-ad)^{2/3} \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \\
 & \left. \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3} (bc-ad)^{2/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} + \right.} \right. \\
 & \left. \left. 4b^{2/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{2/3} \right) /} \right. \\
 & \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) \\
 & \text{EllipticF} \left[ \text{ArcSin} \left[ \left( (1-\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) / \right. \right. \\
 & \left. \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( \sqrt{2} b^{2/3} d (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right) /} \right. \\
 & \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) \Big)
 \end{aligned}$$

Result (type 5, 94 leaves):

$$\begin{aligned}
 & \left( 3 \left( \frac{b(c+dx)}{bc-ad} \right)^{1/3} (ad+b(c+2dx))^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{ad+b(c+2dx)}{-bc+ad} \right] \right) / \\
 & \left( 2 \times 2^{2/3} b d (c+dx)^{1/3} \right)
 \end{aligned}$$

**Problem 3014: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)(c+dx)^{1/3}(bc+ad+2bdx)^{1/3}} dx$$

Optimal (type 3, 178 leaves, 1 step):

$$\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{2/3}(c+dx)^{2/3}}{\sqrt{3}(bc-ad)^{1/3}(bc+ad+2bdx)^{1/3}}\right]}{2b^{2/3}(bc-ad)^{2/3}} + \frac{\operatorname{Log}[a+bx]}{2b^{2/3}(bc-ad)^{2/3}} + \frac{3 \operatorname{Log}\left[\frac{b^{2/3}(c+dx)^{2/3}}{(bc-ad)^{1/3}} - (bc+ad+2bdx)^{1/3}\right]}{4b^{2/3}(bc-ad)^{2/3}}$$

Result (type 6, 276 leaves):

$$\begin{aligned} & - \left( \left( 15d(a+bx) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{-bc+ad}{d(a+bx)}, -\frac{bc-ad}{2ad+2bdx}\right] \right) / \left( b(c+dx)^{1/3} \right. \right. \\ & \quad \left. \left. (ad+b(c+2dx))^{1/3} \left( 10d(a+bx) \operatorname{AppellF1}\left[\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{-bc+ad}{d(a+bx)}, -\frac{bc-ad}{2ad+2bdx}\right] - \right. \right. \right. \\ & \quad \left. \left. (bc-ad) \left( \operatorname{AppellF1}\left[\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \frac{-bc+ad}{d(a+bx)}, -\frac{bc-ad}{2ad+2bdx}\right] + \right. \right. \right. \\ & \quad \left. \left. \left. 2 \operatorname{AppellF1}\left[\frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{-bc+ad}{d(a+bx)}, -\frac{bc-ad}{2ad+2bdx}\right] \right) \right) \right) \right) \end{aligned}$$

**Problem 3015: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^2(c+dx)^{1/3}(bc+ad+2bdx)^{1/3}} dx$$

Optimal (type 4, 1510 leaves, 8 steps):

$$\begin{aligned} & - \frac{(c+dx)^{2/3}(bc+ad+2bdx)^{2/3}}{(bc-ad)^2(a+bx)} + \\ & \left( \left( (c+dx)(bc+ad+2bdx)^{1/3} \sqrt{d^2(3bc+ad+4bdx)^2} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right) / \right. \\ & \quad \left( b^{2/3}d(bc-ad)^2(c+dx)^{1/3}(bc+ad+2bdx)^{1/3}(3bc+ad+4bdx) \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right) + \\ & \frac{\sqrt{3}d \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2b^{2/3}(c+dx)^{2/3}}{\sqrt{3}(bc-ad)^{1/3}(bc+ad+2bdx)^{1/3}}\right]}{2b^{2/3}(bc-ad)^{5/3}} - \\ & \left( 3^{1/4} \sqrt{2-\sqrt{3}} d((c+dx)(bc+ad+2bdx))^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\ & \quad \left. \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3}(bc-ad)^{2/3} \right)} \right) \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{\left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} + 4b^{2/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{2/3}}{\left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right)^2} \right) \\
 & \text{EllipticE} \left[ \text{ArcSin} \left[ \left( (1-\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right) \right. \right. \\
 & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right) \right], -7-4\sqrt{3} \right] \right) / \\
 & \left( 2b^{2/3} (bc-ad)^{4/3} (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \right. \\
 & \left. \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \left. \sqrt{\left( \left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right) \right) \right) \right. \right. \\
 & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) \right) + \\
 & \left( \sqrt{2} d \left( (c+dx)(bc+ad+2bdx) \right)^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \left. \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right) \right. \\
 & \left. \sqrt{\left( \left( (bc-ad)^{4/3} - 2b^{1/3} (bc-ad)^{2/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} + \right. \right. \right. \right. \\
 & \left. \left. \left. 4b^{2/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{2/3} \right) \right) \right. \right. \\
 & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) \right) \\
 & \text{EllipticF} \left[ \text{ArcSin} \left[ \left( (1-\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right) \right. \right. \\
 & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right) \right], -7-4\sqrt{3} \right] \right) / \\
 & \left( 3^{1/4} b^{2/3} (bc-ad)^{4/3} (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \right. \\
 & \left. \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \left. \sqrt{\left( \left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right) \right) \right) \right. \right. \\
 & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx)(ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) \right) + \\
 & \frac{d \text{Log}[a+bx]}{2b^{2/3} (bc-ad)^{5/3}} - \frac{3d \text{Log} \left[ \frac{b^{2/3} (c+dx)^{2/3}}{(bc-ad)^{1/3}} - (bc+ad+2bdx)^{1/3} \right]}{4b^{2/3} (bc-ad)^{5/3}}
 \end{aligned}$$

Result (type 6, 593 leaves):

$$\frac{1}{5 (bc - ad)^2} (c + dx)^{2/3} (ad + b(c + 2dx))^{2/3} \left( -\frac{5}{a+bx} + \frac{1}{bc+ad+2bdx} d \left( 10 - \frac{5c}{c+dx} + \frac{5ad}{bc+bdx} + \right. \right. \\ \left. \left. \left( 100b (bc - ad) (c + dx) \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) / \right. \right. \\ \left. \left. \left( d (a + bx) \left( 10b (c + dx) \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \right. \right. \right. \right. \\ \left. \left. \left( bc - ad \right) \left( 6 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \right. \right. \right. \\ \left. \left. \left. \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) \right) \right) - \right. \\ \left. \left( 16 (bc - ad)^2 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) / \right. \\ \left. \left( d (a + bx) \left( 16b (c + dx) \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \right. \right. \right. \\ \left. \left. \left( bc - ad \right) \left( 6 \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \right. \right. \right. \\ \left. \left. \left. \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) \right) \right) \right) \right)$$

**Problem 3016: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^3 (c+dx)^{1/3} (bc+ad+2bdx)^{1/3}} dx$$

Optimal (type 4, 1558 leaves, 9 steps):

$$-\frac{(c+dx)^{2/3} (bc+ad+2bdx)^{2/3}}{2 (bc - ad)^2 (a+bx)^2} + \frac{2d (c+dx)^{2/3} (bc+ad+2bdx)^{2/3}}{(bc - ad)^3 (a+bx)} - \\ \left( 2 \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{d^2 (3bc+ad+4bdx)^2} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right) / \\ \left( b^{2/3} (bc - ad)^3 (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \right. \\ \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad + b(c + 2dx)) \right)^{1/3} \right) - \right. \\ \left. \frac{2d^2 \operatorname{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2b^{2/3} (c+dx)^{2/3}}{\sqrt{3} (bc - ad)^{1/3} (bc+ad+2bdx)^{1/3}} \right]}{\sqrt{3} b^{2/3} (bc - ad)^{8/3}} + \right. \\ \left. \left( 3^{1/4} \sqrt{2 - \sqrt{3}} d^2 \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \right. \\ \left. \left. \left( (bc - ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad + b(c + 2dx)) \right) \right)^{1/3} \sqrt{\left( (bc - ad)^{4/3} - 2b^{1/3} (bc - ad)^{2/3} \right. \right. \right. \\ \left. \left. \left. \left( (c+dx) (ad + b(c + 2dx)) \right)^{1/3} + 4b^{2/3} \left( (c+dx) (ad + b(c + 2dx)) \right)^{2/3} \right) \right) / \right. \\ \left. \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad + b(c + 2dx)) \right) \right)^{1/3} \right)^2 \right)$$

$$\begin{aligned}
 & \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \left( (1 - \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right) / \right. \right. \right. \\
 & \quad \left. \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right) \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \left( b^{2/3} (bc - ad)^{7/3} (c + dx)^{1/3} (bc + ad + 2bdx)^{1/3} (3bc + ad + 4bdx) \sqrt{d^2 (3bc + ad + 4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right)^2 \right) - \\
 & \left( 2\sqrt{2} d^2 ((c + dx)(bc + ad + 2bdx))^{1/3} \sqrt{(d(3bc + ad) + 4bd^2x)^2} \right. \\
 & \quad \left. \left( (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right) \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{4/3} - 2b^{1/3} (bc - ad)^{2/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} + \right. \right. \\
 & \quad \quad \left. \left. 4b^{2/3} ((c + dx)(ad + b(c + 2dx)))^{2/3} \right) /} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right)^2 \right) \\
 & \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \left( (1 - \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right) / \right. \right. \right. \right. \\
 & \quad \left. \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right) \right], -7 - 4\sqrt{3} \right] \right) / \\
 & \left( 3^{1/4} b^{2/3} (bc - ad)^{7/3} (c + dx)^{1/3} (bc + ad + 2bdx)^{1/3} (3bc + ad + 4bdx) \right. \\
 & \quad \left. \sqrt{d^2 (3bc + ad + 4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2b^{1/3} ((c + dx)(ad + b(c + 2dx)))^{1/3} \right)^2 \right) - \\
 & \frac{2d^2 \text{Log}[a + bx]}{3b^{2/3} (bc - ad)^{8/3}} + \frac{d^2 \text{Log} \left[ \frac{b^{2/3} (c + dx)^{2/3}}{(bc - ad)^{1/3}} - (bc + ad + 2bdx)^{1/3} \right]}{b^{2/3} (bc - ad)^{8/3}}
 \end{aligned}$$

Result (type 6, 620 leaves):

$$\frac{1}{10} (c+dx)^{2/3} (ad+b(c+2dx))^{2/3} \left( \frac{5(-bc+5ad+4bdx)}{(bc-ad)^3 (a+bx)^2} + \right. \\ \left. \left( 4d^2 \left( 10 - \frac{5c}{c+dx} + \frac{5ad}{bc+bdx} + \left( 75b(bc-ad)(c+dx) \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] \right) / \left( d(a+bx) \left( 10b(c+dx) \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right], \frac{bc-ad}{bc+bdx} \right] + (bc-ad) \left( 6 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] + \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] \right) \right) \right) - \right. \\ \left. \left( 16(bc-ad)^2 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] \right) / \left( d(a+bx) \left( 16b(c+dx) \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] + (bc-ad) \left( 6 \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] + \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] \right) \right) \right) \right) / \left( (-bc+ad)^3 (bc+ad+2bdx) \right) \right)$$

**Problem 3017: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^3}{(c+dx)^{1/3} (bc+ad+2bdx)^{4/3}} dx$$

Optimal (type 4, 1388 leaves, 7 steps):

$$\begin{aligned}
 & \frac{3 (a + b x)^2 (c + d x)^{2/3}}{14 d^2 (b c + a d + 2 b d x)^{1/3}} + \frac{9 (b c - a d) (c + d x)^{2/3} (b c - 7 a d - 6 b d x)}{112 d^4 (b c + a d + 2 b d x)^{1/3}} + \\
 & \left( 81 (b c - a d)^2 ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
 & \quad \left. \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right) / \left( 112 b^{2/3} d^6 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} \right. \\
 & \quad \left. (3 b c + a d + 4 b d x) \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right) - \\
 & \left( 81 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (b c - a d)^{8/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right. \\
 & \quad \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} \right.} \\
 & \quad \left. ((c + d x) (a d + b (c + 2 d x)))^{1/3} + 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) / \\
 & \quad \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \Big) / \\
 & \quad \text{EllipticE}\left[\text{ArcSin}\left[\left( (1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right], -7 - 4 \sqrt{3} \right] \Big) / \\
 & \left( 224 b^{2/3} d^4 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
 & \quad \left. \sqrt{\left( (b c - a d)^{2/3} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) + \\
 & \left( 27 \times 3^{3/4} (b c - a d)^{8/3} ((c + d x) (b c + a d + 2 b d x))^{1/3} \sqrt{(d (3 b c + a d) + 4 b d^2 x)^2} \right. \\
 & \quad \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \\
 & \quad \left. \sqrt{\left( (b c - a d)^{4/3} - 2 b^{1/3} (b c - a d)^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} + \right.} \right. \\
 & \quad \left. \left. 4 b^{2/3} ((c + d x) (a d + b (c + 2 d x)))^{2/3} \right) /} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right) \\
 & \quad \text{EllipticF}\left[\text{ArcSin}\left[\left( (1 - \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right], -7 - 4 \sqrt{3} \right] \Big) / \\
 & \left( 56 \sqrt{2} b^{2/3} d^4 (c + d x)^{1/3} (b c + a d + 2 b d x)^{1/3} (3 b c + a d + 4 b d x) \sqrt{d^2 (3 b c + a d + 4 b d x)^2} \right. \\
 & \quad \left. \sqrt{\left( (b c - a d)^{2/3} \left( (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2 b^{1/3} ((c + d x) (a d + b (c + 2 d x)))^{1/3} \right)^2 \right)
 \end{aligned}$$

Result (type 5, 157 leaves):

$$\frac{1}{224 (c+dx)^{1/3}} (ad+b(c+2dx))^{2/3} \left( \frac{6(c+dx) \left( -11bc + 15ad + 4bdx + \frac{14(bc-ad)^2}{ad+b(c+2dx)} \right)}{d^4} + \frac{1}{bd^4} \right)$$

$$81 \times 2^{1/3} (bc-ad)^2 \left( \frac{b(c+dx)}{bc-ad} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{ad+b(c+2dx)}{-bc+ad} \right]$$

**Problem 3018: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^2}{(c+dx)^{1/3} (bc+ad+2bdx)^{4/3}} dx$$

Optimal (type 4, 1366 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{3(bc-ad)(c+dx)^{2/3}}{4d^3(bc+ad+2bdx)^{1/3}} + \frac{3(c+dx)^{2/3}(bc+ad+2bdx)^{2/3}}{16d^3} - \\
 & \left( 9(bc-ad)((c+dx)(bc+ad+2bdx))^{1/3} \sqrt{d^2(3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{(d(3bc+ad)+4bd^2x)^2} \right) / \left( 16b^{2/3}d^5(c+dx)^{1/3}(bc+ad+2bdx)^{1/3} \right. \\
 & \quad \left. (3bc+ad+4bdx) \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right) + \\
 & \left( 9 \times 3^{1/4} \sqrt{2-\sqrt{3}} (bc-ad)^{5/3} ((c+dx)(bc+ad+2bdx))^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3}(bc-ad)^{2/3} \right.} \\
 & \quad \left. ((c+dx)(ad+b(c+2dx)))^{1/3} + 4b^{2/3}((c+dx)(ad+b(c+2dx)))^{2/3} \right) / \\
 & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \Big) / \\
 & \quad \text{EllipticE}\left[\text{ArcSin}\left[\left( (1-\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( 32b^{2/3}d^3(c+dx)^{1/3}(bc+ad+2bdx)^{1/3}(3bc+ad+4bdx) \sqrt{d^2(3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right) / \right.} \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \right) - \\
 & \left( 3 \times 3^{3/4} (bc-ad)^{5/3} ((c+dx)(bc+ad+2bdx))^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \\
 & \quad \left. \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3}(bc-ad)^{2/3}((c+dx)(ad+b(c+2dx)))^{1/3} + \right. \right.} \\
 & \quad \left. \left. 4b^{2/3}((c+dx)(ad+b(c+2dx)))^{2/3} \right) / \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \right) \\
 & \quad \text{EllipticF}\left[\text{ArcSin}\left[\left( (1-\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right], -7-4\sqrt{3} \right] \Big) / \\
 & \left( 8\sqrt{2} b^{2/3}d^3(c+dx)^{1/3}(bc+ad+2bdx)^{1/3}(3bc+ad+4bdx) \sqrt{d^2(3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right) \right) / \right.} \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2b^{1/3}((c+dx)(ad+b(c+2dx)))^{1/3} \right)^2 \right) \Big)
 \end{aligned}$$

Result(type 5, 119leaves):

$$\begin{aligned}
 & - \left( \left( 3 (c+dx)^{2/3} \left( 6bc - 10ad - 4bdx + \frac{1}{\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}} 3 \times 2^{1/3} (ad+b(c+2dx)) \right. \right. \right. \\
 & \quad \left. \left. \left. \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{ad+b(c+2dx)}{-bc+ad}\right]\right) \right) \right) / \left( 32d^3 (ad+b(c+2dx))^{1/3} \right)
 \end{aligned}$$

**Problem 3020: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(c+dx)^{1/3} (bc+ad+2bdx)^{4/3}} dx$$

Optimal (type 4, 1333 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{3 (c+dx)^{2/3}}{d (bc-ad) (bc+ad+2bdx)^{1/3}} + \\
 & \left( \frac{3 \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{d^2 (3bc+ad+4bdx)^2} \sqrt{(d(3bc+ad)+4bd^2x)^2}}{2b^{2/3}d^3 (bc-ad) (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx)} \right. \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) - \right. \\
 & \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3} (bc-ad)^{2/3} \right.} \right. \\
 & \quad \left. \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} + 4b^{2/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{2/3} \right) / \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) \\
 & \quad \text{EllipticE}\left[\text{ArcSin}\left[\left( (1-\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right], -7-4\sqrt{3} \right] \right) / \\
 & \left( 4b^{2/3}d (bc-ad)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \right. \\
 & \quad \left. \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) + \\
 & \left( 3^{3/4} \left( (c+dx) (bc+ad+2bdx) \right)^{1/3} \sqrt{(d(3bc+ad)+4bd^2x)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{4/3} - 2b^{1/3} (bc-ad)^{2/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} + \right.} \right. \\
 & \quad \left. \left. 4b^{2/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{2/3} \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \right) \\
 & \quad \text{EllipticF}\left[\text{ArcSin}\left[\left( (1-\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) / \right. \right. \\
 & \quad \left. \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right], -7-4\sqrt{3} \right] \right) / \\
 & \left( \sqrt{2} b^{2/3}d (bc-ad)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx)^{1/3} (3bc+ad+4bdx) \right. \\
 & \quad \left. \sqrt{d^2 (3bc+ad+4bdx)^2} \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2b^{1/3} \left( (c+dx) (ad+b(c+2dx)) \right)^{1/3} \right)^2 \right)
 \end{aligned}$$

Result (type 5, 127 leaves):

$$\left( 12 b (c+dx) - 3 \times 2^{1/3} \left( \frac{b(c+dx)}{bc-ad} \right)^{1/3} (ad+b(c+2dx)) \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{ad+b(c+2dx)}{-bc+ad} \right] \right) / \left( 4bd(-bc+ad)(c+dx)^{1/3}(ad+b(c+2dx))^{1/3} \right)$$

**Problem 3021: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+bx)(c+dx)^{1/3}(bc+ad+2bdx)^{4/3}} dx$$

Optimal (type 6, 113 leaves, 2 steps):

$$\left( 3(c+dx)^{2/3} \left( -\frac{bc+ad+2bdx}{bc-ad} \right)^{1/3} \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{4}{3}, 1, \frac{5}{3}, \frac{2b(c+dx)}{bc-ad}, \frac{b(c+dx)}{bc-ad} \right] \right) / \left( 2(bc-ad)^2 (bc+ad+2bdx)^{1/3} \right)$$

Result (type 6, 395 leaves):

$$\begin{aligned} & \left( -15b(c+dx) \operatorname{AppellF1} \left[ \frac{2}{3}, -\frac{2}{3}, 1, \frac{5}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] + \right. \\ & 6d(a+bx) \left( 3 \operatorname{AppellF1} \left[ \frac{5}{3}, -\frac{2}{3}, 2, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] - \right. \\ & \left. \left. \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] \right) \right) / \left( 2bd(a+bx)(c+dx)^{1/3} \right) \\ & \left( ad+b(c+2dx) \right)^{1/3} \left( 5b(c+dx) \operatorname{AppellF1} \left[ \frac{2}{3}, -\frac{2}{3}, 1, \frac{5}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] + \right. \\ & \left. (bc-ad) \left( 3 \operatorname{AppellF1} \left[ \frac{5}{3}, -\frac{2}{3}, 2, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] - \right. \right. \\ & \left. \left. \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc-ad}{2bc+2bdx}, \frac{bc-ad}{bc+bdx} \right] \right) \right) \end{aligned}$$

**Problem 3022: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+bx)^2 (c+dx)^{1/3} (bc+ad+2bdx)^{4/3}} dx$$

Optimal (type 6, 114 leaves, 2 steps):

$$\left( - \left( 3d(c+dx)^{2/3} \left( -\frac{bc+ad+2bdx}{bc-ad} \right)^{1/3} \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{4}{3}, 2, \frac{5}{3}, \frac{2b(c+dx)}{bc-ad}, \frac{b(c+dx)}{bc-ad} \right] \right) / \left( 2(bc-ad)^3 (bc+ad+2bdx)^{1/3} \right) \right)$$

Result (type 6, 605 leaves):

$$\frac{1}{5 (a d + b (c + 2 d x))^{1/3}} (c + d x)^{2/3} \left( -\frac{5 (13 a d + b (c + 14 d x))}{(b c - a d)^3 (a + b x)} + \frac{1}{(-b c + a d)^3} \right. \\ \left. d \left( -\left( \left( 400 b (b c - a d) (c + d x) \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) / \right. \right. \right. \\ \left. \left( d (a + b x) \left( 10 b (c + d x) \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) + \right. \right. \\ \left. (b c - a d) \left( 6 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \right. \right. \\ \left. \left. \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right) + 7 \left( -10 + \frac{5 c}{c + d x} - \right. \\ \left. \frac{5 a d}{b c + b d x} + \left( 16 (b c - a d)^2 \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) / \\ \left( d (a + b x) \left( 16 b (c + d x) \operatorname{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \right. \right. \\ \left. (b c - a d) \left( 6 \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] + \right. \right. \\ \left. \left. \operatorname{AppellF1} \left[ \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{b c - a d}{2 b c + 2 b d x}, \frac{b c - a d}{b c + b d x} \right] \right) \right) \right) \right)$$

**Problem 3023: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a + b x)^3 (c + d x)^{1/3} (b c + a d + 2 b d x)^{4/3}} dx$$

Optimal (type 6, 116 leaves, 2 steps):

$$\left( 3 d^2 (c + d x)^{2/3} \left( -\frac{b c + a d + 2 b d x}{b c - a d} \right)^{1/3} \operatorname{AppellF1} \left[ \frac{2}{3}, \frac{4}{3}, 3, \frac{5}{3}, \frac{2 b (c + d x)}{b c - a d}, \frac{b (c + d x)}{b c - a d} \right] \right) / \\ \left( 2 (b c - a d)^4 (b c + a d + 2 b d x)^{1/3} \right)$$

Result (type 6, 638 leaves):

$$\frac{1}{10 (bc - ad)^4} (c + dx)^{2/3} (ad + b(c + 2dx))^{2/3} \left( 5 \left( \frac{-bc + ad}{(a + bx)^2} + \frac{8d}{a + bx} + \frac{48d^2}{bc + ad + 2bdx} \right) - \right. \\ \left. \left( 4d^2 \left( \left( 475b (bc - ad) (c + dx) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) / \right. \right. \right. \\ \left. \left( d (a + bx) \left( 10b (c + dx) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, 1, \frac{5}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \right. \right. \right. \\ \left. \left. (bc - ad) \left( 6 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 2, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \right. \right. \right. \\ \left. \left. \left. \text{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, 1, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) \right) \right) \right) + 8 \left( 10 - \frac{5c}{c + dx} + \right. \\ \left. \frac{5ad}{bc + bdx} - \left( 16 (bc - ad)^2 \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) \right) / \\ \left( d (a + bx) \left( 16b (c + dx) \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, 1, \frac{8}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \right. \right. \\ \left. \left. (bc - ad) \left( 6 \text{AppellF1} \left[ \frac{8}{3}, \frac{1}{3}, 2, \frac{11}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] + \text{AppellF1} \left[ \right. \right. \right. \\ \left. \left. \left. \frac{8}{3}, \frac{4}{3}, 1, \frac{11}{3}, \frac{bc - ad}{2bc + 2bdx}, \frac{bc - ad}{bc + bdx} \right] \right) \right) \right) \right) \right) / (ad + b(c + 2dx)) \Bigg)$$

**Problem 3024: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(d - 3ex)^{1/3} (d + ex) (d + 3ex)^{1/3}} dx$$

Optimal (type 3, 120 leaves, 1 step):

$$\frac{\sqrt{3} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{(d-3ex)^{2/3}}{\sqrt{3} d^{1/3} (d+3ex)^{1/3}} \right]}{4 d^{2/3} e} + \frac{\text{Log}[d + ex]}{4 d^{2/3} e} - \frac{3 \text{Log} \left[ -\frac{(d-3ex)^{2/3}}{2 d^{1/3}} - (d + 3ex)^{1/3} \right]}{8 d^{2/3} e}$$

Result (type 6, 196 leaves):

$$- \left( \left( 45 (d + ex) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{4d}{3(d + ex)}, \frac{2d}{3(d + ex)} \right] \right) / \right. \\ \left( 2e (d - 3ex)^{1/3} (d + 3ex)^{1/3} \left( 15 (d + ex) \text{AppellF1} \left[ \frac{2}{3}, \frac{1}{3}, \frac{1}{3}, \frac{5}{3}, \frac{4d}{3(d + ex)}, \frac{2d}{3(d + ex)} \right] + \right. \right. \\ \left. 2d \left( \text{AppellF1} \left[ \frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{8}{3}, \frac{4d}{3(d + ex)}, \frac{2d}{3(d + ex)} \right] + \right. \right. \\ \left. \left. 2 \text{AppellF1} \left[ \frac{5}{3}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{4d}{3(d + ex)}, \frac{2d}{3(d + ex)} \right] \right) \right) \right) \Bigg)$$

**Problem 3025: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + bx)^{4/3} (e + fx)^2}{(c + dx)^{4/3}} dx$$

Optimal (type 3, 562 leaves, 5 steps):

$$\frac{3 (d e - c f)^2 (a + b x)^{7/3}}{d^2 (b c - a d) (c + d x)^{1/3}} - \frac{1}{27 b d^4}$$

$$4 (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) (a + b x)^{1/3} (c + d x)^{2/3} +$$

$$\frac{1}{9 b d^3 (b c - a d)} (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2))$$

$$(a + b x)^{4/3} (c + d x)^{2/3} + \frac{f^2 (a + b x)^{7/3} (c + d x)^{2/3}}{3 b d^2} - \frac{1}{27 \sqrt{3} b^{5/3} d^{13/3}}$$

$$4 (b c - a d) (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2))$$

$$\text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}}\right] - \frac{1}{81 b^{5/3} d^{13/3}}$$

$$2 (b c - a d) (a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) \text{Log}[a + b x] -$$

$$\frac{1}{27 b^{5/3} d^{13/3}} 2 (b c - a d)$$

$$(a^2 d^2 f^2 - a b d f (9 d e - 7 c f) - b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2)) \text{Log}\left[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}\right]$$

Result (type 5, 282 leaves):

$$\frac{1}{27 b d^4} (a + b x)^{1/3} (c + d x)^{2/3}$$

$$\left( \frac{1}{c + d x} (2 a^2 d^2 f^2 (c + d x) + b^2 (140 c^3 f^2 + 7 c^2 d f (-36 e + 5 f x) + 3 c d^2 (36 e^2 - 21 e f x - 5 f^2 x^2) +$$

$$9 d^3 x (3 e^2 + 3 e f x + f^2 x^2)) +$$

$$a b d (-133 c^2 f^2 + c d f (225 e - 37 f x) + d^2 (-81 e^2 + 63 e f x + 15 f^2 x^2)) +$$

$$\frac{1}{\left(\frac{d(a+bx)}{-bc+ad}\right)^{1/3}} 2 (-a^2 d^2 f^2 + a b d f (9 d e - 7 c f) + b^2 (27 d^2 e^2 - 63 c d e f + 35 c^2 f^2))$$

$$\text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 3026: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{4/3} (e + f x)}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 328 leaves, 4 steps):

$$\frac{3 (d e - c f) (a + b x)^{7/3}}{d (b c - a d) (c + d x)^{1/3}} + \frac{2 (6 b d e - 7 b c f + a d f) (a + b x)^{1/3} (c + d x)^{2/3}}{3 d^3} -$$

$$\frac{(6 b d e - 7 b c f + a d f) (a + b x)^{4/3} (c + d x)^{2/3}}{2 d^2 (b c - a d)} +$$

$$\frac{2 (b c - a d) (6 b d e - 7 b c f + a d f) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}}\right]}{3 \sqrt{3} b^{2/3} d^{10/3}} +$$

$$\frac{(b c - a d) (6 b d e - 7 b c f + a d f) \operatorname{Log}[a + b x]}{9 b^{2/3} d^{10/3}} +$$

$$\frac{(b c - a d) (6 b d e - 7 b c f + a d f) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}\right]}{3 b^{2/3} d^{10/3}}$$

Result (type 5, 137 leaves):

$$\frac{1}{6 d^3} (a + b x)^{1/3} (c + d x)^{2/3} \left( 6 b d e - 10 b c f + 7 a d f + 3 b d f x - \right.$$

$$\left. \frac{18 (b c - a d) (-d e + c f)}{c + d x} + \frac{2 (6 b d e - 7 b c f + a d f) \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right]}{\left(\frac{d (a + b x)}{-b c + a d}\right)^{1/3}} \right)$$

**Problem 3027: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$-\frac{3 (a + b x)^{4/3}}{d (c + d x)^{1/3}} + \frac{4 b (a + b x)^{1/3} (c + d x)^{2/3}}{d^2} + \frac{4 b^{1/3} (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c + d x)^{1/3}}{\sqrt{3} d^{1/3} (a + b x)^{1/3}}\right]}{\sqrt{3} d^{7/3}} +$$

$$\frac{2 b^{1/3} (b c - a d) \operatorname{Log}[a + b x]}{3 d^{7/3}} + \frac{2 b^{1/3} (b c - a d) \operatorname{Log}\left[-1 + \frac{b^{1/3} (c + d x)^{1/3}}{d^{1/3} (a + b x)^{1/3}}\right]}{d^{7/3}}$$

Result (type 5, 95 leaves):

$$\frac{(a + b x)^{1/3} (c + d x)^{2/3} \left( \frac{4 b c - 3 a d + b d x}{c + d x} + \frac{2 b \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right]}{\left(\frac{d (a + b x)}{-b c + a d}\right)^{1/3}} \right)}{d^2}$$

**Problem 3028: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{4/3} (e + f x)} dx$$

Optimal (type 3, 380 leaves, 4 steps):

$$\frac{3 (bc - ad) (a + bx)^{1/3}}{d (de - cf) (c + dx)^{1/3}} - \frac{\sqrt{3} b^{4/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+dx)^{1/3}}{\sqrt{3} d^{1/3} (a+bx)^{1/3}}\right]}{d^{4/3} f} +$$

$$\frac{\sqrt{3} (be - af)^{4/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (be-af)^{1/3} (c+dx)^{1/3}}{\sqrt{3} (de-cf)^{1/3} (a+bx)^{1/3}}\right]}{f (de - cf)^{4/3}} -$$

$$\frac{b^{4/3} \text{Log}[a + bx]}{2 d^{4/3} f} - \frac{(be - af)^{4/3} \text{Log}[e + fx]}{2 f (de - cf)^{4/3}} +$$

$$\frac{3 (be - af)^{4/3} \text{Log}\left[-(a + bx)^{1/3} + \frac{(be-af)^{1/3} (c+dx)^{1/3}}{(de-cf)^{1/3}}\right]}{2 f (de - cf)^{4/3}} - \frac{3 b^{4/3} \text{Log}\left[-1 + \frac{b^{1/3} (c+dx)^{1/3}}{d^{1/3} (a+bx)^{1/3}}\right]}{2 d^{4/3} f}$$

Result (type 6, 559 leaves):

$$\frac{1}{5 d^2 (de - cf) (a + bx)^{2/3} (c + dx)^{1/3}}$$

$$3 \left( -5 d (-bc + ad) (a + bx) - \frac{1}{d (e + fx)} 2 b (bc - ad) (c + dx) \right.$$

$$\left. \left( \left( 5 f (-2 b d e + b c f + a d f) (c + dx) \text{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{bc - ad}{bc + b d x}, \frac{-de + c f}{f (c + dx)}\right] \right) / \right.$$

$$\left( 6 b f (c + dx) \text{AppellF1}\left[1, \frac{2}{3}, 1, 2, \frac{bc - ad}{bc + b d x}, \frac{-de + c f}{f (c + dx)}\right] + \right.$$

$$b (-3 d e + 3 c f) \text{AppellF1}\left[2, \frac{2}{3}, 2, 3, \frac{bc - ad}{bc + b d x}, \frac{-de + c f}{f (c + dx)}\right] +$$

$$\left. \left. 2 (bc - ad) f \text{AppellF1}\left[2, \frac{5}{3}, 1, 3, \frac{bc - ad}{bc + b d x}, \frac{-de + c f}{f (c + dx)}\right] \right) - \right.$$

$$\left. \left( 4 b (de - cf)^2 \text{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c + dx)}{bc - ad}, \frac{f (c + dx)}{-de + c f}\right] \right) / \right.$$

$$\left( -\frac{1}{c + dx} 8 (bc - ad) (-de + cf) \text{AppellF1}\left[\frac{5}{3}, \frac{2}{3}, 1, \frac{8}{3}, \frac{b (c + dx)}{bc - ad}, \frac{f (c + dx)}{-de + c f}\right] + \right.$$

$$\left. (-3 b c f + 3 a d f) \text{AppellF1}\left[\frac{8}{3}, \frac{2}{3}, 2, \frac{11}{3}, \frac{b (c + dx)}{bc - ad}, \frac{f (c + dx)}{-de + c f}\right] + \right.$$

$$\left. \left. 2 b (de - cf) \text{AppellF1}\left[\frac{8}{3}, \frac{5}{3}, 1, \frac{11}{3}, \frac{b (c + dx)}{bc - ad}, \frac{f (c + dx)}{-de + c f}\right] \right) \right)$$

**Problem 3029: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + bx)^{4/3}}{(c + dx)^{4/3} (e + fx)^2} dx$$

Optimal (type 3, 301 leaves, 3 steps):

$$\begin{aligned}
 & - \frac{3 (a+bx)^{4/3}}{(de-cf)(c+dx)^{1/3}(e+fx)} + \frac{4 (be-af)(a+bx)^{1/3}(c+dx)^{2/3}}{(de-cf)^2(e+fx)} + \\
 & \frac{4 (bc-ad)(be-af)^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 (be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3} (de-cf)^{1/3}(a+bx)^{1/3}}\right]}{\sqrt{3} (de-cf)^{7/3}} - \\
 & \frac{2 (bc-ad)(be-af)^{1/3} \operatorname{Log}[e+fx]}{3 (de-cf)^{7/3}} + \\
 & \frac{2 (bc-ad)(be-af)^{1/3} \operatorname{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}}\right]}{(de-cf)^{7/3}}
 \end{aligned}$$

Result (type 5, 160 leaves):

$$\begin{aligned}
 & \left( (a+bx)^{1/3} \right. \\
 & \left. \left( b(4ce+dex+3cfx) - a(3de+cf+4dfx) - 4(bc-ad) \left( \frac{(be-af)(c+dx)}{(bc-ad)(e+fx)} \right)^{1/3} (e+fx) \right. \right. \\
 & \left. \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right] \right) \right) / \left( (de-cf)^2 (c+dx)^{1/3} (e+fx) \right)
 \end{aligned}$$

**Problem 3030: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}(e+fx)^3} dx$$

Optimal (type 3, 434 leaves, 4 steps):

$$\begin{aligned}
 & \frac{3d(a+bx)^{7/3}}{(bc-ad)(de-cf)(c+dx)^{1/3}(e+fx)^2} - \\
 & \frac{(6bde+bcf-7adf)(a+bx)^{4/3}(c+dx)^{2/3}}{2(bc-ad)(de-cf)^2(e+fx)^2} + \frac{2(6bde+bcf-7adf)(a+bx)^{1/3}(c+dx)^{2/3}}{3(de-cf)^3(e+fx)} + \\
 & \left( 2(bc-ad)(6bde+bcf-7adf) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3}(de-cf)^{1/3}(a+bx)^{1/3}}\right] \right) / \\
 & \left( 3\sqrt{3}(be-af)^{2/3}(de-cf)^{10/3} \right) - \frac{(bc-ad)(6bde+bcf-7adf) \operatorname{Log}[e+fx]}{9(be-af)^{2/3}(de-cf)^{10/3}} + \\
 & \left( (bc-ad)(6bde+bcf-7adf) \operatorname{Log}\left[-(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}}\right] \right) / \\
 & \left( 3(be-af)^{2/3}(de-cf)^{10/3} \right)
 \end{aligned}$$

Result (type 5, 208 leaves):

$$\left( (a+bx)^{1/3} \left( 18d(bc-ad) + \frac{3(be-af)(de-cf)(c+dx)}{(e+fx)^2} + \frac{(3bde+7bcf-10adf)(c+dx)}{e+fx} - \left( 4(6bde+bcf-7adf)(c+dx) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right] \right) \right) \right) / \left( \left( \frac{(be-af)(c+dx)}{(bc-ad)(e+fx)} \right)^{2/3} (e+fx) \right) / (6(de-cf)^3(c+dx)^{1/3})$$

**Problem 3031: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}(e+fx)^4} dx$$

Optimal (type 3, 645 leaves, 6 steps):

$$\frac{3(bc-ad)(a+bx)^{1/3}}{d(de-cf)(c+dx)^{1/3}(e+fx)^3} + \frac{(bde+9bcf-10adf)(a+bx)^{1/3}(c+dx)^{2/3}}{3d(de-cf)^2(e+fx)^3} + \frac{(3bde+32bcf-35adf)(a+bx)^{1/3}(c+dx)^{2/3}}{9(de-cf)^3(e+fx)^2} + \left( (140a^2d^2f^2 - 7abdf(21de+19cf) + b^2(9d^2e^2 + 129cdef + 2c^2f^2)) (a+bx)^{1/3}(c+dx)^{2/3} \right) / (27(be-af)(de-cf)^4(e+fx)) + \left( 4(bc-ad)(35a^2d^2f^2 - 7abdf(9de+cf) + b^2(27d^2e^2 + 9cdef - c^2f^2)) \operatorname{ArcTan}\left[ \frac{1}{\sqrt{3}} + \frac{2(be-af)^{1/3}(c+dx)^{1/3}}{\sqrt{3}(de-cf)^{1/3}(a+bx)^{1/3}} \right] \right) / (27\sqrt{3}(be-af)^{5/3}(de-cf)^{13/3}) - (2(bc-ad)(35a^2d^2f^2 - 7abdf(9de+cf) + b^2(27d^2e^2 + 9cdef - c^2f^2)) \operatorname{Log}[e+fx]) / (81(be-af)^{5/3}(de-cf)^{13/3}) + \left( 2(bc-ad)(35a^2d^2f^2 - 7abdf(9de+cf) + b^2(27d^2e^2 + 9cdef - c^2f^2)) \operatorname{Log}\left[ -(a+bx)^{1/3} + \frac{(be-af)^{1/3}(c+dx)^{1/3}}{(de-cf)^{1/3}} \right] \right) / (27(be-af)^{5/3}(de-cf)^{13/3})$$

Result (type 5, 371 leaves):

$$\frac{1}{27 (b e - a f)^2 (d e - c f)^4 (c + d x)^{1/3} (e + f x)^3} (a + b x)^{1/3}$$

$$\left( (b e - a f) (9 (b e - a f)^2 (d e - c f)^2 (c + d x) + 3 (b e - a f) (d e - c f) (3 b d e + 5 b c f - 8 a d f) \right.$$

$$\left. (c + d x) (e + f x) + (59 a^2 d^2 f^2 - 2 a b d f (33 d e + 26 c f) + b^2 (9 d^2 e^2 + 48 c d e f + 2 c^2 f^2)) \right.$$

$$\left. (c + d x) (e + f x)^2 + 81 d^2 (b c - a d) (b e - a f) (e + f x)^3 \right) +$$

$$4 (b c - a d) (-35 a^2 d^2 f^2 + 7 a b d f (9 d e + c f) + b^2 (-27 d^2 e^2 - 9 c d e f + c^2 f^2))$$

$$\left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{1/3} (e + f x)^3 \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(-d e + c f) (a + b x)}{(b c - a d) (e + f x)} \right]$$

**Problem 3032: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x) \sqrt{c + d x} (e + f x)^{1/4}} dx$$

Optimal (type 4, 266 leaves, 5 steps):

$$\left( 2 (d e - c f)^{1/4} \sqrt{-\frac{f (c + d x)}{d e - c f}} \text{EllipticPi} \left[ -\frac{\sqrt{b} \sqrt{d e - c f}}{\sqrt{d} \sqrt{b e - a f}}, \text{ArcSin} \left[ \frac{d^{1/4} (e + f x)^{1/4}}{(d e - c f)^{1/4}} \right], -1 \right] \right) /$$

$$(\sqrt{b} d^{1/4} \sqrt{b e - a f} \sqrt{c + d x}) -$$

$$\left( 2 (d e - c f)^{1/4} \sqrt{-\frac{f (c + d x)}{d e - c f}} \text{EllipticPi} \left[ \frac{\sqrt{b} \sqrt{d e - c f}}{\sqrt{d} \sqrt{b e - a f}}, \text{ArcSin} \left[ \frac{d^{1/4} (e + f x)^{1/4}}{(d e - c f)^{1/4}} \right], -1 \right] \right) /$$

$$(\sqrt{b} d^{1/4} \sqrt{b e - a f} \sqrt{c + d x})$$

Result (type 6, 270 leaves):

$$- \left( \left( 28 d f (a + b x) \text{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \frac{7}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) / \right.$$

$$\left( 3 b \sqrt{c + d x} (e + f x)^{1/4} \left( 7 d f (a + b x) \text{AppellF1} \left[ \frac{3}{4}, \frac{1}{2}, \frac{1}{4}, \frac{7}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right.$$

$$\left. (-b d e + a d f) \text{AppellF1} \left[ \frac{7}{4}, \frac{1}{2}, \frac{5}{4}, \frac{11}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right.$$

$$\left. \left. 2 (-b c + a d) f \text{AppellF1} \left[ \frac{7}{4}, \frac{3}{2}, \frac{1}{4}, \frac{11}{4}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) \right) \right)$$

**Problem 3033: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x) \sqrt{c + d x} (e + f x)^{3/4}} dx$$

Optimal (type 4, 252 leaves, 5 steps):

$$\begin{aligned}
 & - \left( \left( 2 (de - cf)^{1/4} \sqrt{-\frac{f(c+dx)}{de - cf}} \operatorname{EllipticPi} \left[ -\frac{\sqrt{b} \sqrt{de - cf}}{\sqrt{d} \sqrt{be - af}}, \operatorname{ArcSin} \left[ \frac{d^{1/4} (e + fx)^{1/4}}{(de - cf)^{1/4}} \right], -1 \right] \right) / \right. \\
 & \quad \left. (d^{1/4} (be - af) \sqrt{c + dx}) \right) - \\
 & \left( 2 (de - cf)^{1/4} \sqrt{-\frac{f(c+dx)}{de - cf}} \operatorname{EllipticPi} \left[ \frac{\sqrt{b} \sqrt{de - cf}}{\sqrt{d} \sqrt{be - af}}, \operatorname{ArcSin} \left[ \frac{d^{1/4} (e + fx)^{1/4}}{(de - cf)^{1/4}} \right], -1 \right] \right) / \\
 & \quad (d^{1/4} (be - af) \sqrt{c + dx})
 \end{aligned}$$

Result (type 6, 271 leaves):

$$\begin{aligned}
 & - \left( \left( 36 df (a + bx) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{4}, \frac{-bc + ad}{d(a + bx)}, \frac{-be + af}{f(a + bx)} \right] \right) / \right. \\
 & \quad \left( 5 b \sqrt{c + dx} (e + fx)^{3/4} \left( 9 df (a + bx) \operatorname{AppellF1} \left[ \frac{5}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{4}, \frac{-bc + ad}{d(a + bx)}, \frac{-be + af}{f(a + bx)} \right] + \right. \right. \\
 & \quad \left. \left. (-3 b d e + 3 a d f) \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{1}{2}, \frac{7}{4}, \frac{13}{4}, \frac{-bc + ad}{d(a + bx)}, \frac{-be + af}{f(a + bx)} \right] + \right. \right. \\
 & \quad \left. \left. 2 (-bc + ad) f \operatorname{AppellF1} \left[ \frac{9}{4}, \frac{3}{2}, \frac{3}{4}, \frac{13}{4}, \frac{-bc + ad}{d(a + bx)}, \frac{-be + af}{f(a + bx)} \right] \right) \right) \right)
 \end{aligned}$$

### Problem 3035: Result unnecessarily involves higher level functions.

$$\int (a + bx) (c + dx)^n (e + fx)^{-n} dx$$

Optimal (type 5, 134 leaves, 3 steps):

$$\begin{aligned}
 & \frac{b (c + dx)^{1+n} (e + fx)^{1-n}}{2 df} + \frac{1}{2 d^2 f (1+n)} (2 a d f - b (c f (1-n) + d e (1+n))) \\
 & (c + dx)^{1+n} (e + fx)^{-n} \left( \frac{d (e + fx)}{d e - c f} \right)^n \operatorname{Hypergeometric2F1} \left[ n, 1+n, 2+n, -\frac{f (c + dx)}{d e - c f} \right]
 \end{aligned}$$

Result (type 6, 201 leaves):

$$\begin{aligned}
 & (c + dx)^n (e + fx)^{-n} \left( \left( 3 b c e x^2 \operatorname{AppellF1} \left[ 2, -n, n, 3, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) / \right. \\
 & \quad \left( 6 c e \operatorname{AppellF1} \left[ 2, -n, n, 3, -\frac{dx}{c}, -\frac{fx}{e} \right] + 2 n x \right. \\
 & \quad \left. \left( d e \operatorname{AppellF1} \left[ 3, 1-n, n, 4, -\frac{dx}{c}, -\frac{fx}{e} \right] - c f \operatorname{AppellF1} \left[ 3, -n, 1+n, 4, -\frac{dx}{c}, -\frac{fx}{e} \right] \right) \right) - \\
 & \quad \frac{1}{f (-1+n)} a \left( \frac{f (c + dx)}{-d e + c f} \right)^{-n} (e + fx) \operatorname{Hypergeometric2F1} \left[ 1-n, -n, 2-n, \frac{d (e + fx)}{d e - c f} \right]
 \end{aligned}$$

**Problem 3041: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^{-n} (c+dx) (e+fx)^n dx$$

Optimal (type 5, 135 leaves, 3 steps):

$$\frac{d (a+bx)^{1-n} (e+fx)^{1+n}}{2bf} + \frac{1}{2bf^2(1+n)} (b(2cf-de(1-n)) - adf(1+n))$$

$$(a+bx)^{-n} \left( -\frac{f(a+bx)}{be-af} \right)^n (e+fx)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{b(e+fx)}{be-af}\right]$$

Result (type 6, 192 leaves):

$$(a+bx)^{-n} (e+fx)^n \left( \left( 3ade x^2 \text{AppellF1}\left[2, n, -n, 3, -\frac{bx}{a}, -\frac{fx}{e}\right] \right) / \right.$$

$$\left( 6ae \text{AppellF1}\left[2, n, -n, 3, -\frac{bx}{a}, -\frac{fx}{e}\right] + 2nx \right.$$

$$\left. \left( af \text{AppellF1}\left[3, n, 1-n, 4, -\frac{bx}{a}, -\frac{fx}{e}\right] - be \text{AppellF1}\left[3, 1+n, -n, 4, -\frac{bx}{a}, -\frac{fx}{e}\right] \right) \right) +$$

$$\left. \frac{c \left( \frac{f(a+bx)}{-be+af} \right)^n (e+fx) \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{b(e+fx)}{be-af}\right]}{f(1+n)} \right)$$

**Problem 3047: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^{-m} (e+fx)^p dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$\frac{1}{b(1+m)} (a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m (e+fx)^p$$

$$\left( \frac{b(e+fx)}{be-af} \right)^{-p} \text{AppellF1}\left[1+m, m, -p, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]$$

Result (type 6, 290 leaves):

$$\left( (bc - ad) (be - af) (2 + m) (a + bx)^{1+m} (c + dx)^{-m} \right. \\ \left. (e + fx)^p \operatorname{AppellF1}\left[1 + m, m, -p, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] \right) / \\ \left( b(1 + m) \left( (bc - ad) (be - af) (2 + m) \operatorname{AppellF1}\left[1 + m, m, -p, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] - \right. \right. \\ \left. (a + bx) \left( (-bc + ad) f p \operatorname{AppellF1}\left[2 + m, m, 1 - p, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] + \right. \right. \\ \left. \left. d (be - af) m \operatorname{AppellF1}\left[2 + m, 1 + m, -p, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] \right) \right) \right)$$

**Problem 3048: Result unnecessarily involves higher level functions.**

$$\int (5 - 4x)^4 (1 + 2x)^{-m} (2 + 3x)^m dx$$

Optimal (type 5, 188 leaves, 4 steps):

$$-\frac{1}{45} (88 - m) (5 - 4x)^2 (1 + 2x)^{1-m} (2 + 3x)^{1+m} - \frac{2}{15} (5 - 4x)^3 (1 + 2x)^{1-m} (2 + 3x)^{1+m} - \\ \frac{1}{1215} (1 + 2x)^{1-m} (2 + 3x)^{1+m} (386850 - 25441m + 426m^2 - 2m^3 - 24(4359 - 154m + m^2)x) + \\ \frac{1}{1215(1 - m)} 2^{-1-m} (3528363 - 639760m + 29050m^2 - 440m^3 + 2m^4) \\ (1 + 2x)^{1-m} \operatorname{Hypergeometric2F1}\left[1 - m, -m, 2 - m, -3(1 + 2x)\right]$$

Result (type 6, 155 leaves):

$$\left( 483 \times 2^{-1-m} (-5 + 4x)^5 (2 + 4x)^{-m} (8 + 12x)^m \operatorname{AppellF1}\left[5, -m, m, 6, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)\right] \right) / \\ \left( 5 \left( 966 \operatorname{AppellF1}\left[5, -m, m, 6, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)\right] + \right. \right. \\ \left. m(-5 + 4x) \left( 21 \operatorname{AppellF1}\left[6, 1 - m, m, 7, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)\right] - \right. \right. \\ \left. \left. 23 \operatorname{AppellF1}\left[6, -m, 1 + m, 7, \frac{3}{23}(5 - 4x), \frac{1}{7}(5 - 4x)\right] \right) \right) \right)$$

**Problem 3049: Result unnecessarily involves higher level functions.**

$$\int (a + bx)^m (c + dx)^{-m} (e + fx)^3 dx$$

Optimal (type 5, 432 leaves, 4 steps):

$$\frac{f (a+bx)^{1+m} (c+dx)^{1-m} (e+fx)^2}{4bd} + \frac{1}{24b^3d^3}$$

$$f (a+bx)^{1+m} (c+dx)^{1-m} (a^2d^2f^2(6-5m+m^2) - 2abdf(6de(2-m) - cf(3-m^2)) + b^2(30d^2e^2 - 12cdef(2+m) + c^2f^2(6+5m+m^2)) - 2bdf(adf(3-m) - b(6de - cf(3+m)))x) -$$

$$\frac{1}{24b^4d^3(1+m)} (a^3d^3f^3(6-11m+6m^2-m^3) - 3a^2bd^2f^2(2-3m+m^2)(4de - cf(1+m)) + 3ab^2df(1-m)(12d^2e^2 - 8cdef(1+m) + c^2f^2(2+3m+m^2)) - b^3(24d^3e^3 - 36cd^2e^2f(1+m) + 12c^2def^2(2+3m+m^2) - c^3f^3(6+11m+6m^2+m^3)))$$

$$(a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m \text{Hypergeometric2F1}\left[m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad}\right]$$

Result (type 6, 440 leaves):

$$(a+bx)^m (c+dx)^{-m} \left( \left( 9ac e^2 f x^2 \text{AppellF1}\left[2, -m, m, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \right.$$

$$\left( 6ac \text{AppellF1}\left[2, -m, m, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + 2mx \right.$$

$$\left. \left( bc \text{AppellF1}\left[3, 1-m, m, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] - ad \text{AppellF1}\left[3, -m, 1+m, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right) +$$

$$\left( 4ac e f^2 x^3 \text{AppellF1}\left[3, -m, m, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) /$$

$$\left( 4ac \text{AppellF1}\left[3, -m, m, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + mx \right.$$

$$\left. \left( bc \text{AppellF1}\left[4, 1-m, m, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] - ad \text{AppellF1}\left[4, -m, 1+m, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right) +$$

$$\left( 5ac f^3 x^4 \text{AppellF1}\left[4, -m, m, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) /$$

$$\left( 20ac \text{AppellF1}\left[4, -m, m, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] + 4bcmx \text{AppellF1}\left[5, 1-m, m, 6, -\frac{bx}{a}, -\frac{dx}{c}\right] - \right.$$

$$\left. 4adm x \text{AppellF1}\left[5, -m, 1+m, 6, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) - \frac{1}{d(-1+m)}$$

$$e^3 \left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \text{Hypergeometric2F1}\left[1-m, -m, 2-m, \frac{b(c+dx)}{bc-ad}\right]$$

**Problem 3050: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^m (c+dx)^{-m} (e+fx)^2 dx$$

Optimal (type 5, 250 leaves, 4 steps):

$$\begin{aligned} & - \frac{f (a d f (2 - m) - b (4 d e - c f (2 + m))) (a + b x)^{1+m} (c + d x)^{1-m}}{6 b^2 d^2} + \\ & \frac{f (a + b x)^{1+m} (c + d x)^{1-m} (e + f x)}{3 b d} + \frac{1}{6 b^3 d^2 (1 + m)} (a^2 d^2 f^2 (2 - 3 m + m^2) - \\ & 2 a b d f (1 - m) (3 d e - c f (1 + m)) + b^2 (6 d^2 e^2 - 6 c d e f (1 + m) + c^2 f^2 (2 + 3 m + m^2))) \\ & (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1} \left[ m, 1 + m, 2 + m, - \frac{d (a + b x)}{b c - a d} \right] \end{aligned}$$

Result (type 6, 320 leaves):

$$\begin{aligned} & (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c e f x^2 \text{AppellF1} \left[ 2, -m, m, 3, - \frac{b x}{a}, - \frac{d x}{c} \right] \right) / \right. \\ & \left( 3 a c \text{AppellF1} \left[ 2, -m, m, 3, - \frac{b x}{a}, - \frac{d x}{c} \right] + m x \right. \\ & \left. \left( b c \text{AppellF1} \left[ 3, 1 - m, m, 4, - \frac{b x}{a}, - \frac{d x}{c} \right] - a d \text{AppellF1} \left[ 3, -m, 1 + m, 4, - \frac{b x}{a}, - \frac{d x}{c} \right] \right) \right) + \\ & \left( 4 a c f^2 x^3 \text{AppellF1} \left[ 3, -m, m, 4, - \frac{b x}{a}, - \frac{d x}{c} \right] \right) / \\ & \left( 12 a c \text{AppellF1} \left[ 3, -m, m, 4, - \frac{b x}{a}, - \frac{d x}{c} \right] + 3 b c m x \text{AppellF1} \left[ 4, 1 - m, m, 5, - \frac{b x}{a}, - \frac{d x}{c} \right] - \right. \\ & \left. 3 a d m x \text{AppellF1} \left[ 4, -m, 1 + m, 5, - \frac{b x}{a}, - \frac{d x}{c} \right] \right) - \frac{1}{d (-1 + m)} \\ & e^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1} \left[ 1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d} \right] \end{aligned}$$

### Problem 3051: Result unnecessarily involves higher level functions.

$$\int (a + b x)^m (c + d x)^{-m} (e + f x) dx$$

Optimal (type 5, 135 leaves, 3 steps):

$$\begin{aligned} & \frac{f (a + b x)^{1+m} (c + d x)^{1-m}}{2 b d} - \frac{1}{2 b^2 d (1 + m)} (a d f (1 - m) - b (2 d e - c f (1 + m))) \\ & (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1} \left[ m, 1 + m, 2 + m, - \frac{d (a + b x)}{b c - a d} \right] \end{aligned}$$

Result (type 6, 201 leaves):

$$\begin{aligned} & (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c f x^2 \text{AppellF1} \left[ 2, -m, m, 3, - \frac{b x}{a}, - \frac{d x}{c} \right] \right) / \right. \\ & \left( 6 a c \text{AppellF1} \left[ 2, -m, m, 3, - \frac{b x}{a}, - \frac{d x}{c} \right] + 2 m x \right. \\ & \left. \left( b c \text{AppellF1} \left[ 3, 1 - m, m, 4, - \frac{b x}{a}, - \frac{d x}{c} \right] - a d \text{AppellF1} \left[ 3, -m, 1 + m, 4, - \frac{b x}{a}, - \frac{d x}{c} \right] \right) \right) - \\ & \frac{1}{d (-1 + m)} e \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1} \left[ 1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d} \right] \end{aligned}$$

**Problem 3053: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{-m}}{e+fx} dx$$

Optimal (type 5, 128 leaves, 5 steps):

$$-\frac{(a+bx)^m (c+dx)^{-m} \operatorname{Hypergeometric2F1}\left[1, m, 1+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] + \frac{1}{fm}}{(a+bx)^m (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \operatorname{Hypergeometric2F1}\left[m, m, 1+m, -\frac{d(a+bx)}{bc-ad}\right]}$$

Result (type 6, 292 leaves):

$$-\left(\left((bc-ad)(be-af)^2(2+m)(a+bx)^{1+m}(c+dx)^{-m} \operatorname{AppellF1}\left[1+m, m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right) / \left(b(-be+af)(1+m)(e+fx)\right) + \left((bc-ad)(be-af)(2+m) \operatorname{AppellF1}\left[1+m, m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right) + (a+bx) \left(\left(-bcf+adf\right) \operatorname{AppellF1}\left[2+m, m, 2, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right) + d(-be+af)^m \operatorname{AppellF1}\left[2+m, 1+m, 1, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right)$$

**Problem 3055: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{-m}}{(e+fx)^3} dx$$

Optimal (type 5, 174 leaves, 2 steps):

$$-\frac{f(a+bx)^{1+m}(c+dx)^{1-m}}{2(be-af)(de-cf)(e+fx)^2} + \left(\left((bc-ad)(b(2de-cf(1-m))-adf(1+m))(a+bx)^{1+m}(c+dx)^{-1-m} \operatorname{Hypergeometric2F1}\left[2, 1+m, 2+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]\right) / \left(2(be-af)^3(de-cf)(1+m)\right)\right)$$

Result (type 5, 432 leaves):



$$\begin{aligned}
 & 6 f (-b e + a f) m^2 (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + \\
 & 2 f^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - \\
 & f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - \\
 & 2 f^2 m^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] + \\
 & f^2 m^3 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m\right] - \\
 & 6 (b e - a f)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - \\
 & 6 (b e - a f)^2 m \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \\
 & 12 f (b e - a f) m (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \\
 & 12 f (b e - a f) m^2 (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \\
 & 3 f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] - \\
 & 3 f^2 m^3 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \\
 & 6 f (-b e + a f) (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \\
 & 12 f (-b e + a f) m (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \\
 & 6 f (-b e + a f) m^2 (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \\
 & 3 f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \\
 & 6 f^2 m^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \\
 & 3 f^2 m^3 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] - \\
 & 2 f^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] - \\
 & 5 f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] -
 \end{aligned}$$

$$\begin{aligned}
 & 4 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] \Bigg) / \\
 & \left( 3 (1 + m) (e + f x)^3 \left( (b e - a f) (c + d x) (a^2 f^2 (2 + 3 m + m^2) + 2 a b f (1 + m) (-2 e + f m x) + \right. \right. \\
 & \quad \left. \left. b^2 (2 e^2 - 4 e f m x + f^2 (-1 + m) m x^2) \right) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \right. \\
 & (a + b x) \left( (a^2 f^2 (2 + 3 m + m^2) (-3 c f + d (e - 2 f x)) - 2 a b f (1 + m) (c f (-e (6 + m) + 2 f m x) + \right. \right. \\
 & \quad \left. \left. d (2 e^2 - 2 e f (2 + m) x + f^2 m x^2) \right) + b^2 (c f (-2 e^2 (3 + 2 m) + 2 e f m (3 + m) x - \right. \\
 & \quad \left. \left. f^2 (-1 + m) m x^2) + d e (2 e^2 - 4 e f (1 + 2 m) x + f^2 m (1 + 3 m) x^2) \right) \right) \\
 & \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + f (1 + m) (a + b x) \\
 & \left( (a f (2 + m) (-2 d e + 3 c f + d f x) + b c f (-e (6 + m) + 2 f m x) + b d e (4 e - f (2 + 3 m) x) \right) \\
 & \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\
 & f (d e - c f) (2 + m) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] \Bigg) \Bigg)
 \end{aligned}$$

**Problem 3057: Result unnecessarily involves higher level functions.**

$$\int \frac{(1 + 2 x)^{-m} (2 + 3 x)^m}{(5 - 4 x)^5} dx$$

Optimal (type 5, 179 leaves, 5 steps):

$$\begin{aligned}
 & \frac{(1 + 2 x)^{1-m} (2 + 3 x)^{1+m}}{322 (5 - 4 x)^4} + \frac{(66 + m) (1 + 2 x)^{1-m} (2 + 3 x)^{1+m}}{77763 (5 - 4 x)^3} + \\
 & \frac{(4359 + 220 m + 2 m^2) (1 + 2 x)^{1-m} (2 + 3 x)^{1+m}}{25039686 (5 - 4 x)^2} + \left( (32010 + 4358 m + 132 m^2 + m^3) (1 + 2 x)^{1-m} \right. \\
 & \left. (2 + 3 x)^{-1+m} \text{Hypergeometric2F1} \left[ 2, 1 - m, 2 - m, \frac{23 (1 + 2 x)}{14 (2 + 3 x)} \right] \right) / (2453889228 (1 - m))
 \end{aligned}$$

Result (type 6, 153 leaves):

$$\left( 15 \times 2^{-4-m} (2+4x)^{-m} (8+12x)^m \text{AppellF1}\left[4, -m, m, 5, \frac{23}{15-12x}, \frac{7}{5-4x}\right] \right) /$$

$$\left( (-5+4x)^3 \left( 15 (-5+4x) \text{AppellF1}\left[4, -m, m, 5, \frac{23}{15-12x}, \frac{7}{5-4x}\right] + \right. \right.$$

$$m \left( 23 \text{AppellF1}\left[5, 1-m, m, 6, \frac{23}{15-12x}, \frac{7}{5-4x}\right] - \right.$$

$$\left. \left. 21 \text{AppellF1}\left[5, -m, 1+m, 6, \frac{23}{15-12x}, \frac{7}{5-4x}\right] \right) \right) \right)$$

**Problem 3058: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^{-1-m} (e+fx)^p dx$$

Optimal (type 6, 130 leaves, 3 steps):

$$\left( (a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m (e+fx)^p \left( \frac{b(e+fx)}{be-af} \right)^{-p} \right.$$

$$\left. \text{AppellF1}\left[1+m, 1+m, -p, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right] \right) / ((bc-ad)(1+m))$$

Result (type 6, 300 leaves):

$$\left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^{-1-m} \right.$$

$$\left. (e+fx)^p \text{AppellF1}\left[1+m, 1+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) /$$

$$\left( b(1+m) \left( (bc-ad)(be-af)(2+m) \text{AppellF1}\left[1+m, 1+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right.$$

$$(a+bx) \left( (-bc+ad) f^p \text{AppellF1}\left[2+m, 1+m, 1-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right.$$

$$\left. \left. d(be-af)(1+m) \text{AppellF1}\left[2+m, 2+m, -p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right)$$

**Problem 3059: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (5-4x)^3 (1+2x)^{-1-m} (2+3x)^m dx$$

Optimal (type 5, 142 leaves, 3 steps):

$$-\frac{2}{9} (5-4x)^2 (1+2x)^{-m} (2+3x)^{1+m} -$$

$$\frac{(1+2x)^{-m} (2+3x)^{1+m} (9261 - 512m + 4m^2 - 4(109-2m)mx)}{27m} + \frac{1}{27(1-m)m}$$

$$2^{-1-m} (27783 - 8324m + 390m^2 - 4m^3) (1+2x)^{1-m} \text{Hypergeometric2F1}\left[1-m, -m, 2-m, -3(1+2x)\right]$$

Result (type 6, 395 leaves):

$$\frac{7}{4} \left( \left( 483 (5-4x)^2 (4+8x)^{-m} (8+12x)^m \text{AppellF1} \left[ 2, -m, m, 3, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] \right) / \right. \\ \left( 483 \text{AppellF1} \left[ 2, -m, m, 3, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] + \right. \\ \left. m (-5+4x) \left( 21 \text{AppellF1} \left[ 3, 1-m, m, 4, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] - \right. \right. \\ \left. \left. 23 \text{AppellF1} \left[ 3, -m, 1+m, 4, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] \right) \right) - \\ \left( 23 \times 2^{3+m} (2+3x)^m (-5+4x)^3 (2+4x)^{-m} \text{AppellF1} \left[ 3, -m, m, 4, -\frac{3}{23} (-5+4x), \frac{1}{7} (5-4x) \right] \right) / \\ \left( 3 \left( 644 \text{AppellF1} \left[ 3, -m, m, 4, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] + \right. \right. \\ \left. \left. m (-5+4x) \left( 21 \text{AppellF1} \left[ 4, 1-m, m, 5, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] - \right. \right. \right. \\ \left. \left. \left. 23 \text{AppellF1} \left[ 4, -m, 1+m, 5, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] \right) \right) \right) + \\ \frac{7 \times 2^{2-m} (1+2x)^{1-m} \text{Hypergeometric2F1} [1-m, -m, 2-m, -3-6x]}{-1+m} - \\ \frac{1}{1+m} \\ 196 (-3-6x)^m (1+2x)^{-m} (2+3x)^{1+m} \\ \left. \text{Hypergeometric2F1} [1+m, 1+m, 2+m, 4+6x] \right)$$

**Problem 3060: Result unnecessarily involves higher level functions.**

$$\int (5-4x)^2 (1+2x)^{-1-m} (2+3x)^m dx$$

Optimal (type 5, 121 leaves, 3 steps):

$$-\frac{7(21-m)(1+2x)^{-m}(2+3x)^{1+m}}{3m} - \frac{1}{3}(5-4x)(1+2x)^{-m}(2+3x)^{1+m} + \frac{1}{3(1-m)m} \\ 2^{-1-m}(441-86m+2m^2)(1+2x)^{1-m} \text{Hypergeometric2F1} [1-m, -m, 2-m, -3(1+2x)]$$

Result (type 6, 241 leaves):

$$\frac{7}{4} \left( \left( 69 (5-4x)^2 (4+8x)^{-m} (8+12x)^m \text{AppellF1} \left[ 2, -m, m, 3, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] \right) / \right. \\ \left( 483 \text{AppellF1} \left[ 2, -m, m, 3, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] + \right. \\ \left. m (-5+4x) \left( 21 \text{AppellF1} \left[ 3, 1-m, m, 4, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] - \right. \right. \\ \left. \left. 23 \text{AppellF1} \left[ 3, -m, 1+m, 4, \frac{3}{23} (5-4x), \frac{1}{7} (5-4x) \right] \right) \right) + \\ \frac{2^{2-m} (1+2x)^{1-m} \text{Hypergeometric2F1} [1-m, -m, 2-m, -3-6x]}{-1+m} - \frac{1}{1+m} \\ \left. 28 (-3-6x)^m (1+2x)^{-m} (2+3x)^{1+m} \text{Hypergeometric2F1} [1+m, 1+m, 2+m, 4+6x] \right)$$

**Problem 3063: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{-1-m}}{e+fx} dx$$

Optimal (type 5, 72 leaves, 1 step):

$$\frac{(a+bx)^m (c+dx)^{-m} \text{Hypergeometric2F1} \left[ 1, -m, 1-m, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)} \right]}{(de-cf)m}$$

Result (type 6, 362 leaves):

$$\frac{1}{de-cf} (a+bx)^m (c+dx)^{-m} \\ \left( \left( (bc-ad) f (be-af)^2 (2+m) (a+bx) \text{AppellF1} \left[ 1+m, m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \right. \\ \left( b (-be+af) (1+m) (e+fx) \right. \\ \left( (bc-ad) (be-af) (2+m) \text{AppellF1} \left[ 1+m, m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \\ (a+bx) \left( (-bcf+adf) \text{AppellF1} \left[ 2+m, m, 2, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \\ \left. \left. d (-be+af) m \text{AppellF1} \left[ 2+m, 1+m, 1, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right) - \\ \frac{\left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} \text{Hypergeometric2F1} \left[ -m, -m, 1-m, \frac{b(c+dx)}{bc-ad} \right]}{m}$$

**Problem 3065: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{-1-m}}{(e+fx)^3} dx$$

Optimal (type 5, 283 leaves, 4 steps):

$$\begin{aligned} & -\frac{f(a+bx)^{1+m}(c+dx)^{-m}}{2(b e - a f)(d e - c f)(e+fx)^2} - \frac{f(b(3 d e - c f(1-m)) - a d f(2+m))(a+bx)^{1+m}(c+dx)^{-m}}{2(b e - a f)^2(d e - c f)^2(e+fx)} + \\ & \left( (2 a b d f(1+m)(2 d e + c f m) - b^2(2 d^2 e^2 + 4 c d e f m - c^2 f^2(1-m)m) - a^2 d^2 f^2(2+3 m+m^2)) \right. \\ & \left. (a+bx)^m(c+dx)^{-m} \operatorname{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f)(c+dx)}{(d e - c f)(a+bx)}\right] \right) / \\ & (2(b e - a f)^2(d e - c f)^3 m) \end{aligned}$$

Result (type 5, 2361 leaves):

$$\begin{aligned} & -\left( (b e - a f)^3 (a+bx)^{1+m}(c+dx)^{-m} \right. \\ & \left( 2(b e - a f)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] + \right. \\ & 2(b e - a f)^2 m \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] + \\ & 4 f(-b e + a f) m(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] + \\ & 4 f(-b e + a f) m^2(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] - \\ & f^2 m(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] + \\ & f^2 m^3(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] + \\ & 4 f(b e - a f)(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 2+m\right] + \\ & 8 f(b e - a f) m(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 2+m\right] + \\ & 4 f(b e - a f) m^2(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 2+m\right] - \\ & 2 f^2 m(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 2+m\right] - \\ & \left. 4 f^2 m^2(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 2+m\right] \right) \end{aligned}$$

$$\begin{aligned}
 & 2 f^2 m^3 (a+bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] + \\
 & 2 f^2 (a+bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & 5 f^2 m (a+bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & 4 f^2 m^2 (a+bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & f^2 m^3 (a+bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] \Big) \Big) / \\
 & \left( 2 (-be+af)^3 (1+m) (e+fx)^2 \left( b^3 c e^3 - a b^2 c e^2 f + b^3 d e^3 x + 2 b^3 c e^2 f x - a b^2 d e^2 f x - \right. \right. \\
 & \quad 2 a b^2 c e f^2 x + 2 b^3 d e^2 f x^2 + b^3 c e f^2 x^2 - 2 a b^2 d e f^2 x^2 - a b^2 c f^3 x^2 + b^3 d e f^2 x^3 - \\
 & \quad \left. \left. a b^2 d f^3 x^3 - f (-be+af) (1+m) (a+bx) (c+dx) (af(2+m) + b(-2e+fm x)) \right) \right. \\
 & \quad \left. \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m \right] + f (1+m) (a+bx)^2 \right. \\
 & \quad \left. (af(2+m) (-de+2cf+dfx) + bcf(-e(4+m) + fm x) + 2bde(e-f(1+m)x)) \right. \\
 & \quad \left. \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] + \right. \\
 & \quad 2 a^3 d e f^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] - \\
 & \quad 2 a^3 c f^3 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & \quad 3 a^3 d e f^2 m \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] - \\
 & \quad 3 a^3 c f^3 m \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & \quad a^3 d e f^2 m^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] - \\
 & \quad a^3 c f^3 m^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & \quad 6 a^2 b d e f^2 x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] - \\
 & \quad 6 a^2 b c f^3 x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & \quad 9 a^2 b d e f^2 m x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] - \\
 & \quad 9 a^2 b c f^3 m x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] +
 \end{aligned}$$

$$\begin{aligned}
 & 3 a^2 b d e f^2 m^2 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
 & 3 a^2 b c f^3 m^2 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] + \\
 & 6 a b^2 d e f^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
 & 6 a b^2 c f^3 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] + \\
 & 9 a b^2 d e f^2 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
 & 9 a b^2 c f^3 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] + \\
 & 3 a b^2 d e f^2 m^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
 & 3 a b^2 c f^3 m^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] + \\
 & 2 b^3 d e f^2 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
 & 2 b^3 c f^3 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] + \\
 & 3 b^3 d e f^2 m x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
 & 3 b^3 c f^3 m x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] + \\
 & b^3 d e f^2 m^2 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
 & b^3 c f^3 m^2 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] \Big) \Big) \Big)
 \end{aligned}$$

**Problem 3066: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{-1-m}}{(e + f x)^4} dx$$

Optimal (type 5, 498 leaves, 5 steps):

$$\frac{f (a+bx)^{1+m} (c+dx)^{-m}}{3 (b e - a f) (d e - c f) (e+fx)^3} - \frac{f (b (5 d e - c f (2-m)) - a d f (3+m)) (a+bx)^{1+m} (c+dx)^{-m}}{6 (b e - a f)^2 (d e - c f)^2 (e+fx)^2} -$$

$$\frac{f (a^2 d^2 f^2 (6+5m+m^2) - a b d f (d e (15+8m) - c f (3-2m-2m^2))) + b^2 (11 d^2 e^2 - c d e f (7-8m) + c^2 f^2 (2-3m+m^2)) (a+bx)^{1+m} (c+dx)^{-m}}{6 (b e - a f)^3 (d e - c f)^3 (e+fx)} + \left( (3 a b^2 d f (1+m) (6 d^2 e^2 + 6 c d e f m - c^2 f^2 (1-m) m) - 3 a^2 b d^2 f^2 (3 d e + c f m) (2+3m+m^2) + a^3 d^3 f^3 (6+11m+6m^2+m^3) - b^3 (6 d^3 e^3 + 18 c d^2 e^2 f m - 9 c^2 d e f^2 (1-m) m + c^3 f^3 m (2-3m+m^2))) (a+bx)^m (c+dx)^{-m} \right. \\ \left. \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f) (c+dx)}{(d e - c f) (a+bx)}\right] \right) / (6 (b e - a f)^3 (d e - c f)^4 m)$$

Result (type 5, 7153 leaves):

$$\left( (a+bx)^{1+m} (c+dx)^{-m} \right. \\ \left( - (a^3 f^3 (6+11m+6m^2+m^3) + 3 a^2 b f^2 (2+3m+m^2) (-3e+fm x) + 3 a b^2 f (1+m) (6 e^2 - 6 e f m x + f^2 (-1+m) m x^2) + b^3 (-6 e^3 + 18 e^2 f m x - 9 e f^2 (-1+m) m x^2 + f^3 m (2-3m+m^2) x^3)) \right. \\ \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+bx)}{(b e - a f) (c+dx)}, 1, 1+m\right] + f (1+m) (a+bx) \right. \\ \left( 3 (a^2 f^2 (6+5m+m^2) + 2 a b f (2+m) (-3e+fm x) + b^2 (6 e^2 - 6 e f m x + f^2 (-1+m) m x^2)) \right. \\ \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+bx)}{(b e - a f) (c+dx)}, 1, 2+m\right] - f (2+m) (a+bx) \right. \\ \left( 3 (a f (3+m) + b (-3e+fm x)) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+bx)}{(b e - a f) (c+dx)}, 1, 3+m\right] - f (3+m) (a+bx) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+bx)}{(b e - a f) (c+dx)}, 1, 4+m\right] \right) \right) / \\ \left( 3 (e+fx)^3 \left( 2 b^4 c e^4 - 2 a b^3 c e^3 f + 2 b^4 d e^4 x + 6 b^4 c e^3 f x - 2 a b^3 d e^3 f x - 6 a b^3 c e^2 f^2 x + 6 b^4 d e^3 f x^2 + 6 b^4 c e^2 f^2 x^2 - 6 a b^3 d e^2 f^2 x^2 - 6 a b^3 c e f^3 x^2 + 6 b^4 d e^2 f^2 x^3 + 2 b^4 c e f^3 x^3 - 6 a b^3 d e f^3 x^3 - 2 a b^3 c f^4 x^3 + 2 b^4 d e f^3 x^4 - 2 a b^3 d f^4 x^4 - f (b e - a f) (1+m) (a+bx) (c+dx) (a^2 f^2 (6+5m+m^2) + 2 a b f (2+m) (-3e+fm x) + b^2 (6 e^2 - 6 e f m x + f^2 (-1+m) m x^2)) \right. \right. \\ \left. \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a+bx)}{(b e - a f) (c+dx)}, 1, 1+m\right] - f (1+m) (a+bx)^2 (a^2 f^2 (6+5m+m^2) (-d e + 3 c f + 2 d f x) + 2 a b f (2+m) (c f (-e (9+m) + 2 f m x) + d (3 e^2 - 2 e f (3+m) x + f^2 m x^2)) + b^2 (c f (6 e^2 (3+m) - 2 e f m (5+m) x + f^2 (-1+m) m x^2) - 3 d e (2 e^2 - 4 e f (1+m) x + f^2 m (1+m) x^2)) \right) \right)$$

$$\begin{aligned}
 & \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 2 + m\right] + 12 a^3 b d e^2 f^2 \\
 & \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 18 a^3 b c e f^3 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 12 a^4 d e f^3 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] + \\
 & 18 a^4 c f^4 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] + \\
 & 18 a^3 b d e^2 f^2 m \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 29 a^3 b c e f^3 m \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 22 a^4 d e f^3 m \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] + \\
 & 33 a^4 c f^4 m \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] + \\
 & 6 a^3 b d e^2 f^2 m^2 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 12 a^3 b c e f^3 m^2 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 12 a^4 d e f^3 m^2 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] + \\
 & 18 a^4 c f^4 m^2 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & a^3 b c e f^3 m^3 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 2 a^4 d e f^3 m^3 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] + \\
 & 3 a^4 c f^4 m^3 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] + \\
 & 36 a^2 b^2 d e^2 f^2 x \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 54 a^2 b^2 c e f^3 x \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] - \\
 & 42 a^3 b d e f^3 x \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m\right] +
 \end{aligned}$$

$$54 a^3 b c f^4 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$6 a^4 d f^4 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$54 a^2 b^2 d e^2 f^2 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$87 a^2 b^2 c e f^3 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$81 a^3 b d e f^3 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$103 a^3 b c f^4 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$11 a^4 d f^4 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$18 a^2 b^2 d e^2 f^2 m^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$36 a^2 b^2 c e f^3 m^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$48 a^3 b d e f^3 m^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$60 a^3 b c f^4 m^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$6 a^4 d f^4 m^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$3 a^2 b^2 c e f^3 m^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$9 a^3 b d e f^3 m^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$11 a^3 b c f^4 m^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$a^4 d f^4 m^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$36 a b^3 d e^2 f^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$54 a b^3 c e f^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] -$$

$$54 a^2 b^2 d e f^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +$$

$$\begin{aligned}
& 54 a^2 b^2 c f^4 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 18 a^3 b d f^4 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 54 a b^3 d e^2 f^2 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 87 a b^3 c e f^3 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 111 a^2 b^2 d e f^3 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 111 a^2 b^2 c f^4 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 33 a^3 b d f^4 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 18 a b^3 d e^2 f^2 m^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 36 a b^3 c e f^3 m^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 72 a^2 b^2 d e f^3 m^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 72 a^2 b^2 c f^4 m^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 18 a^3 b d f^4 m^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 3 a b^3 c e f^3 m^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 15 a^2 b^2 d e f^3 m^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 15 a^2 b^2 c f^4 m^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 3 a^3 b d f^4 m^3 x^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
& 12 b^4 d e^2 f^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 18 b^4 c e f^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] - \\
& 30 a b^3 d e f^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] +
\end{aligned}$$

$$\begin{aligned}
 & 18 a^3 b^3 c f^4 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 18 a^2 b^2 d f^4 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 18 b^4 d e^2 f^2 m x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 29 b^4 c e f^3 m x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 67 a b^3 d e f^3 m x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 45 a b^3 c f^4 m x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 33 a^2 b^2 d f^4 m x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 6 b^4 d e^2 f^2 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 12 b^4 c e f^3 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 48 a b^3 d e f^3 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 36 a b^3 c f^4 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 18 a^2 b^2 d f^4 m^2 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & b^4 c e f^3 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 11 a b^3 d e f^3 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 9 a b^3 c f^4 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 3 a^2 b^2 d f^4 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 6 b^4 d e f^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 6 a b^3 d f^4 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 15 b^4 d e f^3 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] +
 \end{aligned}$$

$$\begin{aligned}
 & 4 b^4 c f^4 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 11 a b^3 d f^4 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 12 b^4 d e f^3 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 6 b^4 c f^4 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 6 a b^3 d f^4 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 3 b^4 d e f^3 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 2 b^4 c f^4 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & a b^3 d f^4 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] + \\
 & 6 a^4 d e f^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 6 a^4 c f^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & 11 a^4 d e f^3 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 11 a^4 c f^4 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & 6 a^4 d e f^3 m^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 6 a^4 c f^4 m^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & a^4 d e f^3 m^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & a^4 c f^4 m^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & 24 a^3 b d e f^3 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 24 a^3 b c f^4 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & 44 a^3 b d e f^3 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] -
 \end{aligned}$$

$$\begin{aligned}
 &44 a^3 b c f^4 m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &24 a^3 b d e f^3 m^2 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &24 a^3 b c f^4 m^2 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &4 a^3 b d e f^3 m^3 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &4 a^3 b c f^4 m^3 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &36 a^2 b^2 d e f^3 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &36 a^2 b^2 c f^4 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &66 a^2 b^2 d e f^3 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &66 a^2 b^2 c f^4 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &36 a^2 b^2 d e f^3 m^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &36 a^2 b^2 c f^4 m^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &6 a^2 b^2 d e f^3 m^3 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &6 a^2 b^2 c f^4 m^3 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &24 a b^3 d e f^3 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &24 a b^3 c f^4 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &44 a b^3 d e f^3 m x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &44 a b^3 c f^4 m x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] + \\
 &24 a b^3 d e f^3 m^2 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] - \\
 &24 a b^3 c f^4 m^2 x^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 4 + m\right] +
 \end{aligned}$$

$$\begin{aligned}
 & 4 a b^3 d e f^3 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 4 a b^3 c f^4 m^3 x^3 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & 6 b^4 d e f^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 6 b^4 c f^4 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & 11 b^4 d e f^3 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 11 b^4 c f^4 m x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & 6 b^4 d e f^3 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & 6 b^4 c f^4 m^2 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] + \\
 & b^4 d e f^3 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] - \\
 & b^4 c f^4 m^3 x^4 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 4 + m \right] \Big) \Big)
 \end{aligned}$$

**Problem 3067: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{-2-m} (e + f x)^p dx$$

Optimal (type 6, 131 leaves, 3 steps):

$$\left( b (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \right. \\
 \left. \text{AppellF1} \left[ 1 + m, 2 + m, -p, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right] \right) / \left( (b c - a d)^2 (1 + m) \right)$$

Result (type 6, 300 leaves):

$$\left( (bc - ad) (be - af) (2+m) (a+bx)^{1+m} (c+dx)^{-2-m} \right. \\ \left. (e+fx)^p \operatorname{AppellF1}\left[1+m, 2+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \\ \left( b(1+m) \left( (bc - ad) (be - af) (2+m) \operatorname{AppellF1}\left[1+m, 2+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right. \\ \left. (a+bx) \left( (-bc+ad) f p \operatorname{AppellF1}\left[2+m, 2+m, 1-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right. \right. \\ \left. \left. d (be - af) (2+m) \operatorname{AppellF1}\left[2+m, 3+m, -p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right)$$

**Problem 3068: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (5-4x)^3 (1+2x)^{-2-m} (2+3x)^m dx$$

Optimal (type 5, 132 leaves, 3 steps):

$$-\frac{1}{3} (5-4x)^2 (1+2x)^{-1-m} (2+3x)^{1+m} - \\ \frac{(1+2x)^{-1-m} (2+3x)^{1+m} (2768 - 315m + 4m^2 - 8(43-m)(1+m)x)}{9(1+m)} + \frac{1}{9m} \\ 2^{-m} (1323 - 128m + 2m^2) (1+2x)^{-m} \operatorname{Hypergeometric2F1}[-m, -m, 1-m, -3(1+2x)]$$

Result (type 6, 273 leaves):

$$\frac{7}{2} \left( -\frac{98(1+2x)^{-1-m} (2+3x)^{1+m}}{1+m} - \right. \\ \left( 69(5-4x)^2 (2+4x)^{-m} (4+6x)^m \operatorname{AppellF1}\left[2, -m, m, 3, -\frac{3}{23}(5-4x), \frac{1}{7}(5-4x)\right] \right) / \\ \left( 483 \operatorname{AppellF1}\left[2, -m, m, 3, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)\right] + \right. \\ \left. m(-5+4x) \left( 21 \operatorname{AppellF1}\left[3, 1-m, m, 4, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)\right] - \right. \right. \\ \left. \left. 23 \operatorname{AppellF1}\left[3, -m, 1+m, 4, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)\right] \right) \right) + \\ \frac{2^{3-m} (1+2x)^{1-m} \operatorname{Hypergeometric2F1}[1-m, -m, 2-m, -3-6x]}{1-m} + \frac{1}{1+m} \\ \left. 84(-1-2x)^m (1+2x)^{-m} (2+3x) (6+9x)^m \operatorname{Hypergeometric2F1}[1+m, 1+m, 2+m, 4+6x] \right)$$

### Problem 3069: Result unnecessarily involves higher level functions.

$$\int (a+bx)^m (c+dx)^{-2-m} (e+fx)^2 dx$$

Optimal (type 5, 204 leaves, 4 steps):

$$\frac{(de - cf) (adf(1+m) + b(de - cf(2+m))) (a+bx)^{1+m} (c+dx)^{-1-m}}{bd^2 (bc - ad) (1+m)} +$$

$$\frac{f(a+bx)^{1+m} (c+dx)^{-1-m} (e+fx)}{bd} - \frac{1}{bd^3 m} f(adfm + b(2de - cf(2+m)))$$

$$(a+bx)^m \left( -\frac{d(a+bx)}{bc - ad} \right)^{-m} (c+dx)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{b(c+dx)}{bc - ad}\right]$$

Result (type 6, 300 leaves):

$$\frac{1}{3} (a+bx)^m (c+dx)^{-2-m}$$

$$\left( \frac{3e^2 (a+bx) (c+dx)}{(bc - ad) (1+m)} - \left( 9ac e f x^2 \text{AppellF1}\left[2, -m, 2+m, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \right.$$

$$\left( -3ac \text{AppellF1}\left[2, -m, 2+m, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] - bcm x \text{AppellF1}\left[3, 1-m, 2+m, \right.$$

$$\left. 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + ad(2+m) x \text{AppellF1}\left[3, -m, 3+m, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) -$$

$$\left( 4ac f^2 x^3 \text{AppellF1}\left[3, -m, 2+m, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \right.$$

$$\left( -4ac \text{AppellF1}\left[3, -m, 2+m, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] - bcm x \text{AppellF1}\left[4, 1-m, 2+m, \right.$$

$$\left. 5, -\frac{bx}{a}, -\frac{dx}{c}\right] + ad(2+m) x \text{AppellF1}\left[4, -m, 3+m, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \Bigg)$$

### Problem 3072: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^m (c+dx)^{-2-m}}{e+fx} dx$$

Optimal (type 5, 120 leaves, 2 steps):

$$\frac{d(a+bx)^{1+m} (c+dx)^{-1-m}}{(bc - ad) (de - cf) (1+m)} + \frac{1}{(de - cf)^2 m}$$

$$f(a+bx)^m (c+dx)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(be - af)(c+dx)}{(de - cf)(a+bx)}\right]$$

Result (type 5, 578 leaves):

$$\begin{aligned}
 & - \left( \left( (a+bx)^{1+m} (c+dx)^{-2-m} \left( 6 \operatorname{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] + \right. \right. \right. \\
 & \quad 5m \operatorname{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] + m^2 \operatorname{HurwitzLerchPhi} \left[ \right. \\
 & \quad \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] - \frac{3f(a+bx) \operatorname{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right]}{-be+af} - \\
 & \quad \left. \frac{fm(a+bx) \operatorname{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right]}{-be+af} + \right. \\
 & \quad \left. \left( (de-cf)(a+bx) \operatorname{Hypergeometric2F1} \left[ 2, 3+m, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \right. \\
 & \quad \left. \left( (be-af)(c+dx) - \left( f(-de+cf)(a+bx)^2 \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Hypergeometric2F1} \left[ 2, 3+m, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \left( (be-af)^2 (c+dx) \right) \right) \right) / \\
 & \quad \left( (-be+af)(3+m) \left( \frac{-ad(1+m) + bc(2+m) + bdx}{bc-ad} - \right. \right. \\
 & \quad \left. \left. \frac{b(2+m)(e+fx) \operatorname{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right]}{be-af} + \right. \right. \\
 & \quad \left. \left. \left( b(-de+cf)(a+bx)(e+fx) \operatorname{Hypergeometric2F1} \left[ 2, 3+m, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \right. \right. \\
 & \quad \left. \left. \left( (be-af)^2 (3+m)(c+dx) \right) \right) \right) \right)
 \end{aligned}$$

### Problem 3073: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^m (c+dx)^{-2-m}}{(e+fx)^2} dx$$

Optimal (type 5, 233 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{d(adf(2+m) - b(de+cf(1+m))) (a+bx)^{1+m} (c+dx)^{-1-m}}{(bc-ad)(be-af)(de-cf)^2(1+m)} - \\
 & \frac{f(a+bx)^{1+m} (c+dx)^{-1-m}}{(be-af)(de-cf)(e+fx)} - \left( f(adf(2+m) - b(2de+cfm)) (a+bx)^m (c+dx)^{-m} \right. \\
 & \quad \left. \operatorname{Hypergeometric2F1} \left[ 1, -m, 1-m, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)} \right] \right) / \left( (be-af)(de-cf)^3 m \right)
 \end{aligned}$$

Result (type 5, 21480 leaves): Display of huge result suppressed!

**Problem 3074: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{-2-m}}{(e + f x)^3} dx$$

Optimal (type 5, 432 leaves, 5 steps):

$$\begin{aligned} & \left( d (a^2 d^2 f^2 (6 + 5 m + m^2) + b^2 (2 d^2 e^2 + 5 c d e f (1 + m) - c^2 f^2 (1 - m^2))) - \right. \\ & \quad \left. a b d f (d e (9 + 5 m) + c f (3 + 5 m + 2 m^2)) \right) (a + b x)^{1+m} (c + d x)^{-1-m} / \\ & \left( 2 (b c - a d) (b e - a f)^2 (d e - c f)^3 (1 + m) \right) - \frac{f (a + b x)^{1+m} (c + d x)^{-1-m}}{2 (b e - a f) (d e - c f) (e + f x)^2} - \\ & \frac{f (b (4 d e - c f (1 - m)) - a d f (3 + m)) (a + b x)^{1+m} (c + d x)^{-1-m}}{2 (b e - a f)^2 (d e - c f)^2 (e + f x)} - \\ & \left( f (2 a b d f (2 + m) (3 d e + c f m) - b^2 (6 d^2 e^2 + 6 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (6 + 5 m + m^2)) \right. \\ & \quad \left. (a + b x)^m (c + d x)^{-m} \operatorname{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right] \right) / \\ & \left( 2 (b e - a f)^2 (d e - c f)^4 m \right) \end{aligned}$$

Result (type 5, 57971 leaves): Display of huge result suppressed!

**Problem 3075: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{-3-m} (e + f x)^p dx$$

Optimal (type 6, 133 leaves, 3 steps):

$$\begin{aligned} & \left( b^2 (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m (e + f x)^p \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \right. \\ & \quad \left. \operatorname{AppellF1}\left[1 + m, 3 + m, -p, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f}\right] \right) / \left( (b c - a d)^3 (1 + m) \right) \end{aligned}$$

Result (type 6, 300 leaves):

$$\left( (bc - ad) (be - af) (2+m) (a+bx)^{1+m} (c+dx)^{-3-m} \right. \\ \left. (e+fx)^p \operatorname{AppellF1}\left[1+m, 3+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \\ \left( b(1+m) \left( (bc - ad) (be - af) (2+m) \operatorname{AppellF1}\left[1+m, 3+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right. \\ \left. (a+bx) \left( (-bc+ad) f p \operatorname{AppellF1}\left[2+m, 3+m, 1-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right. \right. \\ \left. \left. d (be - af) (3+m) \operatorname{AppellF1}\left[2+m, 4+m, -p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right)$$

**Problem 3076: Result unnecessarily involves higher level functions.**

$$\int (5-4x)^4 (1+2x)^{-3-m} (2+3x)^m dx$$

Optimal (type 5, 188 leaves, 4 steps):

$$-\frac{1}{9} (107-2m) (5-4x)^2 (1+2x)^{-2-m} (2+3x)^{1+m} - \\ \frac{1}{3} (5-4x)^3 (1+2x)^{-2-m} (2+3x)^{1+m} + \frac{1}{9(2+3m+m^2)} \\ 7(1+2x)^{-2-m} (2+3x)^{1+m} (3(4638+485m+108m^2-2m^3) + 2(15209+1882m-530m^2+8m^3)x) - \\ \frac{1}{9m} 2^{2-m} (1323-85m+m^2) (1+2x)^{-m} \operatorname{Hypergeometric2F1}[-m, -m, 1-m, -3(1+2x)]$$

Result (type 6, 318 leaves):

$$21 \left( \frac{392(1+2x)^{-1-m} (2+3x)^{1+m}}{3+3m} + \right. \\ \left( 23(5-4x)^2 (2+4x)^{-m} (4+6x)^m \operatorname{AppellF1}\left[2, -m, m, 3, -\frac{3}{23}(-5+4x), \frac{1}{7}(5-4x)\right] \right) / \\ \left( 483 \operatorname{AppellF1}\left[2, -m, m, 3, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)\right] + \right. \\ \left. m(-5+4x) \left( 21 \operatorname{AppellF1}\left[3, 1-m, m, 4, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)\right] - \right. \right. \\ \left. \left. 23 \operatorname{AppellF1}\left[3, -m, 1+m, 4, \frac{3}{23}(5-4x), \frac{1}{7}(5-4x)\right] \right) \right) + \\ \frac{2^{2-m} (1+2x)^{1-m} \operatorname{Hypergeometric2F1}[1-m, -m, 2-m, -3-6x]}{-1+m} - \frac{1}{1+m} \\ 56(-3-6x)^m (1+2x)^{-m} (2+3x)^{1+m} \operatorname{Hypergeometric2F1}[1+m, 1+m, 2+m, 4+6x] - \frac{1}{1+m} \\ \left. 1029(-1-2x)^m (1+2x)^{-m} (2+3x) (6+9x)^m \operatorname{Hypergeometric2F1}[1+m, 3+m, 2+m, 4+6x] \right)$$

**Problem 3078: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{-3-m} (e + f x)^2 dx$$

Optimal (type 5, 205 leaves, 4 steps):

$$\frac{(d e - c f)^2 (a + b x)^{1+m} (c + d x)^{-2-m}}{d^2 (b c - a d) (2 + m)} - \frac{((d e - c f) (2 a d f (2 + m) - b (d e + c f (3 + 2 m))) (a + b x)^{1+m} (c + d x)^{-1-m})}{(d^2 (b c - a d)^2 (1 + m) (2 + m)) - \frac{1}{d^3 m}} f^2 (a + b x)^m \left(-\frac{d (a + b x)}{b c - a d}\right)^{-m} (c + d x)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1 - m, \frac{b (c + d x)}{b c - a d}\right]$$

Result (type 6, 426 leaves):

$$\frac{1}{3} (a + b x)^m (c + d x)^{-3-m} \left( \left( 6 e f \left( \frac{c (a + b x)}{a (c + d x)} \right)^{-m} (c + d x) \right. \right. \\ \left. \left. \left( b^2 c^2 (1 + m) x^2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m - a b c x \left( \frac{c (a + b x)}{a (c + d x)} \right)^m (-c m + d (2 + m) x) + \right. \right. \right. \\ \left. \left. \left. a^2 \left( d^2 x^2 - c^2 \left( -1 + \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) - c d x \left( -2 + 2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) \right) \right) / \\ \left( c (b c - a d)^2 (1 + m) (2 + m) \right) - \left( 4 a c f^2 x^3 \text{AppellF1}\left[3, -m, 3 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \\ \left( -4 a c \text{AppellF1}\left[3, -m, 3 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] - \right. \\ \left. b c m x \text{AppellF1}\left[4, 1 - m, 3 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] + \right. \\ \left. a d (3 + m) x \text{AppellF1}\left[4, -m, 4 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) - \frac{1}{d (2 + m)} \\ 3 e^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1}\left[-2 - m, -m, -1 - m, \frac{b (c + d x)}{b c - a d}\right]$$

**Problem 3081: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{-3-m}}{e + f x} dx$$

Optimal (type 5, 196 leaves, 4 steps):

$$\frac{d (a+bx)^{1+m} (c+dx)^{-2-m}}{(bc-ad)(de-cf)(2+m)} + \frac{d(adf(2+m) + b(de-cf(3+m))) (a+bx)^{1+m} (c+dx)^{-1-m}}{(bc-ad)^2 (de-cf)^2 (1+m)(2+m)} - \frac{1}{(de-cf)^3 m} f^2 (a+bx)^m (c+dx)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)}\right]$$

Result (type 5, 12578 leaves):

$$\left( (a+bx)^{1+2m} (c+dx)^{-6-2m} \left( \frac{-bc-bdx}{-bc+ad} \right)^{3+m} (-be-bfx) \right. \\ \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^{-3-m} \left( 24 \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \right. \\ 26m \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\ 9m^2 \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\ m^3 \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\ \frac{24f(a+bx) \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{be-af} + \\ \frac{14fm(a+bx) \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{be-af} + \\ \frac{2fm^2(a+bx) \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{be-af} + \\ \frac{8f^2(a+bx)^2 \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{(be-af)^2} + \\ \left. \frac{2f^2m(a+bx)^2 \text{HurwitzLerchPhi}\left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{(be-af)^2} \right) + \\ \left( 5(de-cf)(a+bx) \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \\ ((be-af)(c+dx)) + \left( 2(de-cf)m(a+bx) \right. \\ \left. \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / ((be-af)(c+dx)) + \\ \left( 8f(de-cf)(a+bx)^2 \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \\ ((be-af)^2(c+dx)) + \left( 2f(de-cf)m(a+bx)^2 \right.$$



$$\begin{aligned}
 & \frac{1}{de-cf} 14fm(c+dx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \\
 & \left( \frac{1}{1 - \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} + (-3-m) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] \right) + \\
 & \frac{1}{de-cf} 2fm^2(c+dx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \\
 & \left( \frac{1}{1 - \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} + (-3-m) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] \right) + \\
 & \left( 24(be-af)(c+dx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \right. \\
 & \left. \left( \frac{1}{1 - \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} + (-3-m) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] \right) \right) / \\
 & ((de-cf)(a+bx)) + \left( 26(be-af)m(c+dx) \right. \\
 & \left. \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \left( \frac{1}{1 - \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} + (-3-m) \right. \right. \\
 & \left. \left. \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] \right) \right) / ((de-cf)(a+bx)) + \\
 & \left( 9(be-af)m^2(c+dx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \right. \\
 & \left. \left( \frac{1}{1 - \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} + (-3-m) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] \right) \right) / \\
 & ((de-cf)(a+bx)) + \left( (be-af)m^3(c+dx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \right. \right. \\
 & \left. \left. \frac{b(de-cf)}{(be-af)(c+dx)} \right) \left( \frac{1}{1 - \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} + (-3-m) \text{HurwitzLerchPhi} \left[ \right. \right. \\
 & \left. \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] \right) \right) / ((de-cf)(a+bx)) + \left( 8f^2(a+bx) \right. \\
 & \left. (c+dx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \left( \frac{1}{1 - \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}} + (-3-m) \right) \right)
 \end{aligned}$$



$$\begin{aligned}
 & \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right] / \left((b-e-af)(c+dx)^2\right) - \\
 & \left(8df(d-e-cf)(a+bx)^2 \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \\
 & \left((b-e-af)^2(c+dx)^2\right) - \left(2df(d-e-cf)m(a+bx)^2\right. \\
 & \left. \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \left((b-e-af)^2(c+dx)^2\right) - \\
 & \left(3df^2(d-e-cf)(a+bx)^3 \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \\
 & \left((b-e-af)^3(c+dx)^2\right) + \left(5b(d-e-cf)\right. \\
 & \left. \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \left((b-e-af)(c+dx)\right) + \\
 & \left(2b(d-e-cf)m \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \\
 & \left((b-e-af)(c+dx)\right) + \left(16bf(d-e-cf)(a+bx)\right. \\
 & \left. \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \left((b-e-af)^2(c+dx)\right) + \\
 & \left(4bf(d-e-cf)m(a+bx) \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \\
 & \left((b-e-af)^2(c+dx)\right) + \left(9bf^2(d-e-cf)(a+bx)^2\right. \\
 & \left. \text{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right]\right) / \left((b-e-af)^3(c+dx)\right) + \\
 & (4+m) \left( -\frac{d(d-e-cf)(a+bx)}{(b-e-af)(c+dx)^2} + \frac{b(d-e-cf)}{(b-e-af)(c+dx)} \right) \left( -\left( \left( (b-e-af)^2(c+dx)^2 \right. \right. \right. \\
 & \left. \left. \left. (bce+ade-2acf+2bdex-bcfx-adfx) \right) / \left( (-bc+ad)^3(e+fx)^3 \right) \right) - \right. \\
 & \left. \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right] \right) + \\
 & \frac{1}{b-e-af} 2f(4+m)(a+bx) \left( -\frac{d(d-e-cf)(a+bx)}{(b-e-af)(c+dx)^2} + \frac{b(d-e-cf)}{(b-e-af)(c+dx)} \right) \\
 & \left( -\left( \left( (b-e-af)^2(c+dx)^2 (bce+ade-2acf+2bdex-bcfx-adfx) \right) / \right. \right. \\
 & \left. \left. \left( (-bc+ad)^3(e+fx)^3 \right) \right) - \text{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \right. \right. \\
 & \left. \left. \{1, 5+m\}, \frac{(d-e-cf)(a+bx)}{(b-e-af)(c+dx)}\right] \right) + \frac{1}{(b-e-af)^2} f^2(4+m)(a+bx)^2
 \end{aligned}$$

$$\begin{aligned}
 & \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \left( -\left( (b e - a f)^2 (c + d x)^2 \right. \right. \\
 & \quad \left. \left. (b c e + a d e - 2 a c f + 2 b d e x - b c f x - a d f x) \right) / \left( (-b c + a d)^3 (e + f x)^3 \right) \right) - \\
 & \quad \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] - \\
 & \left( d (d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \right. \right. \\
 & \quad \left. \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f) (c + d x)^2 \right) - \left( 2 d f (d e - c f) (a + b x)^2 \right. \\
 & \quad \left. \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f)^2 (c + d x)^2 \right) - \left( d f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[ \right. \right. \\
 & \quad \left. \left. \{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)} \right] \right) / \left( (b e - a f)^3 (c + d x)^2 \right) + \\
 & \left( b (d e - c f) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f) (c + d x) \right) + \left( 4 b f (d e - c f) (a + b x) \text{HypergeometricPFQ}\left[ \right. \right. \\
 & \quad \left. \left. \{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)} \right] \right) / \left( (b e - a f)^2 (c + d x) \right) + \\
 & \left( 3 b f^2 (d e - c f) (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \right. \right. \\
 & \quad \left. \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)} \right] \right) / \left( (b e - a f)^3 (c + d x) \right) \Bigg) - \\
 & \frac{1}{(-b c + a d) (-b e + a f) (1 + m) (2 + m) (4 + m) (e + f x)} d (-3 - m) (a + b x)^{1+m} \\
 & (c + d x)^{-3-m} \left( \frac{-b c - b d x}{-b c + a d} \right)^{3+m} (-b e - b f x) \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-4-m} \\
 & \left( 24 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \right. \\
 & \quad 26 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
 & \quad 9 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
 & \quad \left. m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{24 f (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \\
 & \frac{14 f m (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \\
 & \frac{2 f m^2 (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \\
 & \frac{8 f^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \\
 & \frac{2 f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{(b e - a f)^2} + \\
 & \left( 5 (d e - c f) (a + b x) \operatorname{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f) (c + d x) \right) + \left( 2 (d e - c f) m (a + b x) \right. \\
 & \left. \operatorname{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f) (c + d x) \right) + \\
 & \left( 8 f (d e - c f) (a + b x)^2 \operatorname{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f)^2 (c + d x) \right) + \left( 2 f (d e - c f) m (a + b x)^2 \right. \\
 & \left. \operatorname{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f)^2 (c + d x) \right) + \\
 & \left( 3 f^2 (d e - c f) (a + b x)^3 \operatorname{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f)^3 (c + d x) \right) + \left( (d e - c f) (a + b x) \operatorname{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \right. \right. \\
 & \left. \left. \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f) (c + d x) \right) + \left( 2 f (d e - c f) \right. \\
 & \left. (a + b x)^2 \operatorname{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f)^2 (c + d x) \right) + \left( f^2 (d e - c f) (a + b x)^3 \operatorname{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \right. \right. \\
 & \left. \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f)^3 (c + d x) \right) \Bigg) - \\
 & \frac{1}{(b e + a f) (1 + m) (2 + m) (4 + m) (e + f x)} f (a + b x)^{1+m} (c + d x)^{-3-m}
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{-bc - bdx}{-bc + ad} \right)^{3+m} \left( 1 - \frac{d(a+bx)}{-bc + ad} \right)^{-3-m} \\
 & \left( 24 \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right] + \right. \\
 & 26m \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right] + \\
 & 9m^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right] + \\
 & m^3 \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right] + \\
 & \frac{24f(a+bx) \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right]}{be - af} + \\
 & \frac{14fm(a+bx) \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right]}{be - af} + \\
 & \frac{2fm^2(a+bx) \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right]}{be - af} + \\
 & \frac{8f^2(a+bx)^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right]}{(be - af)^2} + \\
 & \frac{2f^2m(a+bx)^2 \operatorname{HurwitzLerchPhi} \left[ \frac{(de - cf)(a+bx)}{(be - af)(c+dx)}, 1, 3+m \right]}{(be - af)^2} + \\
 & \left( 5(de - cf)(a+bx) \operatorname{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(de - cf)(a+bx)}{(be - af)(c+dx)} \right] \right) / \\
 & ((be - af)(c+dx)) + \left( 2(de - cf)m(a+bx) \right. \\
 & \left. \operatorname{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(de - cf)(a+bx)}{(be - af)(c+dx)} \right] \right) / ((be - af)(c+dx)) + \\
 & \left( 8f(de - cf)(a+bx)^2 \operatorname{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(de - cf)(a+bx)}{(be - af)(c+dx)} \right] \right) / \\
 & ((be - af)^2(c+dx)) + \left( 2f(de - cf)m(a+bx)^2 \right. \\
 & \left. \operatorname{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(de - cf)(a+bx)}{(be - af)(c+dx)} \right] \right) / ((be - af)^2(c+dx)) + \\
 & \left( 3f^2(de - cf)(a+bx)^3 \operatorname{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(de - cf)(a+bx)}{(be - af)(c+dx)} \right] \right) / \\
 & ((be - af)^3(c+dx)) + \left( (de - cf)(a+bx) \operatorname{HypergeometricPFQ} \left[ \{2, 2, 4+m\}, \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left\{ 1, 5+m \right\}, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \Big/ \left( (be-af)(c+dx) \right) + \left( 2f(de-cf) \right. \\
 & \left. (a+bx)^2 \text{HypergeometricPFQ} \left[ \left\{ 2, 2, 4+m \right\}, \left\{ 1, 5+m \right\}, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) \Big/ \\
 & \left( (be-af)^2 (c+dx) \right) + \left( f^2 (de-cf) (a+bx)^3 \text{HypergeometricPFQ} \left[ \right. \right. \\
 & \left. \left. \left\{ 2, 2, 4+m \right\}, \left\{ 1, 5+m \right\}, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) \Big/ \left( (be-af)^3 (c+dx) \right) \Big) - \\
 & \frac{1}{b(-be+af)(1+m)(2+m)(4+m)(e+fx)^2} f(a+bx)^{1+m} (c+dx)^{-3-m} \\
 & \left( \frac{-bc-bdx}{-bc+ad} \right)^{3+m} (-be-bfx) \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^{-3-m} \\
 & \left( 24 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \right. \\
 & 26m \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & 9m^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & m^3 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right] + \\
 & \frac{24f(a+bx) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{be-af} + \\
 & \frac{14fm(a+bx) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{be-af} + \\
 & \frac{2fm^2(a+bx) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{be-af} + \\
 & \frac{8f^2(a+bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{(be-af)^2} + \\
 & \left. \frac{2f^2m(a+bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m \right]}{(be-af)^2} \right) + \\
 & \left( 5(de-cf)(a+bx) \text{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) \Big/ \\
 & \left( (be-af)(c+dx) \right) + \left( 2(de-cf)m(a+bx) \right. \\
 & \left. \text{Hypergeometric2F1} \left[ 2, 4+m, 5+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) \Big/ \left( (be-af)(c+dx) \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( 8 f (d e - c f) (a + b x)^2 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f)^2 (c + d x) \right) + \left( 2 f (d e - c f) m (a + b x)^2 \right. \\
 & \left. \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f)^2 (c + d x) \right) + \\
 & \left( 3 f^2 (d e - c f) (a + b x)^3 \text{Hypergeometric2F1}\left[2, 4 + m, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f)^3 (c + d x) \right) + \left( (d e - c f) (a + b x) \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \right. \right. \\
 & \left. \left. \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f) (c + d x) \right) + \left( 2 f (d e - c f) \right. \\
 & \left. (a + b x)^2 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \\
 & \left( (b e - a f)^2 (c + d x) \right) + \left( f^2 (d e - c f) (a + b x)^3 \text{HypergeometricPFQ}\left[\{2, 2, 4 + m\}, \{1, 5 + m\}, \right. \right. \\
 & \left. \left. \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f)^3 (c + d x) \right) \Bigg) - \\
 & \frac{1}{(-b c + a d) (-b e + a f) (1 + m) (2 + m) (4 + m) (e + f x)} d (3 + m) (a + b x)^{1+m} \\
 & (c + d x)^{-3-m} \left( \frac{-b c - b d x}{-b c + a d} \right)^{2+m} \\
 & (-b e - b f x) \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-3-m} \\
 & \left( 24 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \right. \\
 & 26 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
 & 9 m^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
 & m^3 \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] + \\
 & \left. \frac{24 f (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} + \right. \\
 & \left. \frac{14 f m (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right]}{b e - a f} \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2 f m^2 (a+b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{b e-a f} + \\
 & \frac{8 f^2 (a+b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{(b e-a f)^2} + \\
 & \frac{2 f^2 m (a+b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{(b e-a f)^2} + \\
 & \left( 5 (d e-c f) (a+b x) \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / \\
 & \left( (b e-a f)(c+d x) + \left( 2 (d e-c f) m (a+b x) \right. \right. \\
 & \quad \left. \left. \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / ((b e-a f)(c+d x)) + \right. \\
 & \left. \left( 8 f (d e-c f) (a+b x)^2 \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / \right. \\
 & \left. \left( (b e-a f)^2 (c+d x) + \left( 2 f (d e-c f) m (a+b x)^2 \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / ((b e-a f)^2 (c+d x)) + \right. \\
 & \left. \left( 3 f^2 (d e-c f) (a+b x)^3 \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / \right. \\
 & \left. \left( (b e-a f)^3 (c+d x) + \left( (d e-c f) (a+b x) \operatorname{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \right. \right. \right. \\
 & \quad \left. \left. \{1, 5+m\}, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / ((b e-a f)(c+d x)) + \left( 2 f (d e-c f) \right. \\
 & \quad \left. (a+b x)^2 \operatorname{HypergeometricPFQ}\left[\{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / \right. \\
 & \left. \left( (b e-a f)^2 (c+d x) + \left( f^2 (d e-c f) (a+b x)^3 \operatorname{HypergeometricPFQ}\left[ \right. \right. \right. \\
 & \quad \left. \left. \{2, 2, 4+m\}, \{1, 5+m\}, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right] \right) / ((b e-a f)^3 (c+d x)) \right) + \\
 & \frac{1}{b(-b e+a f)(1+m)(2+m)(4+m)(e+f x)} d(-3-m)(a+b x)^{1+m}(c+d x)^{-4-m} \\
 & \left( \frac{-b c-b d x}{-b c+a d} \right)^{3+m} (-b e-b f x) \left( 1 - \frac{d(a+b x)}{-b c+a d} \right)^{-3-m} \\
 & \left( 24 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right] + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 26 m \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]+ \\
 & 9 m^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]+ \\
 & m^3 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]+ \\
 & \frac{24 f(a+b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{b e-a f}+ \\
 & \frac{14 f m(a+b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{b e-a f}+ \\
 & \frac{2 f m^2(a+b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{b e-a f}+ \\
 & \frac{8 f^2(a+b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{(b e-a f)^2}+ \\
 & \frac{2 f^2 m(a+b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 3+m\right]}{(b e-a f)^2}+ \\
 & \left(5(d e-c f)(a+b x) \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right) / \\
 & ((b e-a f)(c+d x))+\left(2(d e-c f) m(a+b x) \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right) / ((b e-a f)(c+d x))+ \\
 & \left(8 f(d e-c f)(a+b x)^2 \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right) / \\
 & ((b e-a f)^2(c+d x))+\left(2 f(d e-c f) m(a+b x)^2 \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right) / ((b e-a f)^2(c+d x))+ \\
 & \left(3 f^2(d e-c f)(a+b x)^3 \operatorname{Hypergeometric2F1}\left[2, 4+m, 5+m, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right) / \\
 & ((b e-a f)^3(c+d x))+\left((d e-c f)(a+b x) \operatorname{HypergeometricPFQ}\left[\{2, 2, 4+m\},\right.\right. \\
 & \left.\left.\{1, 5+m\}, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right) / ((b e-a f)(c+d x))+\left(2 f(d e-c f) \right. \\
 & \left.(a+b x)^2 \operatorname{HypergeometricPFQ}\left[\{2, 2, 4+m\},\{1, 5+m\}, \frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}\right]\right) /
 \end{aligned}$$

$$\begin{aligned}
 & \left( (be - af)^2 (c + dx) \right) + \left( f^2 (de - cf) (a + bx)^3 \text{HypergeometricPFQ} \left[ \right. \right. \\
 & \quad \left. \left. \{2, 2, 4 + m\}, \{1, 5 + m\}, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \left( (be - af)^3 (c + dx) \right) + \\
 & \frac{1}{(-be + af)(2 + m)(4 + m)(e + fx)} (a + bx)^m (c + dx)^{-3 - m} \left( \frac{-bc - bdx}{-bc + ad} \right)^{3 + m} \\
 & (-be - bfx) \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^{-3 - m} \\
 & \left( 24 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \right. \\
 & 26 m \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
 & 9 m^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
 & m^3 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right] + \\
 & \frac{24 f (a + bx) \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right]}{be - af} + \\
 & \frac{14 f m (a + bx) \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right]}{be - af} + \\
 & \frac{2 f m^2 (a + bx) \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right]}{be - af} + \\
 & \frac{8 f^2 (a + bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right]}{(be - af)^2} + \\
 & \frac{2 f^2 m (a + bx)^2 \text{HurwitzLerchPhi} \left[ \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 3 + m \right]}{(be - af)^2} + \\
 & \left( 5 (de - cf) (a + bx) \text{Hypergeometric2F1} \left[ 2, 4 + m, 5 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \\
 & \left( (be - af) (c + dx) \right) + \left( 2 (de - cf) m (a + bx) \right. \\
 & \quad \left. \text{Hypergeometric2F1} \left[ 2, 4 + m, 5 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \left( (be - af) (c + dx) \right) + \\
 & \left( 8 f (de - cf) (a + bx)^2 \text{Hypergeometric2F1} \left[ 2, 4 + m, 5 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \\
 & \left( (be - af)^2 (c + dx) \right) + \left( 2 f (de - cf) m (a + bx)^2 \right.
 \end{aligned}$$



$$\left( b^3 (a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m (e+fx)^p \left( \frac{b(e+fx)}{be-af} \right)^{-p} \right. \\ \left. \text{AppellF1} \left[ 1+m, 4+m, -p, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right] \right) / \left( (bc-ad)^4 (1+m) \right)$$

Result (type 6, 300 leaves):

$$\left( (bc-ad) (be-af) (2+m) (a+bx)^{1+m} (c+dx)^{-4-m} \right. \\ \left. (e+fx)^p \text{AppellF1} \left[ 1+m, 4+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \\ \left( b(1+m) \left( (bc-ad) (be-af) (2+m) \text{AppellF1} \left[ 1+m, 4+m, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \right. \\ \left. (a+bx) \left( (-bc+ad) f^p \text{AppellF1} \left[ 2+m, 4+m, 1-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \\ \left. \left. d (be-af) (4+m) \text{AppellF1} \left[ 2+m, 5+m, -p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right)$$

**Problem 3085: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^{-4-m} (e+fx)^3 dx$$

Optimal (type 5, 406 leaves, 10 steps):

$$\frac{(de-cf)^3 (a+bx)^{1+m} (c+dx)^{-3-m}}{d^3 (bc-ad) (3+m)} + \frac{3f(de-cf)^2 (a+bx)^{1+m} (c+dx)^{-2-m}}{d^3 (bc-ad) (2+m)} + \\ \frac{2b(de-cf)^3 (a+bx)^{1+m} (c+dx)^{-2-m}}{d^3 (bc-ad)^2 (2+m) (3+m)} + \frac{3f^2(de-cf) (a+bx)^{1+m} (c+dx)^{-1-m}}{d^3 (bc-ad) (1+m)} + \\ \frac{3bf(de-cf)^2 (a+bx)^{1+m} (c+dx)^{-1-m}}{d^3 (bc-ad)^2 (1+m) (2+m)} + \frac{2b^2(de-cf)^3 (a+bx)^{1+m} (c+dx)^{-1-m}}{d^3 (bc-ad)^3 (1+m) (2+m) (3+m)} - \frac{1}{d^4 m} \\ f^3 (a+bx)^m \left( -\frac{d(a+bx)}{bc-ad} \right)^{-m} (c+dx)^{-m} \text{Hypergeometric2F1} \left[ -m, -m, 1-m, \frac{b(c+dx)}{bc-ad} \right]$$

Result (type 6, 1833 leaves):

$$\frac{1}{c(bc-ad)^3 (1+m) (2+m) (3+m)} 3ef^2 (a+bx)^m \left( \frac{c(a+bx)}{a(c+dx)} \right)^{-m} (c+dx)^{-3-m} \\ \left( b^3 c^3 (2+3m+m^2) x^3 \left( \frac{c(a+bx)}{a(c+dx)} \right)^m - a b^2 c^2 (1+m) x^2 \left( \frac{c(a+bx)}{a(c+dx)} \right)^m (-cm+2d(3+m)x) + \right. \\ \left. a^2 b c x \left( \frac{c(a+bx)}{a(c+dx)} \right)^m (-2c^2 m - 2cdm(3+m)x + d^2(6+5m+m^2)x^2) + \right.$$

$$\begin{aligned}
 & a^3 \left( -2 d^3 x^3 + 2 c^3 \left( -1 + \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) + 2 c^2 d x \left( -3 + 3 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) + \right. \\
 & \quad \left. c d^2 x^2 \left( -6 + 6 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + 5 m \left( \frac{c (a + b x)}{a (c + d x)} \right)^m + m^2 \left( \frac{c (a + b x)}{a (c + d x)} \right)^m \right) \right) - \\
 & \left( 5 a c f^3 x^4 (a + b x)^m (c + d x)^{-4-m} \text{AppellF1} \left[ 4, -m, 4 + m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \\
 & \left( 4 \left( -5 a c \text{AppellF1} \left[ 4, -m, 4 + m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] - \right. \right. \\
 & \quad \left. b c m x \text{AppellF1} \left[ 5, 1 - m, 4 + m, 6, -\frac{b x}{a}, -\frac{d x}{c} \right] + \right. \\
 & \quad \left. a d (4 + m) x \text{AppellF1} \left[ 5, -m, 5 + m, 6, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) + \left( 3 e^2 f x^2 (a + b x)^m (c + d x)^{-4-m} \right. \\
 & \quad \left. \left( 1 + \frac{d x}{c} \right) \left( (c + d x) \left( b^3 c^3 m (1 + m) x^3 + a b^2 c^2 m x^2 (c (-3 + m) - 2 d (3 + m) x) - a^2 b c x \right. \right. \right. \\
 & \quad \left. \left. \left( d^2 (3 + m) x^2 \left( -2 - m + 2 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 2 c d (3 + m) x \left( -2 + m + 2 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + \right. \right. \\
 & \quad \left. \left. 2 c^2 \left( -3 + 2 m + 3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m + m \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) + a^3 \left( 2 d^3 m x^3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m - \right. \right. \\
 & \quad \left. \left. 6 c^3 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 2 c^2 d x \left( 6 - 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m + m \left( 2 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) + \right. \\
 & \quad \left. \left. c d^2 x^2 \left( 6 + m^2 - 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m + m \left( 5 + 4 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) \right) \right) \text{Gamma} [1 - m] + \\
 & \quad m (3 c + d x) \left( b^3 c^3 (2 + 3 m + m^2) x^3 + a b^2 c^2 (1 + m) x^2 (c m - 2 d (3 + m) x) + \right. \\
 & \quad \left. a^2 b c x (-2 c^2 m - 2 c d m (3 + m) x + d^2 (6 + 5 m + m^2) x^2) + \right. \\
 & \quad \left. a^3 \left( -2 d^3 x^3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m - 2 c^3 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) - 2 c^2 d x \right. \right. \\
 & \quad \left. \left. \left( -3 - m + 3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) - c d^2 x^2 \left( -6 - 5 m - m^2 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) \right) \text{Gamma} [-m] \right) / \\
 & \left( (c + d x) \left( b^3 c^3 m (2 + 3 m + m^2) x^3 - 3 a b^2 c^2 m (1 + m) x^2 (c + d (3 + m) x) + \right. \right. \\
 & \quad \left. \left. 3 a^2 b c m x (2 c^2 + 2 c d (3 + m) x + d^2 (6 + 5 m + m^2) x^2) + \right. \right. \\
 & \quad \left. a^3 \left( 6 c^3 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 6 c^2 d x \left( -3 - m + 3 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + \right. \right. \\
 & \quad \left. \left. 3 c d^2 x^2 \left( -6 - 5 m - m^2 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + d^3 x^3 \left( -6 - 11 m - 6 m^2 - m^3 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) \right) \\
 & \text{Gamma} [1 - m] + m \left( b^3 c^3 (2 + 3 m + m^2) x^3 (3 c (2 + m) + d m x) - \right. \\
 & \quad \left. 3 a b^2 c^2 (1 + m) x^2 (c^2 m + c d (12 + 14 m + 3 m^2) x + d^2 m (3 + m) x^2) + \right. \\
 & \quad \left. 3 a^2 b c x (2 c^3 m + 2 c^2 d m (4 + m) x + c d^2 (12 + 34 m + 19 m^2 + 3 m^3) x^2 + d^3 m (6 + 5 m + m^2) x^3) + \right.
 \end{aligned}$$

$$\begin{aligned}
 & a^3 \left( 6 c^4 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 6 c^3 d x \left( -4 - m + 4 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + \right. \\
 & d^4 x^4 \left( -6 - 11 m - 6 m^2 - m^3 + 6 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 3 c d^3 x^3 \left( -12 - 16 m - 7 m^2 - m^3 + \right. \\
 & \left. \left. 8 \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) + 3 c^2 d^2 x^2 \left( -7 m - m^2 + 12 \left( -1 + \left( \frac{a (c + d x)}{c (a + b x)} \right)^m \right) \right) \right) \Gamma[-m] - \\
 & \frac{1}{d (3 + m)} e^3 (c + d x)^{-3-m} \left( a - \frac{b c}{d} + \frac{b (c + d x)}{d} \right)^m \left( 1 + \frac{b (c + d x)}{\left( a - \frac{b c}{d} \right) d} \right)^{-m} \\
 & \text{Hypergeometric2F1} \left[ \right. \\
 & \quad -3 - m, \\
 & \quad -m, \\
 & \quad -2 - m, \\
 & \quad \left. - \frac{b (c + d x)}{\left( a - \frac{b c}{d} \right) d} \right]
 \end{aligned}$$

**Problem 3089: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{-4-m}}{e + f x} dx$$

Optimal (type 5, 330 leaves, 5 steps):

$$\begin{aligned}
 & \frac{d (a + b x)^{1+m} (c + d x)^{-3-m}}{(b c - a d) (d e - c f) (3 + m)} + \frac{d (a d f (3 + m) + b (2 d e - c f (5 + m))) (a + b x)^{1+m} (c + d x)^{-2-m}}{(b c - a d)^2 (d e - c f)^2 (2 + m) (3 + m)} + \\
 & \frac{d (a^2 d^2 f^2 (6 + 5 m + m^2) + a b d f (3 + m) (d e - c f (5 + 2 m)) + b^2 (2 d^2 e^2 - c d e f (7 + m) + c^2 f^2 (11 + 6 m + m^2))) (a + b x)^{1+m} (c + d x)^{-1-m}}{(b c - a d)^3 (d e - c f)^3 (1 + m) (2 + m) (3 + m)} + \frac{1}{(d e - c f)^4 m} \\
 & f^3 (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1} \left[ 1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)} \right]
 \end{aligned}$$

Result (type 5, 26263 leaves): Display of huge result suppressed!

**Problem 3090: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{-4-m}}{(e + f x)^2} dx$$

Optimal (type 5, 634 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{d (a d f (4+m) - b (d e + c f (3+m))) (a+b x)^{1+m} (c+d x)^{-3-m}}{(b c - a d) (b e - a f) (d e - c f)^2 (3+m)} - \\
 & \left( d (a^2 d^2 f^2 (12+7 m+m^2) - b^2 (2 d^2 e^2 - 2 c d e f (4+m) - c^2 f^2 (6+5 m+m^2))) - \right. \\
 & \quad \left. 2 a b d f (d e (2+m) + c f (10+6 m+m^2)) \right) (a+b x)^{1+m} (c+d x)^{-2-m} / \\
 & \left( (b c - a d)^2 (b e - a f) (d e - c f)^3 (2+m) (3+m) \right) - \\
 & \quad \frac{1}{(b c - a d)^3 (b e - a f) (d e - c f)^4 (1+m) (2+m) (3+m)} \\
 & d (a^3 d^3 f^3 (24+26 m+9 m^2+m^3) - a^2 b d^2 f^2 (3+m) (d e (4+3 m) + c f (20+15 m+3 m^2))) - \\
 & \quad b^3 (2 d^3 e^3 - 2 c d^2 e^2 f (5+m) + c^2 d e f^2 (26+17 m+3 m^2) + c^3 f^3 (6+11 m+6 m^2+m^3)) - \\
 & \quad a b^2 d f (2 d^2 e^2 (2+m) - 2 c d e f (16+15 m+3 m^2) - c^2 f^2 (44+50 m+21 m^2+3 m^3)) \\
 & (a+b x)^{1+m} (c+d x)^{-1-m} - \frac{f (a+b x)^{1+m} (c+d x)^{-3-m}}{(b e - a f) (d e - c f) (e+f x)} - \\
 & \left( f^3 (a d f (4+m) - b (4 d e + c f m)) (a+b x)^m (c+d x)^{-m} \right. \\
 & \quad \left. \text{Hypergeometric2F1}\left[1, -m, 1-m, \frac{(b e - a f) (c+d x)}{(d e - c f) (a+b x)}\right] \right) / \left( (b e - a f) (d e - c f)^5 m \right)
 \end{aligned}$$

Result (type 5, 64 249 leaves): Display of huge result suppressed!

### Problem 3091: Result more than twice size of optimal antiderivative.

$$\int (a+b x)^m (c+d x)^{-5-m} (e+f x)^p dx$$

Optimal (type 6, 133 leaves, 3 steps):

$$\begin{aligned}
 & \left( b^4 (a+b x)^{1+m} (c+d x)^{-m} \left( \frac{b (c+d x)}{b c - a d} \right)^m (e+f x)^p \left( \frac{b (e+f x)}{b e - a f} \right)^{-p} \right. \\
 & \quad \left. \text{AppellF1}\left[1+m, 5+m, -p, 2+m, -\frac{d (a+b x)}{b c - a d}, -\frac{f (a+b x)}{b e - a f}\right] \right) / \left( (b c - a d)^5 (1+m) \right)
 \end{aligned}$$

Result (type 6, 300 leaves):

$$\begin{aligned}
 & \left( (b c - a d) (b e - a f) (2+m) (a+b x)^{1+m} (c+d x)^{-5-m} \right. \\
 & \quad \left. (e+f x)^p \text{AppellF1}\left[1+m, 5+m, -p, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}\right] \right) / \\
 & \left( b (1+m) \left( (b c - a d) (b e - a f) (2+m) \text{AppellF1}\left[1+m, 5+m, -p, 2+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}\right] - \right. \right. \\
 & \quad \left. (a+b x) \left( (-b c + a d) f p \text{AppellF1}\left[2+m, 5+m, 1-p, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}\right] + \right. \right. \\
 & \quad \left. \left. d (b e - a f) (5+m) \text{AppellF1}\left[2+m, 6+m, -p, 3+m, \frac{d (a+b x)}{-b c + a d}, \frac{f (a+b x)}{-b e + a f}\right] \right) \right) \right)
 \end{aligned}$$

**Problem 3093: Attempted integration timed out after 120 seconds.**

$$\int (a+bx)^m (c+dx)^{-5-m} (e+fx)^4 dx$$

Optimal (type 5, 650 leaves, 14 steps):

$$\begin{aligned} & \frac{(de-cf)^4 (a+bx)^{1+m} (c+dx)^{-4-m}}{d^4 (bc-ad) (4+m)} + \frac{4f (de-cf)^3 (a+bx)^{1+m} (c+dx)^{-3-m}}{d^4 (bc-ad) (3+m)} + \\ & \frac{3b (de-cf)^4 (a+bx)^{1+m} (c+dx)^{-3-m}}{d^4 (bc-ad)^2 (3+m) (4+m)} + \frac{6f^2 (de-cf)^2 (a+bx)^{1+m} (c+dx)^{-2-m}}{d^4 (bc-ad) (2+m)} + \\ & \frac{8bf (de-cf)^3 (a+bx)^{1+m} (c+dx)^{-2-m}}{d^4 (bc-ad)^2 (2+m) (3+m)} + \frac{6b^2 (de-cf)^4 (a+bx)^{1+m} (c+dx)^{-2-m}}{d^4 (bc-ad)^3 (2+m) (3+m) (4+m)} + \\ & \frac{4f^3 (de-cf) (a+bx)^{1+m} (c+dx)^{-1-m}}{d^4 (bc-ad) (1+m)} + \frac{6bf^2 (de-cf)^2 (a+bx)^{1+m} (c+dx)^{-1-m}}{d^4 (bc-ad)^2 (1+m) (2+m)} + \\ & \frac{8b^2 f (de-cf)^3 (a+bx)^{1+m} (c+dx)^{-1-m}}{d^4 (bc-ad)^3 (1+m) (2+m) (3+m)} + \frac{6b^3 (de-cf)^4 (a+bx)^{1+m} (c+dx)^{-1-m}}{d^4 (bc-ad)^4 (1+m) (2+m) (3+m) (4+m)} - \\ & \frac{1}{d^5 m} f^4 (a+bx)^m \left( -\frac{d(a+bx)}{bc-ad} \right)^{-m} (c+dx)^{-m} \text{Hypergeometric2F1} \left[ -m, -m, 1-m, \frac{b(c+dx)}{bc-ad} \right] \end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 3098: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{-5-m}}{e+fx} dx$$

Optimal (type 5, 557 leaves, 6 steps):

$$\begin{aligned} & \frac{d (a+bx)^{1+m} (c+dx)^{-4-m}}{(bc-ad) (de-cf) (4+m)} + \frac{d (adf (4+m) + b (3de-cf (7+m))) (a+bx)^{1+m} (c+dx)^{-3-m}}{(bc-ad)^2 (de-cf)^2 (3+m) (4+m)} + \\ & \left( d (a^2 d^2 f^2 (12+7m+m^2) + 2abd f (4+m) (de-cf (4+m)) + \right. \\ & \quad \left. b^2 (6d^2 e^2 - 2cdef (10+m) + c^2 f^2 (26+9m+m^2))) (a+bx)^{1+m} (c+dx)^{-2-m} \right) / \\ & \left( (bc-ad)^3 (de-cf)^3 (2+m) (3+m) (4+m) \right) + \\ & \left( d (a^3 d^3 f^3 (24+26m+9m^2+m^3) + a^2 b d^2 f^2 (12+7m+m^2) (de-cf (7+3m)) + \right. \\ & \quad \left. a b^2 d f (4+m) (2d^2 e^2 - 2cdef (5+m) + c^2 f^2 (26+17m+3m^2)) + \right. \\ & \quad \left. b^3 (6d^3 e^3 - 2cd^2 e^2 f (13+m) + c^2 def^2 (46+11m+m^2) - c^3 f^3 (50+35m+10m^2+m^3))) \right) \\ & (a+bx)^{1+m} (c+dx)^{-1-m} / \left( (bc-ad)^4 (de-cf)^4 (1+m) (2+m) (3+m) (4+m) \right) - \\ & \frac{1}{(de-cf)^5 m} f^4 (a+bx)^m (c+dx)^{-m} \text{Hypergeometric2F1} \left[ 1, -m, 1-m, \frac{(be-af)(c+dx)}{(de-cf)(a+bx)} \right] \end{aligned}$$

Result (type 5, 50481 leaves): Display of huge result suppressed!

**Problem 3099: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x)^p dx$$

Optimal (type 6, 131 leaves, 3 steps):

$$\frac{1}{b^2 (1+m)} (b c - a d) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m (e + f x)^p$$

$$\left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} \left[ 1+m, -1+m, -p, 2+m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right]$$

Result (type 6, 298 leaves):

$$\left( (b c - a d) (b e - a f) (2+m) (a + b x)^{1+m} (c + d x)^{1-m} (e + f x)^p \right.$$

$$\left. \text{AppellF1} \left[ 1+m, -1+m, -p, 2+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \left( b (1+m) \right.$$

$$\left( (b c - a d) (b e - a f) (2+m) \text{AppellF1} \left[ 1+m, -1+m, -p, 2+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] - \right.$$

$$\left. (a + b x) \left( (-b c + a d) f^p \text{AppellF1} \left[ 2+m, -1+m, 1-p, 3+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + \right.$$

$$\left. \left. d (b e - a f) (-1+m) \text{AppellF1} \left[ 2+m, m, -p, 3+m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) \right)$$

**Problem 3100: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x)^3 dx$$

Optimal (type 5, 445 leaves, 4 steps):

$$\frac{f (a + b x)^{1+m} (c + d x)^{2-m} (e + f x)^2}{5 b d} + \frac{1}{60 b^3 d^3}$$

$$f (a + b x)^{1+m} (c + d x)^{2-m} (a^2 d^2 f^2 (12 - 7 m + m^2) - a b d f (15 d e (3 - m) - c f (9 + 2 m - 2 m^2)) +$$

$$b^2 (48 d^2 e^2 - 15 c d e f (2 + m) + c^2 f^2 (6 + 5 m + m^2))) -$$

$$3 b d f (a d f (4 - m) - b (7 d e - c f (3 + m))) x - \frac{1}{60 b^5 d^3 (1 + m)}$$

$$(b c - a d) (a^3 d^3 f^3 (24 - 26 m + 9 m^2 - m^3) - 3 a^2 b d^2 f^2 (6 - 5 m + m^2) (5 d e - c f (1 + m)) +$$

$$3 a b^2 d f (2 - m) (20 d^2 e^2 - 10 c d e f (1 + m) + c^2 f^2 (2 + 3 m + m^2))) -$$

$$b^3 (60 d^3 e^3 - 60 c d^2 e^2 f (1 + m) + 15 c^2 d e f^2 (2 + 3 m + m^2) - c^3 f^3 (6 + 11 m + 6 m^2 + m^3)))$$

$$(a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1} \left[ -1+m, 1+m, 2+m, -\frac{d (a + b x)}{b c - a d} \right]$$

Result (type 6, 461 leaves):

$$\begin{aligned}
 & \frac{1}{4} (a+bx)^m (c+dx)^{1-m} \left( \left( 18 a c e^2 f x^2 \operatorname{AppellF1} \left[ 2, -m, -1+m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \right. \\
 & \quad \left( 3 a c \operatorname{AppellF1} \left[ 2, -m, -1+m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] + b c m x \operatorname{AppellF1} \left[ 3, 1-m, -1+m, \right. \right. \\
 & \quad \quad \left. \left. 4, -\frac{bx}{a}, -\frac{dx}{c} \right] - a d (-1+m) x \operatorname{AppellF1} \left[ 3, -m, m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) + \\
 & \quad \left( 16 a c e f^2 x^3 \operatorname{AppellF1} \left[ 3, -m, -1+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \\
 & \quad \left( 4 a c \operatorname{AppellF1} \left[ 3, -m, -1+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] + b c m x \operatorname{AppellF1} \left[ 4, 1-m, -1+m, \right. \right. \\
 & \quad \quad \left. \left. 5, -\frac{bx}{a}, -\frac{dx}{c} \right] - a d (-1+m) x \operatorname{AppellF1} \left[ 4, -m, m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) + \\
 & \quad \left( 5 a c f^3 x^4 \operatorname{AppellF1} \left[ 4, -m, -1+m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \\
 & \quad \left( 5 a c \operatorname{AppellF1} \left[ 4, -m, -1+m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] + \right. \\
 & \quad \quad b c m x \operatorname{AppellF1} \left[ 5, 1-m, -1+m, 6, -\frac{bx}{a}, -\frac{dx}{c} \right] - \\
 & \quad \quad \left. a d (-1+m) x \operatorname{AppellF1} \left[ 5, -m, m, 6, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) - \frac{1}{d (-2+m)} \\
 & \quad \left. 4 e^3 \left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \operatorname{Hypergeometric2F1} \left[ 2-m, -m, 3-m, \frac{b(c+dx)}{bc-ad} \right] \right)
 \end{aligned}$$

**Problem 3101: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^m (c+dx)^{1-m} (e+fx)^2 dx$$

Optimal (type 5, 260 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{f(a d f(3-m) - b(5 d e - c f(2+m))) (a+bx)^{1+m} (c+dx)^{2-m}}{12 b^2 d^2} + \\
 & \frac{f(a+bx)^{1+m} (c+dx)^{2-m} (e+fx)}{4 b d} + \frac{1}{12 b^4 d^2 (1+m)} (b c - a d) (a^2 d^2 f^2 (6 - 5 m + m^2) - \\
 & \quad 2 a b d f (2 - m) (4 d e - c f (1 + m)) + b^2 (12 d^2 e^2 - 8 c d e f (1 + m) + c^2 f^2 (2 + 3 m + m^2))) \\
 & (a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m \operatorname{Hypergeometric2F1} \left[ -1+m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad} \right]
 \end{aligned}$$

Result (type 6, 510 leaves):

$$\begin{aligned}
 & c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a e (d e + 2 c f) x^2 \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \right. \\
 & \quad \left( 6 a c \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c} \right] + 2 m x \right. \\
 & \quad \left. \left( b c \operatorname{AppellF1} \left[ 3, 1 - m, m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] - a d \operatorname{AppellF1} \left[ 3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) \right) + \\
 & \quad \left( 4 a f (2 d e + c f) x^3 \operatorname{AppellF1} \left[ 3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \\
 & \quad \left( 3 \left( 4 a c \operatorname{AppellF1} \left[ 3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] + b c m x \operatorname{AppellF1} \left[ 4, 1 - m, m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] - \right. \right. \\
 & \quad \left. \left. a d m x \operatorname{AppellF1} \left[ 4, -m, 1 + m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) \right) + \\
 & \quad \left( 5 a d f^2 x^4 \operatorname{AppellF1} \left[ 4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \\
 & \quad \left( 20 a c \operatorname{AppellF1} \left[ 4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] + 4 b c m x \operatorname{AppellF1} \left[ 5, 1 - m, m, 6, -\frac{b x}{a}, -\frac{d x}{c} \right] - \right. \\
 & \quad \left. 4 a d m x \operatorname{AppellF1} \left[ 5, -m, 1 + m, 6, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) - \\
 & \quad \frac{c e^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} \operatorname{Hypergeometric2F1} \left[ 1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d} \right]}{d (-1 + m)} - \\
 & \quad \left. \frac{e^2 x \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} \operatorname{Hypergeometric2F1} \left[ 1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d} \right]}{-1 + m} \right)
 \end{aligned}$$

**Problem 3102: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{1-m} (e + f x) dx$$

Optimal (type 5, 145 leaves, 3 steps):

$$\begin{aligned}
 & \frac{f (a + b x)^{1+m} (c + d x)^{2-m}}{3 b d} - \frac{1}{3 b^3 d (1 + m)} (b c - a d) (a d f (2 - m) - b (3 d e - c f (1 + m))) \\
 & (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \operatorname{Hypergeometric2F1} \left[ -1 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d} \right]
 \end{aligned}$$

Result (type 6, 322 leaves):

$$\begin{aligned}
 & c (a+bx)^m (c+dx)^{-m} \left( \left( 3 a (de+cf) x^2 \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \right. \\
 & \quad \left( 6 a c \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] + 2 m x \right. \\
 & \quad \left. \left( b c \operatorname{AppellF1} \left[ 3, 1-m, m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] - a d \operatorname{AppellF1} \left[ 3, -m, 1+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) \right) + \\
 & \quad \left( 4 a d f x^3 \operatorname{AppellF1} \left[ 3, -m, m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \\
 & \quad \left( 12 a c \operatorname{AppellF1} \left[ 3, -m, m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] + 3 b c m x \operatorname{AppellF1} \left[ 4, 1-m, m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] - \right. \\
 & \quad \left. 3 a d m x \operatorname{AppellF1} \left[ 4, -m, 1+m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) - \frac{1}{d(-1+m)} \\
 & e \left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \operatorname{Hypergeometric2F1} \left[ 1-m, -m, 2-m, \frac{b(c+dx)}{bc-ad} \right]
 \end{aligned}$$

**Problem 3103: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^{1-m} dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$\begin{aligned}
 & \frac{1}{b^2(1+m)} (bc-ad) (a+bx)^{1+m} (c+dx)^{-m} \\
 & \left( \frac{b(c+dx)}{bc-ad} \right)^m \operatorname{Hypergeometric2F1} \left[ -1+m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad} \right]
 \end{aligned}$$

Result (type 6, 202 leaves):

$$\begin{aligned}
 & \frac{1}{d} c (a+bx)^m (c+dx)^{-m} \\
 & \left( \left( 3 a d^2 x^2 \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \left( 6 a c \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] + 2 m x \right. \right. \\
 & \quad \left. \left( b c \operatorname{AppellF1} \left[ 3, 1-m, m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] - a d \operatorname{AppellF1} \left[ 3, -m, 1+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) \right) - \\
 & \quad \left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \operatorname{Hypergeometric2F1} \left[ 1-m, -m, 2-m, \frac{b(c+dx)}{bc-ad} \right] \\
 & \quad \quad \quad -1+m
 \end{aligned}$$

**Problem 3104: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{1-m}}{e+fx} dx$$

Optimal (type 5, 230 leaves, 6 steps):

$$\frac{d (d e - c f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^2 m} - \frac{1}{f^2 m}$$

$$(d e - c f) (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right] +$$

$$\left(d (b (d e - c f (1 - m)) - a d f m) (a + b x)^{1+m} (c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]\right) / (b (b c - a d) f^2 m (1 + m))$$

Result (type 6, 622 leaves):

$$\left((a + b x)^m (c + d x)^{-m} \left(-d (-b c + a d) e (b e - a f) (-1 + m) (2 + m) (a + b x) \text{AppellF1}\left[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] - c (b c - a d) f (b e - a f) (-1 + m) (2 + m) (a + b x) \text{AppellF1}\left[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + b (1 + m) \left(\frac{d (a + b x)}{-b c + a d}\right)^{-m} (c + d x) (e + f x) \left((b c - a d) (b e - a f) (2 + m) \text{AppellF1}\left[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + (a + b x) \left((-b c f + a d f) \text{AppellF1}\left[2 + m, m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + d (-b e + a f) m \text{AppellF1}\left[2 + m, 1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right]\right)\right) \text{Hypergeometric2F1}\left[1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d}\right]\right) / (b f (1 - m) (1 + m) (e + f x) \left((b c - a d) (b e - a f) (2 + m) \text{AppellF1}\left[1 + m, m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + (a + b x) \left((-b c f + a d f) \text{AppellF1}\left[2 + m, m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + d (-b e + a f) m \text{AppellF1}\left[2 + m, 1 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right]\right)\right))$$

**Problem 3105: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{(e + f x)^2} dx$$

Optimal (type 5, 190 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{(a+bx)^m (c+dx)^{1-m}}{f(e+fx)} + \frac{1}{f^2 (be-af)^m} (adf(1-m) - b(de-cfm)) \\
 & (a+bx)^m (c+dx)^{-m} \operatorname{Hypergeometric2F1}\left[1, m, 1+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] + \frac{1}{f^2 m} \\
 & d(a+bx)^m (c+dx)^{-m} \left(\frac{b(c+dx)}{bc-ad}\right)^m \operatorname{Hypergeometric2F1}\left[m, m, 1+m, -\frac{d(a+bx)}{bc-ad}\right]
 \end{aligned}$$

Result (type 6, 461 leaves):

$$\begin{aligned}
 & \frac{1}{(be-af)(1+m)(e+fx)} (a+bx)^{1+m} (c+dx)^{-m} \\
 & \left(-\left(\left(d(bc-ad)(be-af)^3(2+m) \operatorname{AppellF1}\left[1+m, m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right) / \right. \\
 & \quad \left. \left(bf(-be+af)\right.\right. \\
 & \quad \left.\left.\left(\left((bc-ad)(be-af)(2+m) \operatorname{AppellF1}\left[1+m, m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right) + \right.\right. \\
 & \quad \left.\left.\left(a+bx\right)\left(\left(-bcf+adf\right) \operatorname{AppellF1}\left[2+m, m, 2, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right) + \right.\right. \\
 & \quad \left.\left.\left.d(-be+af)^m \operatorname{AppellF1}\left[2+m, 1+m, 1, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right]\right)\right)\right) + \\
 & c \left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^m \operatorname{Hypergeometric2F1}\left[m, 1+m, 2+m, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right] - \\
 & \frac{1}{f} \\
 & de \left(\frac{(be-af)(c+dx)}{(bc-ad)(e+fx)}\right)^m \\
 & \operatorname{Hypergeometric2F1}\left[m, 1+m, 2+m, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)}\right]
 \end{aligned}$$

**Problem 3106: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{1-m}}{(e+fx)^3} dx$$

Optimal (type 5, 85 leaves, 1 step):

$$\begin{aligned}
 & \left(\frac{(bc-ad)^2 (a+bx)^{1+m} (c+dx)^{-1-m} \operatorname{Hypergeometric2F1}\left[3, 1+m, 2+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af)^3 (1+m)}\right) /
 \end{aligned}$$

Result (type 5, 933 leaves):

$$\begin{aligned}
 & \left( (a+bx)^{1+m} (c+dx)^{-m} \left( -de (be-af)^2 (1+m) (c+dx) \right. \right. \\
 & \quad \left( (-2be+af(1+m)+bf(-1+m)x) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \right. \\
 & \quad 2(af(1+m)+b(-e+fm x)) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m \right] + \\
 & \quad \left. \left. f(1+m)(a+bx) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] \right) + cf (be-af)^2 (1+m) \right. \\
 & \quad (c+dx) \left( (-2be+af(1+m)+bf(-1+m)x) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \right. \\
 & \quad 2(af(1+m)+b(-e+fm x)) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m \right] + \\
 & \quad \left. \left. f(1+m)(a+bx) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] \right) + \right. \\
 & \quad d(2be-2af) \left( \frac{(be-af)(c+dx)}{(bc-ad)(e+fx)} \right)^m (e+fx) \left( (be-af)(c+dx) \right. \\
 & \quad \left. (af(1+m)+b(-e+fm x)) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - (a+bx) \right. \\
 & \quad \left( (af(1+m)(-2cf+d(e-fx))+b(cf(e(2+m)-fm x)+de(-e+f(1+2m)x))) \right. \\
 & \quad \left. \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m \right] + \right. \\
 & \quad \left. \left. f(-de+cf)(1+m)(a+bx) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] \right) \right) \\
 & \quad \left. \left. \text{Hypergeometric2F1} \left[ m, 1+m, 2+m, \frac{(-de+cf)(a+bx)}{(bc-ad)(e+fx)} \right] \right) \right) / \\
 & \left( f(2be-2af)(be-af)(1+m) \right. \\
 & \quad (e+fx)^2 \\
 & \quad \left( (be-af)(c+dx)(af(1+m)+b(-e+fm x)) \right. \\
 & \quad \left. \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \right. \\
 & \quad (a+bx) \left( (af(1+m)(-2cf+d(e-fx))+b(cf(e(2+m)-fm x)+de(-e+f(1+2m)x))) \right. \\
 & \quad \left. \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m \right] + \right. \\
 & \quad \left. \left. f(-de+cf)(1+m)(a+bx) \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m \right] \right) \right) \right)
 \end{aligned}$$

### Problem 3107: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^m (c+dx)^{1-m}}{(e+fx)^4} dx$$

Optimal (type 5, 176 leaves, 2 steps):

$$-\frac{f(a+bx)^{1+m}(c+dx)^{2-m}}{3(b e - a f)(d e - c f)(e+fx)^3} + \left( (b c - a d)^2 (b(3 d e - c f(2-m)) - a d f(1+m)) (a+bx)^{1+m} (c+dx)^{-1-m} \right. \\ \left. \text{Hypergeometric2F1}\left[3, 1+m, 2+m, \frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}\right] \right) / (3(b e - a f)^4 (d e - c f)(1+m))$$

Result (type 5, 3837 leaves):

$$\left( d(b e - a f)^4 (a+bx)^{1+m} (c+dx)^{-m} \right. \\ \left( (-2 b e + a f(1+m) + b f(-1+m)x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, m\right] - \right. \\ \left. 2(a f(1+m) + b(-e+fx)) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] + \right. \\ \left. f(1+m)(a+bx) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 2+m\right] \right) / \\ \left( f(2 b e - 2 a f)(-b e + a f)^3 (e+fx)^2 \left( (b e - a f)(-a f(1+m) + b(e-fmx)) \right. \right. \\ \left. \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, m\right] + \frac{1}{c+dx}(a+bx) \right. \right. \\ \left. \left. \left( (a f(1+m)(-2 c f + d(e-fx)) + b(c f(e(2+m) - fmx) + d e(-e+fx(1+2m)x))) \right. \right. \right. \\ \left. \left. \left. \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 1+m\right] + \right. \right. \\ \left. \left. \left. f(-d e + c f)(1+m)(a+bx) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, 2+m\right] \right) \right) \right) + \\ \left( c(a+bx)^{1+m} (c+dx)^{1-m} \left( 6(b e - a f)^2 \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, m\right] + \right. \right. \\ \left. \left. 6(b e - a f)^2 m \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, m\right] + \right. \right. \\ \left. \left. 6 f(b e - a f)(a+bx) \text{HurwitzLerchPhi}\left[\frac{(d e - c f)(a+bx)}{(b e - a f)(c+dx)}, 1, m\right] + \right. \right.$$

$$\begin{aligned}
& 6 f (-b e + a f) m^2 (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, m\right] + \\
& 2 f^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, m\right] - \\
& f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, m\right] - \\
& 2 f^2 m^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, m\right] + \\
& f^2 m^3 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, m\right] - \\
& 6 (b e - a f)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 1 + m\right] - \\
& 6 (b e - a f)^2 m \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 1 + m\right] + \\
& 12 f (b e - a f) m (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 1 + m\right] + \\
& 12 f (b e - a f) m^2 (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 1 + m\right] + \\
& 3 f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 1 + m\right] - \\
& 3 f^2 m^3 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 1 + m\right] + \\
& 6 f (-b e + a f) (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 2 + m\right] + \\
& 12 f (-b e + a f) m (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 2 + m\right] + \\
& 6 f (-b e + a f) m^2 (a + b x) \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 2 + m\right] + \\
& 3 f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 2 + m\right] + \\
& 6 f^2 m^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 2 + m\right] + \\
& 3 f^2 m^3 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 2 + m\right] - \\
& 2 f^2 (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] - \\
& 5 f^2 m (a + b x)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e - c f)(a + b x)}{(b e - a f)(c + d x)}, 1, 3 + m\right] -
\end{aligned}$$

$$\begin{aligned}
 & 4 f^2 m^2 (a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m\right] - \\
 & f^2 m^3 (a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m\right] \Big) \Big) / \\
 & \left( 3(1+m)(e+fx)^3 \left( (be-af)(c+dx)(a^2 f^2(2+3m+m^2) + 2abf(1+m)(-2e+fm)x + \right. \right. \\
 & \quad \left. \left. b^2(2e^2 - 4efmx + f^2(-1+m)mx^2)) \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] - \right. \right. \\
 & \quad (a+bx) \left( (a^2 f^2(2+3m+m^2)(-3cf+d(e-2fx)) - 2abf(1+m) \right. \\
 & \quad \left. (cf(-e(6+m) + 2fm)x) + d(2e^2 - 2ef(2+m)x + f^2 mx^2)) + \right. \\
 & \quad \left. b^2(cf(-2e^2(3+2m) + 2efm(3+m)x - f^2(-1+m)mx^2) + de(2e^2 - 4ef(1+2m)x + \right. \\
 & \quad \left. f^2 m(1+3m)x^2)) \right) \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m\right] + \\
 & \quad f(1+m)(a+bx) \left( (af(2+m)(-2de+3cf+dfx) + bcf(-e(6+m) + 2fm)x + \right. \\
 & \quad \left. bde(4e-f(2+3m)x)) \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m\right] + \right. \\
 & \quad \left. f(de-cf)(2+m)(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 3+m\right] \right) \Big) \Big) - \\
 & \left( de(a+bx)^{1+m}(c+dx)^{1-m} \left( 6(be-af)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \right. \right. \\
 & \quad 6(be-af)^2 m \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \\
 & \quad 6f(be-af)(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \\
 & \quad 6f(-be+af)m^2(a+bx) \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \\
 & \quad 2f^2(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] - \\
 & \quad f^2 m(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] - \\
 & \quad 2f^2 m^2(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \\
 & \quad f^2 m^3(a+bx)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] - \\
 & \quad \left. 6(be-af)^2 \operatorname{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m\right] - \right.
 \end{aligned}$$

$$\begin{aligned}
 & 6 (b e - a f)^2 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 12 f (b e - a f) m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 12 f (b e - a f) m^2 (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 3 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \\
 & 3 f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 6 f (-b e + a f) (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\
 & 12 f (-b e + a f) m (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\
 & 6 f (-b e + a f) m^2 (a + b x) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\
 & 3 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\
 & 6 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] + \\
 & 3 f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m \right] - \\
 & 2 f^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 5 f^2 m (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & 4 f^2 m^2 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] - \\
 & f^2 m^3 (a + b x)^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m \right] \Big) \Big) / \\
 & \left( 3 f (1 + m) (e + f x)^3 \left( (b e - a f) (c + d x) (a^2 f^2 (2 + 3 m + m^2) + 2 a b f (1 + m) (-2 e + f m x) + \right. \right. \\
 & \quad \left. \left. b^2 (2 e^2 - 4 e f m x + f^2 (-1 + m) m x^2) \right) \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \right. \\
 & \quad (a + b x) \left( (a^2 f^2 (2 + 3 m + m^2) (-3 c f + d (e - 2 f x)) - 2 a b f (1 + m) \right. \\
 & \quad \left. \left. (c f (-e (6 + m) + 2 f m x) + d (2 e^2 - 2 e f (2 + m) x + f^2 m x^2)) + \right. \right. \\
 & \quad \left. \left. b^2 (c f (-2 e^2 (3 + 2 m) + 2 e f m (3 + m) x - f^2 (-1 + m) m x^2) + d e (2 e^2 - 4 e f (1 + 2 m) x + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & f^2 m (1 + 3 m) x^2)) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m\right] + \\
 & f (1 + m) (a + b x) \left( (a f (2 + m) (-2 d e + 3 c f + d f x) + b c f (-e (6 + m) + 2 f m x) + \right. \\
 & \quad \left. b d e (4 e - f (2 + 3 m) x)) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 2 + m\right] + \right. \\
 & \quad \left. f (d e - c f) (2 + m) (a + b x) \text{HurwitzLerchPhi}\left[\frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 3 + m\right] \right) \right)
 \end{aligned}$$

**Problem 3108: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{(e + f x)^5} dx$$

Optimal (type 5, 311 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{f (a + b x)^{1+m} (c + d x)^{2-m}}{4 (b e - a f) (d e - c f) (e + f x)^4} - \frac{f (b (5 d e - c f (3 - m)) - a d f (2 + m)) (a + b x)^{1+m} (c + d x)^{2-m}}{12 (b e - a f)^2 (d e - c f)^2 (e + f x)^3} - \\
 & \left( (b c - a d)^2 (2 a b d f (4 d e - c f (2 - m)) (1 + m) - a^2 d^2 f^2 (2 + 3 m + m^2) - \right. \\
 & \quad \left. b^2 (12 d^2 e^2 - 8 c d e f (2 - m) + c^2 f^2 (6 - 5 m + m^2))) (a + b x)^{1+m} (c + d x)^{-1-m} \right. \\
 & \quad \left. \text{Hypergeometric2F1}\left[3, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / (12 (b e - a f)^5 (d e - c f)^2 (1 + m))
 \end{aligned}$$

Result (type 5, 63464 leaves): Display of huge result suppressed!

**Problem 3109: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^m (c + d x)^{1-m}}{(e + f x)^6} dx$$

Optimal (type 5, 542 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{f (a + b x)^{1+m} (c + d x)^{2-m}}{5 (b e - a f) (d e - c f) (e + f x)^5} - \frac{f (b (7 d e - c f (4 - m)) - a d f (3 + m)) (a + b x)^{1+m} (c + d x)^{2-m}}{20 (b e - a f)^2 (d e - c f)^2 (e + f x)^4} \\
 & \left( f (a^2 d^2 f^2 (6 + 5 m + m^2) - a b d f (3 d e (7 + 4 m) - c f (9 + 2 m - 2 m^2)) + \right. \\
 & \quad \left. b^2 (27 d^2 e^2 - 3 c d e f (11 - 4 m) + c^2 f^2 (12 - 7 m + m^2))) (a + b x)^{1+m} (c + d x)^{2-m} \right) / \\
 & \left( 60 (b e - a f)^3 (d e - c f)^3 (e + f x)^3 \right) + \frac{1}{60 (b e - a f)^6 (d e - c f)^3 (1 + m)} \\
 & \left( b c - a d \right)^2 \left( 3 a^2 b d^2 f^2 (5 d e - c f (2 - m)) (2 + 3 m + m^2) - a^3 d^3 f^3 (6 + 11 m + 6 m^2 + m^3) - \right. \\
 & \quad \left. 3 a b^2 d f (1 + m) (20 d^2 e^2 - 10 c d e f (2 - m) + c^2 f^2 (6 - 5 m + m^2)) + \right. \\
 & \quad \left. b^3 (60 d^3 e^3 - 60 c d^2 e^2 f (2 - m) + 15 c^2 d e f^2 (6 - 5 m + m^2) - c^3 f^3 (24 - 26 m + 9 m^2 - m^3)) \right) \\
 & (a + b x)^{1+m} (c + d x)^{-1-m} \text{Hypergeometric2F1} \left[ 3, 1 + m, 2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)} \right]
 \end{aligned}$$

Result (type 5, 136671 leaves): Display of huge result suppressed!

**Problem 3110: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{2-m} (e + f x)^p dx$$

Optimal (type 6, 133 leaves, 3 steps):

$$\begin{aligned}
 & \frac{1}{b^3 (1 + m)} (b c - a d)^2 (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m (e + f x)^p \\
 & \left( \frac{b (e + f x)}{b e - a f} \right)^{-p} \text{AppellF1} \left[ 1 + m, -2 + m, -p, 2 + m, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right]
 \end{aligned}$$

Result (type 6, 300 leaves):

$$\begin{aligned}
 & \left( (b c - a d) (b e - a f) (2 + m) (a + b x)^{1+m} (c + d x)^{2-m} (e + f x)^p \right. \\
 & \quad \left. \text{AppellF1} \left[ 1 + m, -2 + m, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \left( b (1 + m) \right. \\
 & \quad \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1} \left[ 1 + m, -2 + m, -p, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] - \right. \\
 & \quad \left. (a + b x) \left( (-b c + a d) f^p \text{AppellF1} \left[ 2 + m, -2 + m, 1 - p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + \right. \\
 & \quad \left. \left. d (b e - a f) (-2 + m) \text{AppellF1} \left[ 2 + m, -1 + m, -p, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) \right) \left. \right)
 \end{aligned}$$

**Problem 3111: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (c + d x)^{2-m} (e + f x)^3 dx$$

Optimal (type 5, 447 leaves, 4 steps):

$$\frac{f (a+bx)^{1+m} (c+dx)^{3-m} (e+fx)^2}{6bd} + \frac{1}{120b^3d^3}$$

$$f (a+bx)^{1+m} (c+dx)^{3-m} (a^2d^2f^2(20-9m+m^2) - 2abdf(9de(4-m) - cf(6+2m-m^2)) + b^2(70d^2e^2 - 18cdef(2+m) + c^2f^2(6+5m+m^2)) - 4bdf(adf(5-m) - b(8de - cf(3+m))))x - \frac{1}{120b^6d^3(1+m)}$$

$$(bc-ad)^2(a^3d^3f^3(60-47m+12m^2-m^3) - 3a^2bd^2f^2(12-7m+m^2)(6de - cf(1+m)) + 3ab^2df(3-m)(30d^2e^2 - 12cdef(1+m) + c^2f^2(2+3m+m^2)) - b^3(120d^3e^3 - 90cd^2e^2f(1+m) + 18c^2def^2(2+3m+m^2) - c^3f^3(6+11m+6m^2+m^3)))$$

$$(a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc-ad} \right)^m \text{Hypergeometric2F1} \left[ -2+m, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad} \right]$$

Result (type 6, 467 leaves):

$$\frac{1}{4} (a+bx)^m (c+dx)^{2-m} \left( \left( 18ac e^2 f x^2 \text{AppellF1} \left[ 2, -m, -2+m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) / \right.$$

$$\left( 3ac \text{AppellF1} \left[ 2, -m, -2+m, 3, -\frac{bx}{a}, -\frac{dx}{c} \right] + bcm x \text{AppellF1} \left[ 3, 1-m, -2+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] - ad(-2+m) x \text{AppellF1} \left[ 3, -m, -1+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) +$$

$$\left( 16ace f^2 x^3 \text{AppellF1} \left[ 3, -m, -2+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) /$$

$$\left( 4ac \text{AppellF1} \left[ 3, -m, -2+m, 4, -\frac{bx}{a}, -\frac{dx}{c} \right] + bcm x \text{AppellF1} \left[ 4, 1-m, -2+m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] - ad(-2+m) x \text{AppellF1} \left[ 4, -m, -1+m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) +$$

$$\left( 5ac f^3 x^4 \text{AppellF1} \left[ 4, -m, -2+m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) /$$

$$\left( 5ac \text{AppellF1} \left[ 4, -m, -2+m, 5, -\frac{bx}{a}, -\frac{dx}{c} \right] + bcm x \text{AppellF1} \left[ 5, 1-m, -2+m, 6, -\frac{bx}{a}, -\frac{dx}{c} \right] - ad(-2+m) x \text{AppellF1} \left[ 5, -m, -1+m, 6, -\frac{bx}{a}, -\frac{dx}{c} \right] \right) - \frac{1}{d(-3+m)}$$

$$4e^3 \left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \text{Hypergeometric2F1} \left[ 3-m, -m, 4-m, \frac{b(c+dx)}{bc-ad} \right]$$

**Problem 3112: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^m (c+dx)^{2-m} (e+fx)^2 dx$$

Optimal (type 5, 262 leaves, 4 steps):

$$\begin{aligned} & - \frac{f (a d f (4 - m) - b (6 d e - c f (2 + m))) (a + b x)^{1+m} (c + d x)^{3-m}}{20 b^2 d^2} + \\ & \frac{f (a + b x)^{1+m} (c + d x)^{3-m} (e + f x)}{5 b d} + \frac{1}{20 b^5 d^2 (1 + m)} (b c - a d)^2 (a^2 d^2 f^2 (12 - 7 m + m^2) - \\ & 2 a b d f (3 - m) (5 d e - c f (1 + m)) + b^2 (20 d^2 e^2 - 10 c d e f (1 + m) + c^2 f^2 (2 + 3 m + m^2))) \\ & (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1} \left[ -2 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d} \right] \end{aligned}$$

Result (type 6, 340 leaves):

$$\begin{aligned} & \frac{1}{3} (a + b x)^m (c + d x)^{2-m} \left( \left( 9 a c e f x^2 \text{AppellF1} \left[ 2, -m, -2 + m, 3, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \right. \\ & \left( 3 a c \text{AppellF1} \left[ 2, -m, -2 + m, 3, -\frac{b x}{a}, -\frac{d x}{c} \right] + b c m x \text{AppellF1} \left[ 3, 1 - m, -2 + m, \right. \right. \\ & \left. \left. 4, -\frac{b x}{a}, -\frac{d x}{c} \right] - a d (-2 + m) x \text{AppellF1} \left[ 3, -m, -1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) + \\ & \left( 4 a c f^2 x^3 \text{AppellF1} \left[ 3, -m, -2 + m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \\ & \left( 4 a c \text{AppellF1} \left[ 3, -m, -2 + m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] + \right. \\ & \left. b c m x \text{AppellF1} \left[ 4, 1 - m, -2 + m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] - \right. \\ & \left. a d (-2 + m) x \text{AppellF1} \left[ 4, -m, -1 + m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) - \frac{1}{d (-3 + m)} \\ & 3 e^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \text{Hypergeometric2F1} \left[ 3 - m, -m, 4 - m, \frac{b (c + d x)}{b c - a d} \right] \end{aligned}$$

**Problem 3113: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{2-m} (e + f x) dx$$

Optimal (type 5, 147 leaves, 3 steps):

$$\begin{aligned} & \frac{f (a + b x)^{1+m} (c + d x)^{3-m}}{4 b d} - \frac{1}{4 b^4 d (1 + m)} (b c - a d)^2 (a d f (3 - m) - b (4 d e - c f (1 + m))) \\ & (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \text{Hypergeometric2F1} \left[ -2 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d} \right] \end{aligned}$$

Result (type 6, 509 leaves):

$$\begin{aligned}
 & c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c (2 d e + c f) x^2 \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \right. \\
 & \quad \left( 6 a c \operatorname{AppellF1} \left[ 2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c} \right] + 2 m x \right. \\
 & \quad \left. \left( b c \operatorname{AppellF1} \left[ 3, 1 - m, m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] - a d \operatorname{AppellF1} \left[ 3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) \right) + \\
 & \quad \left( 4 a d (d e + 2 c f) x^3 \operatorname{AppellF1} \left[ 3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \\
 & \quad \left( 3 \left( 4 a c \operatorname{AppellF1} \left[ 3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c} \right] + b c m x \operatorname{AppellF1} \left[ 4, 1 - m, m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] - \right. \right. \\
 & \quad \left. \left. a d m x \operatorname{AppellF1} \left[ 4, -m, 1 + m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) \right) + \\
 & \quad \left( 5 a d^2 f x^4 \operatorname{AppellF1} \left[ 4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) / \\
 & \quad \left( 20 a c \operatorname{AppellF1} \left[ 4, -m, m, 5, -\frac{b x}{a}, -\frac{d x}{c} \right] + 4 b c m x \operatorname{AppellF1} \left[ 5, 1 - m, m, 6, -\frac{b x}{a}, -\frac{d x}{c} \right] - \right. \\
 & \quad \left. 4 a d m x \operatorname{AppellF1} \left[ 5, -m, 1 + m, 6, -\frac{b x}{a}, -\frac{d x}{c} \right] \right) - \\
 & \quad \frac{c^2 e \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} \operatorname{Hypergeometric2F1} \left[ 1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d} \right]}{d (-1 + m)} - \\
 & \quad \left. \frac{c e x \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} \operatorname{Hypergeometric2F1} \left[ 1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d} \right]}{-1 + m} \right)
 \end{aligned}$$

**Problem 3114: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{2-m} dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\begin{aligned}
 & \frac{1}{b^3 (1 + m)} (b c - a d)^2 (a + b x)^{1+m} (c + d x)^{-m} \\
 & \quad \left( \frac{b (c + d x)}{b c - a d} \right)^m \operatorname{Hypergeometric2F1} \left[ -2 + m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d} \right]
 \end{aligned}$$

Result (type 6, 319 leaves):

$$\frac{1}{d} c (a + b x)^m (c + d x)^{-m} \left( \left( 3 a c d^2 x^2 \operatorname{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \left( 3 a c \operatorname{AppellF1}\left[2, -m, m, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + m x \left( b c \operatorname{AppellF1}\left[3, 1 - m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] - a d \operatorname{AppellF1}\left[3, -m, 1 + m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) \right) + \left( 4 a d^3 x^3 \operatorname{AppellF1}\left[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \left( 12 a c \operatorname{AppellF1}\left[3, -m, m, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + 3 b c m x \operatorname{AppellF1}\left[4, 1 - m, m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] - 3 a d m x \operatorname{AppellF1}\left[4, -m, 1 + m, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) - \frac{1}{-1 + m} c \left( \frac{d (a + b x)}{-b c + a d} \right)^{-m} (c + d x) \operatorname{Hypergeometric2F1}\left[1 - m, -m, 2 - m, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 3115: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^m (c + d x)^{2-m}}{e + f x} dx$$

Optimal (type 5, 370 leaves, 6 steps):

$$\begin{aligned} & - \left( \left( d (2 a b c d f^2 m - a^2 d^2 f^2 m - b^2 (2 d^2 e^2 - 4 c d e f + c^2 f^2 (2 + m))) (a + b x)^{1+m} (c + d x)^{-m} \right) / \right. \\ & \quad \left. (2 b^2 (b c - a d) f^3 m) \right) + \frac{d^2 (a + b x)^{2+m} (c + d x)^{-m}}{2 b^2 f} + \frac{1}{f^3 m} \\ & \quad (d e - c f)^2 (a + b x)^m (c + d x)^{-m} \operatorname{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right] + \\ & \quad \left( d (2 a b d f (d e - c f (2 - m)) m + a^2 d^2 f^2 (1 - m) m - \right. \\ & \quad \left. b^2 (2 d^2 e^2 - 2 c d e f (2 - m) + c^2 f^2 (2 - 3 m + m^2))) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \right. \\ & \quad \left. \operatorname{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right] \right) / (2 b^2 (b c - a d) f^3 m (1 + m)) \end{aligned}$$

Result (type 6, 303 leaves):

$$\begin{aligned}
 & - \left( \left( (bc - ad) (be - af)^2 (2+m) (a+bx)^{1+m} (c+dx)^{2-m} \right. \right. \\
 & \quad \left. \left. \text{AppellF1} \left[ 1+m, -2+m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(-be+af)(1+m)(e+fx) \right) \right. \\
 & \quad \left( (bc - ad) (be - af) (2+m) \text{AppellF1} \left[ 1+m, -2+m, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \\
 & \quad (a+bx) \left( (-bcf + adf) \text{AppellF1} \left[ 2+m, -2+m, 2, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \\
 & \quad \left. \left. \left. \left. d (be - af) (-2+m) \text{AppellF1} \left[ 2+m, -1+m, 1, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right] \right) \right) \right)
 \end{aligned}$$

**Problem 3116: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{(e+fx)^2} dx$$

Optimal (type 5, 316 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{2d^2 (de - cf) (a+bx)^{1+m} (c+dx)^{-m}}{(bc - ad) f^3 m} + \frac{(de - cf)^2 (a+bx)^{1+m} (c+dx)^{-m}}{f^2 (be - af) (e+fx)} + \\
 & \frac{1}{f^3 (be - af) m} (de - cf) (adf(2-m) - b(2de - cfm)) (a+bx)^m \\
 & (c+dx)^{-m} \text{Hypergeometric2F1} \left[ 1, -m, 1-m, \frac{(be - af)(c+dx)}{(de - cf)(a+bx)} \right] + \\
 & \left( d^2 (b(2de - cf(2-m)) - adfm) (a+bx)^{1+m} (c+dx)^{-m} \left( \frac{b(c+dx)}{bc - ad} \right)^m \right. \\
 & \quad \left. \text{Hypergeometric2F1} \left[ m, 1+m, 2+m, -\frac{d(a+bx)}{bc - ad} \right] \right) / (b(bc - ad) f^3 m (1+m))
 \end{aligned}$$

Result (type 6, 291 leaves):

$$\begin{aligned}
 & \left( (bc - ad) (be - af) (2+m) (a+bx)^{1+m} (c+dx)^{2-m} \right. \\
 & \quad \left. \text{AppellF1} \left[ 1+m, -2+m, 2, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(1+m)(e+fx)^2 \right) \\
 & \quad \left( (bc - ad) (be - af) (2+m) \text{AppellF1} \left[ 1+m, -2+m, 2, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \\
 & \quad (a+bx) \left( (-2bcf + 2adf) \text{AppellF1} \left[ 2+m, -2+m, 3, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \\
 & \quad \left. \left. \left. \left. d (be - af) (-2+m) \text{AppellF1} \left[ 2+m, -1+m, 2, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right] \right) \right) \right)
 \end{aligned}$$

### Problem 3117: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{(e+fx)^3} dx$$

Optimal (type 5, 362 leaves, 7 steps):

$$\frac{(be-af)(a+bx)^{-1+m}(c+dx)^{2-m}}{2f^2(e+fx)^2} + \frac{(adf(2-m) - b(3de-cf(1+m)))(a+bx)^{-1+m}(c+dx)^{2-m}}{2f^2(de-cf)(e+fx)}$$

$$\left( (2abd f(2-m)(de-cfm) - b^2(2d^2e^2 - 2cdefm - c^2f^2(1-m)m) - a^2d^2f^2(2-3m+m^2)) \right.$$

$$\left. (a+bx)^{-1+m}(c+dx)^{1-m} \text{Hypergeometric2F1}\left[1, -1+m, m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) /$$

$$(2f^3(be-af)(de-cf)(1-m)) - \frac{1}{f^3(1-m)}d(bc-ad)(a+bx)^{-1+m}(c+dx)^{-m}$$

$$\left( \frac{b(c+dx)}{bc-ad} \right)^m \text{Hypergeometric2F1}\left[-1+m, -1+m, m, -\frac{d(a+bx)}{bc-ad}\right]$$

Result (type 6, 304 leaves):

$$- \left( \left( (bc-ad)(be-af)^4(2+m)(a+bx)^{1+m}(c+dx)^{2-m} \right. \right.$$

$$\left. \left. \text{AppellF1}\left[1+m, -2+m, 3, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \left( b(-be+af)^3(1+m)(e+fx)^3 \right. \right.$$

$$\left. \left( (bc-ad)(be-af)(2+m) \text{AppellF1}\left[1+m, -2+m, 3, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right. \right.$$

$$\left. (a+bx) \left( (-3bcf+3adf) \text{AppellF1}\left[2+m, -2+m, 4, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right.$$

$$\left. \left. \left. \left. d(be-af)(-2+m) \text{AppellF1}\left[2+m, -1+m, 3, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right) \right)$$

### Problem 3119: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{(e+fx)^5} dx$$

Optimal (type 5, 176 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{f (a+bx)^{1+m} (c+dx)^{3-m}}{4 (be-af) (de-cf) (e+fx)^4} + \\
 & \left( (bc-ad)^3 (b(4de-cf(3-m)) - adf(1+m)) (a+bx)^{1+m} (c+dx)^{-1-m} \right. \\
 & \quad \left. \text{Hypergeometric2F1}\left[4, 1+m, 2+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \left( 4 (be-af)^5 (de-cf) (1+m) \right)
 \end{aligned}$$

Result (type 5, 3314 leaves):

$$\begin{aligned}
 & - \left( \left( (a+bx)^{1+m} (c+dx)^{3-m} \right. \right. \\
 & \quad \left( (-4be+af(1+m) + bf(-3+m)x) \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, -2+m\right] + \right. \\
 & \quad 4(3be-af(1+m) - bf(-2+m)x) \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, -1+m\right] - \\
 & \quad 12be \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \\
 & \quad 6af \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \\
 & \quad 6afm \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] - 6bf x \text{HurwitzLerchPhi}\left[ \right. \\
 & \quad \quad \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + 6bfm x \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m\right] + \\
 & \quad 4be \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m\right] - \\
 & \quad 4af \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m\right] - \\
 & \quad 4afm \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m\right] - \\
 & \quad 4bfm x \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 1+m\right] + \\
 & \quad af \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m\right] + \\
 & \quad afm \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m\right] + \\
 & \quad bfx \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m\right] + \\
 & \quad \left. \left. \left. bfmx \text{HurwitzLerchPhi}\left[\frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, 2+m\right] \right) \right) \right) /
 \end{aligned}$$

$$\begin{aligned}
& \left( 4 (e+fx)^4 \left( (be-af)(c+dx)(3be-af(1+m) - bf(-2+m)x) \right. \right. \\
& \quad \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, -2+m \right] + (a^2 f(1+m)(-4cf+d(e-3fx)) - \\
& \quad b^2 (dex(9e+f(5-4m)x) + c(6e^2 - 3efmx + f^2(-2+m)x^2)) + \\
& \quad \left. \left. ab(cf(3e(4+m) + f(4-5m)x) + d(-3e^2 + ef(8+5m)x - 3f^2(-1+m)x^2)) \right) \right) \\
& \quad \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, -1+m \right] + \\
& \quad 3b^2 c e^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] + \\
& \quad 6abd e^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 12abc e f \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 3a^2 d e f \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] + \\
& \quad 6a^2 c f^2 \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 3abc e f m \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 3a^2 d e f m \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] + \\
& \quad 6a^2 c f^2 m \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] + \\
& \quad 9b^2 d e^2 x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 6b^2 c e f x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 6abd e f x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] + \\
& \quad 3a^2 d f^2 x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 3b^2 c e f m x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] - \\
& \quad 9abd e f m x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] + \\
& \quad 9abc f^2 m x \text{HurwitzLerchPhi} \left[ \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}, 1, m \right] +
\end{aligned}$$

$$\begin{aligned}
 & 3 a^2 d f^2 m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
 & 3 b^2 d e f x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
 & 3 b^2 c f^2 x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
 & 6 b^2 d e f m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
 & 3 b^2 c f^2 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] + \\
 & 3 a b d f^2 m x^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, m \right] - \\
 & 3 a b d e^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 4 a b c e f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 3 a^2 d e f \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \\
 & 4 a^2 c f^2 \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & a b c e f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 3 a^2 d e f m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \\
 & 4 a^2 c f^2 m \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \\
 & 3 b^2 d e^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 4 b^2 c e f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & 4 a b d e f x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \\
 & 4 a b c f^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] - \\
 & a^2 d f^2 x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] + \\
 & b^2 c e f m x \text{HurwitzLerchPhi} \left[ \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}, 1, 1 + m \right] +
 \end{aligned}$$

$$\begin{aligned}
& 7 a b d e f m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] - \\
& 7 a b c f^2 m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] - \\
& a^2 d f^2 m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] + \\
& b^2 d e f x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] - \\
& a b d f^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] + \\
& 4 b^2 d e f m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] - \\
& 3 b^2 c f^2 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] - \\
& a b d f^2 m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 1+m\right] - \\
& a^2 d e f \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] + \\
& a^2 c f^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] - \\
& a^2 d e f m \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] + \\
& a^2 c f^2 m \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] - \\
& 2 a b d e f x \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] + \\
& 2 a b c f^2 x \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] - \\
& 2 a b d e f m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] + \\
& 2 a b c f^2 m x \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] - \\
& b^2 d e f x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] + \\
& b^2 c f^2 x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] - \\
& b^2 d e f m x^2 \operatorname{HurwitzLerchPhi}\left[\frac{(d e-c f)(a+b x)}{(b e-a f)(c+d x)}, 1, 2+m\right] +
\end{aligned}$$

$$b^2 c f^2 m x^2 \text{HurwitzLerchPhi}\left[\frac{(de - cf)(a + bx)}{(be - af)(c + dx)}, 1, 2 + m\right]\right)$$

**Problem 3120: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + bx)^m (c + dx)^{2-m}}{(e + fx)^6} dx$$

Optimal (type 5, 311 leaves, 4 steps):

$$\begin{aligned} & - \frac{f (a + bx)^{1+m} (c + dx)^{3-m}}{5 (be - af) (de - cf) (e + fx)^5} - \frac{f (b (6de - cf(4 - m)) - adf(2 + m)) (a + bx)^{1+m} (c + dx)^{3-m}}{20 (be - af)^2 (de - cf)^2 (e + fx)^4} \\ & \left( (bc - ad)^3 (2abdf(5de - cf(3 - m)) (1 + m) - a^2 d^2 f^2 (2 + 3m + m^2)) - \right. \\ & \quad \left. b^2 (20d^2 e^2 - 10cdef(3 - m) + c^2 f^2 (12 - 7m + m^2)) \right) (a + bx)^{1+m} (c + dx)^{-1-m} \\ & \text{Hypergeometric2F1}\left[4, 1 + m, 2 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] / \left(20 (be - af)^6 (de - cf)^2 (1 + m)\right) \end{aligned}$$

Result (type 5, 29088 leaves): Display of huge result suppressed!

**Problem 3121: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + bx)^m (c + dx)^{2-m}}{(e + fx)^7} dx$$

Optimal (type 5, 541 leaves, 5 steps):

$$\begin{aligned} & - \frac{f (a + bx)^{1+m} (c + dx)^{3-m}}{6 (be - af) (de - cf) (e + fx)^6} - \frac{f (b (8de - cf(5 - m)) - adf(3 + m)) (a + bx)^{1+m} (c + dx)^{3-m}}{30 (be - af)^2 (de - cf)^2 (e + fx)^5} \\ & \left( f (a^2 d^2 f^2 (6 + 5m + m^2) - 2abdf(de(12 + 7m) - cf(6 + 2m - m^2))) + \right. \\ & \quad \left. b^2 (38d^2 e^2 - 2cdef(26 - 7m) + c^2 f^2 (20 - 9m + m^2)) \right) (a + bx)^{1+m} (c + dx)^{3-m} / \\ & \left( 120 (be - af)^3 (de - cf)^3 (e + fx)^4 \right) + \frac{1}{120 (be - af)^7 (de - cf)^3 (1 + m)} \\ & \left( (bc - ad)^3 (3a^2 b d^2 f^2 (6de - cf(3 - m)) (2 + 3m + m^2) - a^3 d^3 f^3 (6 + 11m + 6m^2 + m^3)) - \right. \\ & \quad \left. 3ab^2 df(1 + m) (30d^2 e^2 - 12cdef(3 - m) + c^2 f^2 (12 - 7m + m^2)) + \right. \\ & \quad \left. b^3 (120d^3 e^3 - 90c d^2 e^2 f(3 - m) + 18c^2 def^2 (12 - 7m + m^2) - c^3 f^3 (60 - 47m + 12m^2 - m^3)) \right) \\ & (a + bx)^{1+m} (c + dx)^{-1-m} \text{Hypergeometric2F1}\left[4, 1 + m, 2 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \end{aligned}$$

Result (type 5, 79140 leaves): Display of huge result suppressed!

**Problem 3122: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + bx)^m (c + dx)^{3-m}}{e + fx} dx$$

Optimal (type 5, 488 leaves, 7 steps):

$$\frac{b (b e - a f)^3 (a + b x)^{-3+m} (c + d x)^{4-m}}{(b c - a d) f^4 (3 - m)} -$$

$$\frac{b (b (3 d e - c f (1 - m)) - a d f (2 + m)) (a + b x)^{-2+m} (c + d x)^{4-m}}{6 d^2 f^2} +$$

$$\frac{b (a + b x)^{-1+m} (c + d x)^{4-m}}{3 d f} - \frac{1}{f^4 (3 - m)}$$

$$(b e - a f)^3 (a + b x)^{-3+m} (c + d x)^{3-m} \text{Hypergeometric2F1}\left[1, -3 + m, -2 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] -$$

$$\frac{1}{6 b^3 d^2 f^4 (2 - m) (3 - m)} (b c - a d)^2 (3 a^2 b d^2 f^2 (d e - c f (3 - m)) (1 - m) m +$$

$$a^3 d^3 f^3 m (2 - 3 m + m^2) + 3 a b^2 d f m (2 d^2 e^2 - 2 c d e f (3 - m) + c^2 f^2 (6 - 5 m + m^2)) -$$

$$b^3 (6 d^3 e^3 - 6 c d^2 e^2 f (3 - m) + 3 c^2 d e f^2 (6 - 5 m + m^2) - c^3 f^3 (6 - 11 m + 6 m^2 - m^3))) (a + b x)^{-2+m}$$

$$(c + d x)^{-m} \left(\frac{b (c + d x)}{b c - a d}\right)^m \text{Hypergeometric2F1}\left[-3 + m, -2 + m, -1 + m, -\frac{d (a + b x)}{b c - a d}\right]$$

Result (type 6, 303 leaves):

$$- \left( \left( (b c - a d) (b e - a f)^2 (2 + m) (a + b x)^{1+m} (c + d x)^{3-m} \right. \right.$$

$$\left. \left. \text{AppellF1}\left[1 + m, -3 + m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] \right) / \left( b (-b e + a f) (1 + m) (e + f x) \right. \right.$$

$$\left. \left. \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}\left[1 + m, -3 + m, 1, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] + \right. \right.$$

$$\left. \left. (a + b x) \left( (-b c f + a d f) \text{AppellF1}\left[2 + m, -3 + m, 2, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] - \right. \right.$$

$$\left. \left. \left. \left. d (b e - a f) (-3 + m) \text{AppellF1}\left[2 + m, -2 + m, 1, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] \right) \right) \right) \right)$$

**Problem 3123: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^m (c + d x)^{3-m}}{(e + f x)^2} dx$$

Optimal (type 5, 397 leaves, 2 steps):



$$\begin{aligned}
 & - \frac{3 d^3 (d e - c f) (a + b x)^{1+m} (c + d x)^{-m}}{(b c - a d) f^4 m} - \frac{(d e - c f)^3 (a + b x)^{1+m} (c + d x)^{-m}}{2 f^3 (b e - a f) (e + f x)^2} + \\
 & \left( (d e - c f)^2 (b (5 d e + c f (1 - m)) - a d f (6 - m)) (a + b x)^{1+m} (c + d x)^{-m} \right) / \\
 & \left( 2 f^3 (b e - a f)^2 (e + f x) \right) + \frac{1}{2 f^4 (b e - a f)^2 m} (d e - c f) \\
 & (2 a b d f (3 - m) (2 d e - c f m) - b^2 (6 d^2 e^2 - 4 c d e f m - c^2 f^2 (1 - m) m) - a^2 d^2 f^2 (6 - 5 m + m^2)) \\
 & (a + b x)^m (c + d x)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, \frac{(b e - a f) (c + d x)}{(d e - c f) (a + b x)}\right] + \\
 & \left( d^3 (b (3 d e - c f (3 - m)) - a d f m) (a + b x)^{1+m} (c + d x)^{-m} \left( \frac{b (c + d x)}{b c - a d} \right)^m \right. \\
 & \left. \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right] \right) / (b (b c - a d) f^4 m (1 + m))
 \end{aligned}$$

Result (type 6, 304 leaves):

$$\begin{aligned}
 & - \left( \left( (b c - a d) (b e - a f)^4 (2 + m) (a + b x)^{1+m} (c + d x)^{3-m} \right. \right. \\
 & \left. \left. \text{AppellF1}\left[1 + m, -3 + m, 3, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] \right) / \left( b (-b e + a f)^3 (1 + m) (e + f x)^3 \right. \right. \\
 & \left. \left( (b c - a d) (b e - a f) (2 + m) \text{AppellF1}\left[1 + m, -3 + m, 3, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] \right) + \right. \\
 & \left. (a + b x) \left( (-3 b c f + 3 a d f) \text{AppellF1}\left[2 + m, -3 + m, 4, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] - \right. \right. \\
 & \left. \left. d (b e - a f) (-3 + m) \text{AppellF1}\left[2 + m, -2 + m, 3, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f}\right] \right) \right) \right)
 \end{aligned}$$

**Problem 3125: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{1-n} (c + d x)^{1+n}}{b c + a d + 2 b d x} dx$$

Optimal (type 5, 245 leaves, 6 steps):

$$\begin{aligned}
 & \frac{(b c - a d) (3 - 2 n) (a + b x)^{2-n} (c + d x)^{-1+n}}{8 b^3 (1 - n)} + \frac{d (a + b x)^{3-n} (c + d x)^{-1+n}}{4 b^3} + \frac{1}{8 b^3 d (1 - n)} \\
 & (b c - a d)^2 (a + b x)^{1-n} (c + d x)^{-1+n} \text{Hypergeometric2F1}\left[1, -1 + n, n, -\frac{b (c + d x)}{d (a + b x)}\right] - \\
 & \frac{1}{8 b^2 d^2 (1 - n) n} (b c - a d)^2 (1 - 2 n^2) (a + b x)^{-n} \left( -\frac{d (a + b x)}{b c - a d} \right)^n \\
 & (c + d x)^n \text{Hypergeometric2F1}\left[-1 + n, n, 1 + n, \frac{b (c + d x)}{b c - a d}\right]
 \end{aligned}$$

Result (type 6, 1073 leaves):

$$\begin{aligned}
 & \frac{1}{4} (a+bx)^{-n} (c+dx)^n \\
 & \left( \left( 3acx^2 \operatorname{AppellF1}\left[2, n, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \left( 3ac \operatorname{AppellF1}\left[2, n, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + nx \right. \right. \\
 & \quad \left. \left. \left( ad \operatorname{AppellF1}\left[3, n, 1-n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] - bc \operatorname{AppellF1}\left[3, 1+n, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right) + \right. \\
 & \quad \left( 2ac(bc-ad)(-2+n)(a+bx) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] \right) / \\
 & \quad \left( b(-1+n)(ad+b(c+2dx)) \left( - (bc-ad)(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{d(a+bx)}{-bc+ad}, \right. \right. \right. \\
 & \quad \quad \left. \left. \frac{2d(a+bx)}{-bc+ad} \right] + d(a+bx) \left( n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] \right) - \right. \\
 & \quad \quad \left. \left. 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] \right) \right) \right) - \\
 & \quad \left( c^2(bc-ad)(-2+n)(a+bx) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] \right) / \\
 & \quad \left( d(-1+n)(ad+b(c+2dx)) \right. \\
 & \quad \left( - (bc-ad)(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] + \right. \\
 & \quad \left. d(a+bx) \left( n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] - \right. \right. \\
 & \quad \quad \left. \left. 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] \right) \right) \right) + \\
 & \quad \left( a^2d(-bc+ad)(-2+n)(a+bx) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] \right) / \\
 & \quad \left( b^2(-1+n)(ad+b(c+2dx)) \right. \\
 & \quad \left( - (bc-ad)(-2+n) \operatorname{AppellF1}\left[1-n, -n, 1, 2-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] + \right. \\
 & \quad \left. d(a+bx) \left( n \operatorname{AppellF1}\left[2-n, 1-n, 1, 3-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] - \right. \right. \\
 & \quad \quad \left. \left. 2 \operatorname{AppellF1}\left[2-n, -n, 2, 3-n, \frac{d(a+bx)}{-bc+ad}, \frac{2d(a+bx)}{-bc+ad}\right] \right) \right) \right) + \\
 & \quad \frac{c \left( \frac{d(a+bx)}{-bc+ad} \right)^n (c+dx) \operatorname{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{b(c+dx)}{bc-ad}\right]}{d^2(1+n)} + \\
 & \quad \left. \frac{a \left( \frac{d(a+bx)}{-bc+ad} \right)^n (c+dx) \operatorname{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{b(c+dx)}{bc-ad}\right]}{bd(1+n)} \right)
 \end{aligned}$$

Problem 3126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^{1-n} (c+dx)^{1+n}}{(bc+ad+2bdx)^2} dx$$

Optimal (type 5, 154 leaves, 4 steps):

$$-\frac{1}{4b^3d(1-n)}(bc-ad)(a+bx)^{1-n}(c+dx)^{-1+n} \text{Hypergeometric2F1}\left[2, 1-n, 2-n, -\frac{d(a+bx)}{b(c+dx)}\right] +$$

$$\frac{1}{4bd^2(1+n)}(a+bx)^{-n} \left(-\frac{d(a+bx)}{bc-ad}\right)^n (c+dx)^{1+n} \text{Hypergeometric2F1}\left[n, 1+n, 2+n, \frac{b(c+dx)}{bc-ad}\right]$$

Result (type 6, 904 leaves):

$$\begin{aligned}
 & \left( (a+bx)^{-n} (c+dx)^n \left( -b^2 c^2 (-1+n) \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \\
 & \quad 2abcd(-1+n) \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] - \\
 & \quad \left. a^2 d^2 (-1+n) \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + ad \left( \frac{b(c+dx)}{bc-ad} \right)^{-n} \right. \\
 & \quad \left( 2(ad+b(c+2dx)) \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \\
 & \quad \left. (bc-ad)n \left( \operatorname{AppellF1}\left[2, 1-n, n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[2, -n, 1+n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \\
 & \quad \operatorname{Hypergeometric2F1}\left[1-n, -n, 2-n, \frac{d(a+bx)}{-bc+ad}\right] + bdx \left( \frac{b(c+dx)}{bc-ad} \right)^{-n} \\
 & \quad \left( 2(ad+b(c+2dx)) \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \\
 & \quad \left. (bc-ad)n \left( \operatorname{AppellF1}\left[2, 1-n, n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[2, -n, 1+n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \\
 & \quad \left. \operatorname{Hypergeometric2F1}\left[1-n, -n, 2-n, \frac{d(a+bx)}{-bc+ad}\right] \right) \Big/ \left( 4b^2 d^2 (1-n) \right. \\
 & \quad \left( 2(ad+b(c+2dx)) \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \\
 & \quad \left. (bc-ad)n \left( \operatorname{AppellF1}\left[2, 1-n, n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \\
 & \quad \left. \left. \operatorname{AppellF1}\left[2, -n, 1+n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \Big)
 \end{aligned}$$

**Problem 3127: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^{1-n} (c+dx)^{1+n}}{(bc+ad+2bdx)^3} dx$$

Optimal (type 5, 230 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{(bc - ad)(a + bx)^{1-n}(c + dx)^n}{8b^2d(bc + ad + 2bdx)^2} - \frac{(1 + 2n)(a + bx)^{1-n}(c + dx)^n}{8b^2d(bc + ad + 2bdx)} - \frac{1}{8b^2d^2n} \\
 & (1 - 2n^2)(a + bx)^{-n}(c + dx)^n \text{Hypergeometric2F1}\left[1, n, 1 + n, -\frac{b(c + dx)}{d(a + bx)}\right] + \frac{1}{8b^2d^2n} \\
 & (a + bx)^{-n} \left(-\frac{d(a + bx)}{bc - ad}\right)^n (c + dx)^n \text{Hypergeometric2F1}\left[n, n, 1 + n, \frac{b(c + dx)}{bc - ad}\right]
 \end{aligned}$$

Result (type 6, 1027 leaves):

$$\begin{aligned}
 & \frac{1}{16 (a d + b (c + 2 d x))} \\
 & (a + b x)^{-n} (c + d x)^n \left( \left( 3 a^2 \operatorname{AppellF1} \left[ 2, -n, n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) / \right. \\
 & \left( b^2 \left( 3 (a d + b (c + 2 d x)) \operatorname{AppellF1} \left[ 2, -n, n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \right. \right. \\
 & \left. (b c - a d) n \left( \operatorname{AppellF1} \left[ 3, 1 - n, n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ 3, -n, 1 + n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) \right) \right) + \\
 & \left( 3 c^2 \operatorname{AppellF1} \left[ 2, -n, n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) / \\
 & \left( d^2 \left( 3 (a d + b (c + 2 d x)) \operatorname{AppellF1} \left[ 2, -n, n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \right. \right. \\
 & \left. (b c - a d) n \left( \operatorname{AppellF1} \left[ 3, 1 - n, n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ 3, -n, 1 + n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) \right) \right) - \\
 & \left( 6 a c \operatorname{AppellF1} \left[ 2, -n, n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) / \\
 & \left( b d \left( 3 (a d + b (c + 2 d x)) \operatorname{AppellF1} \left[ 2, -n, n, 3, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \right. \right. \\
 & \left. (b c - a d) n \left( \operatorname{AppellF1} \left[ 3, 1 - n, n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] + \right. \right. \\
 & \left. \left. \operatorname{AppellF1} \left[ 3, -n, 1 + n, 4, \frac{-b c + a d}{a d + b (c + 2 d x)}, \frac{b c - a d}{b c + a d + 2 b d x} \right] \right) \right) \right) + \\
 & \left( 4 (b c - a d) (2 + n) (c + d x) \operatorname{AppellF1} \left[ 1 + n, n, 1, 2 + n, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] \right) / \\
 & \left( b d^2 (1 + n) \left( (b c - a d) (2 + n) \operatorname{AppellF1} \left[ 1 + n, n, 1, 2 + n, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] + \right. \right. \\
 & \left. b (c + d x) \left( 2 \operatorname{AppellF1} \left[ 2 + n, n, 2, 3 + n, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] + \right. \right. \\
 & \left. \left. n \operatorname{AppellF1} \left[ 2 + n, 1 + n, 1, 3 + n, \frac{b (c + d x)}{b c - a d}, \frac{2 b (c + d x)}{b c - a d} \right] \right) \right) \right) \right)
 \end{aligned}$$

**Problem 3128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^{1-n} (c+dx)^{1+n}}{(bc+ad+2bdx)^4} dx$$

Optimal (type 5, 71 leaves, 1 step):

$$\frac{(a+bx)^{2-n} (c+dx)^{-2+n} \operatorname{Hypergeometric2F1}\left[4, 2-n, 3-n, -\frac{d(a+bx)}{b(c+dx)}\right]}{b^4 (bc-ad) (2-n)}$$

Result (type 6, 543 leaves):

$$\begin{aligned} & \frac{1}{12b^2d^2} (a+bx)^{-n} (c+dx)^n \left( - \left( \left( \left( 3 \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \right) / \right. \\ & \left( 2(ad+b(c+2dx)) \operatorname{AppellF1}\left[1, -n, n, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \\ & (bc-ad)n \left( \operatorname{AppellF1}\left[2, 1-n, n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \\ & \left. \left. \operatorname{AppellF1}\left[2, -n, 1+n, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \left. \right) + \\ & \left( 2(bc-ad)^2 \operatorname{AppellF1}\left[3, -n, n, 4, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) / \\ & \left( (ad+b(c+2dx))^2 \right. \\ & \left( 4(ad+b(c+2dx)) \operatorname{AppellF1}\left[3, -n, n, 4, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \\ & (bc-ad)n \left( \operatorname{AppellF1}\left[4, 1-n, n, 5, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \\ & \left. \left. \operatorname{AppellF1}\left[4, -n, 1+n, 5, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \left. \right) \end{aligned}$$

**Problem 3129: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{bc+ad+2bdx} dx$$

Optimal (type 5, 231 leaves, 6 steps):

$$\frac{(bc - ad)(1 + 2m)(a + bx)^{1+m}(c + dx)^{-m}}{8b^3m} + \frac{d(a + bx)^{2+m}(c + dx)^{-m}}{4b^3} + \frac{1}{8b^3dm}$$

$$(bc - ad)^2(a + bx)^m(c + dx)^{-m} \text{Hypergeometric2F1}\left[1, -m, 1 - m, -\frac{b(c + dx)}{d(a + bx)}\right] -$$

$$\frac{1}{8b^3m(1 + m)}(bc - ad)(1 - 4m + 2m^2)(a + bx)^{1+m}(c + dx)^{-m}$$

$$\left(\frac{b(c + dx)}{bc - ad}\right)^m \text{Hypergeometric2F1}\left[m, 1 + m, 2 + m, -\frac{d(a + bx)}{bc - ad}\right]$$

Result (type 6, 269 leaves):

$$- \left( \left( (bc - ad)(2 + m)(a + bx)^{1+m}(c + dx)^{2-m} \right. \right.$$

$$\left. \text{AppellF1}\left[1 + m, -2 + m, 1, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{2d(a + bx)}{-bc + ad}\right] \right) / \left( b(1 + m)(ad + b(c + 2dx)) \right)$$

$$\left( - (bc - ad)(2 + m) \text{AppellF1}\left[1 + m, -2 + m, 1, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{2d(a + bx)}{-bc + ad}\right] + \right.$$

$$d(a + bx) \left( 2 \text{AppellF1}\left[2 + m, -2 + m, 2, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{2d(a + bx)}{-bc + ad}\right] + \right.$$

$$\left. \left. \left. (-2 + m) \text{AppellF1}\left[2 + m, -1 + m, 1, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{2d(a + bx)}{-bc + ad}\right] \right) \right) \right)$$

Problem 3130: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a + bx)^m(c + dx)^{2-m}}{(bc + ad + 2bdx)^2} dx$$

Optimal (type 5, 144 leaves, 4 steps):

$$-\frac{1}{4b^3dm}(bc - ad)(a + bx)^m(c + dx)^{-m} \text{Hypergeometric2F1}\left[2, m, 1 + m, -\frac{d(a + bx)}{b(c + dx)}\right] + \frac{1}{4b^3dm}$$

$$(bc - ad)(a + bx)^m(c + dx)^{-m} \left(\frac{b(c + dx)}{bc - ad}\right)^m \text{Hypergeometric2F1}\left[-1 + m, m, 1 + m, -\frac{d(a + bx)}{bc - ad}\right]$$

Result (type 6, 1377 leaves):

$$\frac{1}{4b^3}(a + bx)^m(c + dx)^{-m} \left( \left( 2abc \text{AppellF1}\left[1, m, -m, 2, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}\right] \right) / \right.$$

$$\left( 2(ad + b(c + 2dx)) \text{AppellF1}\left[1, m, -m, 2, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}\right] - \right.$$

$$\left. \left. (bc - ad)m \left( \text{AppellF1}\left[2, m, 1 - m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx}\right] + \right. \right.$$

$$\begin{aligned}
 & \left. \left. \left. \text{AppellF1}\left[2, 1+m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right]\right)\right) - \\
 & \left( b^2 c^2 \text{AppellF1}\left[1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right]\right) / \\
 & \left( d \left( 2 (ad+b(c+2dx)) \text{AppellF1}\left[1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] - \right. \right. \\
 & \quad \left. \left. (bc-ad) m \left( \text{AppellF1}\left[2, m, 1-m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \text{AppellF1}\left[2, 1+m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right]\right)\right)\right) - \\
 & \left( a^2 d \text{AppellF1}\left[1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right]\right) / \\
 & \left( 2 (ad+b(c+2dx)) \text{AppellF1}\left[1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] - \right. \\
 & \quad \left. (bc-ad) m \left( \text{AppellF1}\left[2, m, 1-m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \\
 & \quad \left. \left. \text{AppellF1}\left[2, 1+m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right]\right)\right) - \\
 & \left( 2 b^2 c (bc-ad) (-2+m) (c+dx) \text{AppellF1}\left[1-m, -m, 1, 2-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right]\right) / \\
 & \left( d (-1+m) (ad+b(c+2dx)) \right. \\
 & \quad \left( (bc-ad) (-2+m) \text{AppellF1}\left[1-m, -m, 1, 2-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right] + \right. \\
 & \quad \left. b(c+dx) \left( m \text{AppellF1}\left[2-m, 1-m, 1, 3-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right] - \right. \right. \\
 & \quad \left. \left. 2 \text{AppellF1}\left[2-m, -m, 2, 3-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right]\right)\right) - \\
 & \left( 2 a b (-bc+ad) (-2+m) (c+dx) \text{AppellF1}\left[1-m, -m, 1, 2-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right]\right) / \\
 & \left( (-1+m) (ad+b(c+2dx)) \right. \\
 & \quad \left( (bc-ad) (-2+m) \text{AppellF1}\left[1-m, -m, 1, 2-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right] + \right. \\
 & \quad \left. b(c+dx) \left( m \text{AppellF1}\left[2-m, 1-m, 1, 3-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right] - \right. \right. \\
 & \quad \left. \left. 2 \text{AppellF1}\left[2-m, -m, 2, 3-m, \frac{b(c+dx)}{bc-ad}, \frac{2b(c+dx)}{bc-ad}\right]\right)\right) +
 \end{aligned}$$

$$\frac{(a+bx) \left( \frac{b(c+dx)}{bc-ad} \right)^m \text{Hypergeometric2F1} \left[ m, 1+m, 2+m, \frac{d(a+bx)}{-bc+ad} \right]}{1+m}$$

**Problem 3131: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{(bc+ad+2bdx)^3} dx$$

Optimal (type 5, 261 leaves, 7 steps):

$$\frac{(bc-ad)(a+bx)^{-1+m}(c+dx)^{2-m}}{8bd^2(bc+ad+2bdx)^2} + \frac{(1-2m)(a+bx)^{-1+m}(c+dx)^{2-m}}{8bd^2(bc+ad+2bdx)} - \frac{1}{8b^2d^2(1-m)}$$

$$(1-4m+2m^2)(a+bx)^{-1+m}(c+dx)^{1-m} \text{Hypergeometric2F1} \left[ 1, -1+m, m, -\frac{d(a+bx)}{b(c+dx)} \right] -$$

$$\frac{1}{8b^3d^2(1-m)}(bc-ad)(a+bx)^{-1+m}(c+dx)^{-m}$$

$$\left( \frac{b(c+dx)}{bc-ad} \right)^m \text{Hypergeometric2F1} \left[ -1+m, -1+m, m, -\frac{d(a+bx)}{bc-ad} \right]$$

Result (type 6, 1593 leaves):

$$\frac{1}{16b^3} (a+bx)^m (c+dx)^{-m} \left( \left( 8a \text{AppellF1} \left[ 1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] \right) / \right.$$

$$\left( 2(ad+b(c+2dx)) \text{AppellF1} \left[ 1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] - \right.$$

$$(bc-ad)m \left( \text{AppellF1} \left[ 2, m, 1-m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] + \right.$$

$$\left. \left. \text{AppellF1} \left[ 2, 1+m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] \right) \right) -$$

$$\left( 8bc \text{AppellF1} \left[ 1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] \right) /$$

$$\left( d \left( 2(ad+b(c+2dx)) \text{AppellF1} \left[ 1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] - \right.$$

$$(bc-ad)m \left( \text{AppellF1} \left[ 2, m, 1-m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] + \right.$$

$$\left. \left. \text{AppellF1} \left[ 2, 1+m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] \right) \right) \right) +$$

$$\left( 6abc \text{AppellF1} \left[ 2, m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] \right) / \left( (ad+b(c+2dx)) \right.$$

$$\left. \left( 3(ad+b(c+2dx)) \text{AppellF1} \left[ 2, m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx} \right] - \right.$$

$$\begin{aligned}
 & (bc - ad) m \left( \text{AppellF1} \left[ 3, m, 1 - m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] + \right. \\
 & \quad \left. \text{AppellF1} \left[ 3, 1 + m, -m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] \right) - \\
 & \left( 3b^2 c^2 \text{AppellF1} \left[ 2, m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] \right) / \\
 & \left( d(ad + b(c + 2dx)) \right) \\
 & \left( 3(ad + b(c + 2dx)) \text{AppellF1} \left[ 2, m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] - \right. \\
 & \quad (bc - ad) m \left( \text{AppellF1} \left[ 3, m, 1 - m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] + \right. \\
 & \quad \left. \text{AppellF1} \left[ 3, 1 + m, -m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] \right) - \\
 & \left( 3a^2 d \text{AppellF1} \left[ 2, m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] \right) / \\
 & \left( (ad + b(c + 2dx)) \right) \\
 & \left( 3(ad + b(c + 2dx)) \text{AppellF1} \left[ 2, m, -m, 3, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] - \right. \\
 & \quad (bc - ad) m \left( \text{AppellF1} \left[ 3, m, 1 - m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] + \right. \\
 & \quad \left. \text{AppellF1} \left[ 3, 1 + m, -m, 4, \frac{-bc + ad}{ad + b(c + 2dx)}, \frac{bc - ad}{bc + ad + 2bdx} \right] \right) + \\
 & \left( 4b(-bc + ad)(-2 + m)(c + dx) \text{AppellF1} \left[ 1 - m, -m, 1, 2 - m, \frac{b(c + dx)}{bc - ad}, \frac{2b(c + dx)}{bc - ad} \right] \right) / \\
 & \left( d(-1 + m)(ad + b(c + 2dx)) \right) \\
 & \left( (bc - ad)(-2 + m) \text{AppellF1} \left[ 1 - m, -m, 1, 2 - m, \frac{b(c + dx)}{bc - ad}, \frac{2b(c + dx)}{bc - ad} \right] + \right. \\
 & \quad b(c + dx) \left( m \text{AppellF1} \left[ 2 - m, 1 - m, 1, 3 - m, \frac{b(c + dx)}{bc - ad}, \frac{2b(c + dx)}{bc - ad} \right] - \right. \\
 & \quad \left. \left. 2 \text{AppellF1} \left[ 2 - m, -m, 2, 3 - m, \frac{b(c + dx)}{bc - ad}, \frac{2b(c + dx)}{bc - ad} \right] \right) \right)
 \end{aligned}$$

**Problem 3132:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{(a+bx)^m (c+dx)^{2-m}}{(bc+ad+2bdx)^4} dx$$

Optimal (type 5, 65 leaves, 1 step):

$$\frac{(a+bx)^{1+m} (c+dx)^{-1-m} \text{Hypergeometric2F1}\left[4, 1+m, 2+m, -\frac{d(a+bx)}{b(c+dx)}\right]}{b^4 (bc-ad) (1+m)}$$

Result (type 6, 812 leaves):

$$\frac{1}{24 b^3 d} (a+bx)^m (c+dx)^{-m} \left( - \left( \left( 6 \text{AppellF1}\left[1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) / \left( 2(ad+b(c+2dx)) \right) \right. \right. \\ \left. \left. \text{AppellF1}\left[1, m, -m, 2, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] - (bc-ad)m \right. \right. \\ \left. \left. \left( \text{AppellF1}\left[2, m, 1-m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \text{AppellF1}\left[2, 1+m, \right. \right. \right. \\ \left. \left. \left. -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \right) + \frac{1}{(ad+b(c+2dx))^2} (bc-ad) \\ \left( - \left( \left( 9(ad+b(c+2dx)) \text{AppellF1}\left[2, m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) / \right. \right. \\ \left. \left( 3(ad+b(c+2dx)) \text{AppellF1}\left[2, m, -m, 3, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] - \right. \right. \\ \left. \left. (bc-ad)m \left( \text{AppellF1}\left[3, m, 1-m, 4, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \right. \\ \left. \left. \left. \text{AppellF1}\left[3, 1+m, -m, 4, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \right) - \\ \left( 4(bc-ad) \text{AppellF1}\left[3, m, -m, 4, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) / \\ \left( 4(ad+b(c+2dx)) \text{AppellF1}\left[3, m, -m, 4, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] - \right. \\ \left. (bc-ad)m \left( \text{AppellF1}\left[4, m, 1-m, 5, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] + \right. \right. \\ \left. \left. \left. \text{AppellF1}\left[4, 1+m, -m, 5, \frac{-bc+ad}{ad+b(c+2dx)}, \frac{bc-ad}{bc+ad+2bdx}\right] \right) \right) \right)$$

**Problem 3133: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^{n+p} dx$$

Optimal (type 6, 139 leaves, 3 steps):

$$\frac{1}{b(1+m)} (a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{b(c+dx)}{bc-ad} \right)^{m+n} (e+fx)^{n+p} \left( \frac{b(e+fx)}{be-af} \right)^{-n-p} \text{AppellF1} \left[ 1+m, m+n, -n-p, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right]$$

Result (type 6, 323 leaves):

$$\left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^{-m-n} (e+fx)^{n+p} \text{AppellF1} \left[ 1+m, m+n, -n-p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(1+m) \left( (bc-ad)(be-af)(2+m) \text{AppellF1} \left[ 1+m, m+n, -n-p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - (a+bx) \left( - (bc-ad) f(n+p) \text{AppellF1} \left[ 2+m, m+n, 1-n-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + d(be-af)(m+n) \text{AppellF1} \left[ 2+m, 1+m+n, -n-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right)$$

**Problem 3134: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^{1+n} dx$$

Optimal (type 6, 139 leaves, 3 steps):

$$\frac{1}{b^2(1+m)} (be-af)(a+bx)^{1+m}(c+dx)^{-m-n} \left( \frac{b(c+dx)}{bc-ad} \right)^{m+n} (e+fx)^n \left( \frac{b(e+fx)}{be-af} \right)^{-n} \text{AppellF1} \left[ 1+m, m+n, -1-n, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right]$$

Result (type 6, 312 leaves):

$$\left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^{-m-n}(e+fx)^{1+n} \text{AppellF1} \left[ 1+m, m+n, -1-n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(1+m) \left( (bc-ad)(be-af)(2+m) \text{AppellF1} \left[ 1+m, m+n, -1-n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - (a+bx) \left( - (bc-ad) f(1+n) \text{AppellF1} \left[ 2+m, m+n, -n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + d(be-af)(m+n) \text{AppellF1} \left[ 2+m, 1+m+n, -1-n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right)$$

### Problem 3135: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^n dx$$

Optimal (type 6, 129 leaves, 3 steps):

$$\frac{1}{b(1+m)} (a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{b(c+dx)}{bc-ad} \right)^{m+n} (e+fx)^n$$

$$\left( \frac{b(e+fx)}{be-af} \right)^{-n} \text{AppellF1} \left[ 1+m, m+n, -n, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right]$$

Result (type 6, 303 leaves):

$$\left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^{-m-n} \right. \\ \left. (e+fx)^n \text{AppellF1} \left[ 1+m, m+n, -n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \\ \left( b(1+m) \left( (bc-ad)(be-af)(2+m) \text{AppellF1} \left[ 1+m, m+n, -n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \right. \\ \left. (a+bx) \left( (-bc+ad)fn \text{AppellF1} \left[ 2+m, m+n, 1-n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \\ \left. \left. d(be-af)(m+n) \text{AppellF1} \left[ 2+m, 1+m+n, -n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right)$$

### Problem 3136: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^{-1+n} dx$$

Optimal (type 6, 138 leaves, 3 steps):

$$\left( (a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{b(c+dx)}{bc-ad} \right)^{m+n} (e+fx)^n \left( \frac{b(e+fx)}{be-af} \right)^{-n} \right. \\ \left. \text{AppellF1} \left[ 1+m, m+n, 1-n, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right] \right) / ((be-af)(1+m))$$

Result (type 6, 315 leaves):

$$\begin{aligned}
 & - \left( \left( (bc - ad) (be - af) (2+m) (a+bx)^{1+m} (c+dx)^{-m-n} (e+fx)^{-1+n} \right. \right. \\
 & \quad \left. \text{AppellF1} \left[ 1+m, m+n, 1-n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(1+m) \left( - (bc - ad) \right. \right. \\
 & \quad \left. (be - af) (2+m) \text{AppellF1} \left[ 1+m, m+n, 1-n, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + (a+bx) \right. \\
 & \quad \left. \left( - (bc - ad) f(-1+n) \text{AppellF1} \left[ 2+m, m+n, 2-n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \\
 & \quad \left. \left. d (be - af) (m+n) \text{AppellF1} \left[ 2+m, 1+m+n, 1-n, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right)
 \end{aligned}$$

**Problem 3138: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^{-3+n} dx$$

Optimal (type 5, 237 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{f(a+bx)^{1+m} (c+dx)^{1-m-n} (e+fx)^{-2+n}}{(be-af)(de-cf)(2-n)} - \\
 & \left( (adf(1+m) - b(de(2-n) - cf(1-m-n))) (a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{(be-af)(c+dx)}{(bc-ad)(e+fx)} \right)^{m+n} \right. \\
 & \quad \left. (e+fx)^{-1+n} \text{Hypergeometric2F1} \left[ 1+m, m+n, 2+m, -\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)} \right] \right) / \\
 & \left( (be-af)^2 (de-cf)(1+m)(2-n) \right)
 \end{aligned}$$

Result (type 5, 5197 leaves):

$$\begin{aligned}
 & \left( (a+bx)^{1+2m} (c+dx)^{-2m-2n} \left( \frac{-bc-bdx}{-bc+ad} \right)^{m+n} (e+fx)^{-6+2n} \left( \frac{-be-bfx}{-be+af} \right)^{3-n} \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^{-m-n} \right. \\
 & \quad \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-2+n} \text{Gamma}[2+m] \left( \frac{2 \text{Hypergeometric2F1} \left[ 1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{m \text{Hypergeometric2F1} \left[ 1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{f(a+bx) \text{Hypergeometric2F1} \left[ 1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{(be-af) \text{Gamma}[3+m]} + \right. \\
 & \quad \left. \left( (de-cf)(a+bx) \text{Gamma}[1+m+n] \text{Hypergeometric2F1} \left[ 2, 1+m+n, 4+m, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) \right) / \left( (be-af)(c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n] \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( f(-de+cf)(a+bx)^2 \text{Gamma}[1+m+n] \text{Hypergeometric2F1}\left[2, 1+m+n, 4+m, \right. \right. \\
 & \quad \left. \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \left( (be-af)^2 (c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n] \right) \Bigg) / \\
 & \left( b(1+m) \left( -\frac{1}{(-be+af)(1+m)} f(-2+n)(a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{-bc-bdx}{-bc+ad} \right)^{m+n} \right. \right. \\
 & \quad \left. (e+fx)^{-3+n} \left( \frac{-be-bfx}{-be+af} \right)^{3-n} \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^{-m-n} \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-3+n} \right. \\
 & \quad \left. \text{Gamma}[2+m] \left[ \frac{2 \text{Hypergeometric2F1}\left[1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \right. \\
 & \quad \left. \left. \frac{m \text{Hypergeometric2F1}\left[1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \right. \\
 & \quad \left. \left. \left( f(a+bx) \text{Hypergeometric2F1}\left[1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \left( (be-af) \right. \right. \\
 & \quad \left. \left. \text{Gamma}[3+m] \right) + \left( (de-cf)(a+bx) \text{Gamma}[1+m+n] \text{Hypergeometric2F1}\left[2, 1+m+n, \right. \right. \right. \\
 & \quad \left. \left. \left. 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \left( (be-af)(c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n] \right) - \right. \\
 & \quad \left. \left( f(-de+cf)(a+bx)^2 \text{Gamma}[1+m+n] \text{Hypergeometric2F1}\left[2, 1+m+n, 4+m, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \left( (be-af)^2 (c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n] \right) \right) - \\
 & \frac{1}{(-bc+ad)(1+m)} d(-m-n)(a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{-bc-bdx}{-bc+ad} \right)^{m+n} (e+fx)^{-3+n} \\
 & \quad \left( \frac{-be-bfx}{-be+af} \right)^{3-n} \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^{-1-m-n} \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-2+n} \\
 & \quad \text{Gamma}[2+m] \left[ \frac{2 \text{Hypergeometric2F1}\left[1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{m \text{Hypergeometric2F1}\left[1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{f(a+bx) \text{Hypergeometric2F1}\left[1, m+n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af) \text{Gamma}[3+m]} + \right. \\
 & \quad \left. \left( (de-cf)(a+bx) \text{Gamma}[1+m+n] \text{Hypergeometric2F1}\left[2, 1+m+n, 4+m, \right. \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right) \Big/ \left( (be - af)(c + dx) \Gamma[4 + m] \Gamma[m + n] \right) - \\
 & \left( f(-de + cf)(a + bx)^2 \Gamma[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \right. \right. \\
 & \left. \left. \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) \Big/ \left( (be - af)^2 (c + dx) \Gamma[4 + m] \Gamma[m + n] \right) - \\
 & \frac{1}{(-be + af)(1 + m)} f(3 - n)(a + bx)^{1+m} (c + dx)^{-m-n} \left( \frac{-bc - bdx}{-bc + ad} \right)^{m+n} (e + fx)^{-3+n} \\
 & \left( \frac{-be - bfx}{-be + af} \right)^{2-n} \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^{-m-n} \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2+n} \\
 & \Gamma[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{f(a + bx) \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{(be - af) \Gamma[3 + m]} + \right. \\
 & \left( (de - cf)(a + bx) \Gamma[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \right. \right. \\
 & \left. \left. \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) \Big/ \left( (be - af)(c + dx) \Gamma[4 + m] \Gamma[m + n] \right) - \\
 & \left( f(-de + cf)(a + bx)^2 \Gamma[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \right. \right. \\
 & \left. \left. \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) \Big/ \left( (be - af)^2 (c + dx) \Gamma[4 + m] \Gamma[m + n] \right) + \\
 & \frac{1}{b(1 + m)} f(-3 + n)(a + bx)^{1+m} (c + dx)^{-m-n} \left( \frac{-bc - bdx}{-bc + ad} \right)^{m+n} (e + fx)^{-4+n} \\
 & \left( \frac{-be - bfx}{-be + af} \right)^{3-n} \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^{-m-n} \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2+n} \\
 & \Gamma[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{f(a + bx) \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{(be - af) \Gamma[3 + m]} + \right. \\
 & \left( (de - cf)(a + bx) \Gamma[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right) / \left( (be - af)(c + dx) \Gamma[4 + m] \Gamma[m + n] \right) - \\
 & \left( f(-de + cf)(a + bx)^2 \Gamma[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \right. \right. \\
 & \left. \left. \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \left( (be - af)^2 (c + dx) \Gamma[4 + m] \Gamma[m + n] \right) \Bigg) - \\
 & \frac{1}{(-bc + ad)(1 + m)} d^{(m+n)} (a + bx)^{1+m} (c + dx)^{-m-n} \left( \frac{-bc - bdx}{-bc + ad} \right)^{-1+m+n} \\
 & (e + fx)^{-3+n} \left( \frac{-be - bfx}{-be + af} \right)^{3-n} \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^{-m-n} \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2+n} \\
 & \Gamma[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{f(a + bx) \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{(be - af) \Gamma[3 + m]} + \right. \\
 & \left( (de - cf)(a + bx) \Gamma[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \right. \right. \\
 & \left. \left. \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \left( (be - af)(c + dx) \Gamma[4 + m] \Gamma[m + n] \right) - \\
 & \left( f(-de + cf)(a + bx)^2 \Gamma[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \right. \right. \\
 & \left. \left. \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \left( (be - af)^2 (c + dx) \Gamma[4 + m] \Gamma[m + n] \right) \Bigg) + \\
 & \frac{1}{b(1 + m)} d^{(-m-n)} (a + bx)^{1+m} (c + dx)^{-1-m-n} \left( \frac{-bc - bdx}{-bc + ad} \right)^{m+n} (e + fx)^{-3+n} \\
 & \left( \frac{-be - bfx}{-be + af} \right)^{3-n} \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^{-m-n} \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2+n} \\
 & \Gamma[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\Gamma[3 + m]} + \right. \\
 & \left. \frac{f(a + bx) \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{(be - af) \Gamma[3 + m]} + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( (de - cf) (a + bx) \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)(c + dx) \text{Gamma}[4 + m] \text{Gamma}[m + n] \right) - \\
 & \left( f(-de + cf) (a + bx)^2 \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)^2 (c + dx) \text{Gamma}[4 + m] \text{Gamma}[m + n] \right) \Bigg) + \\
 & (a + bx)^m (c + dx)^{-m-n} \left( \frac{-bc - bdx}{-bc + ad} \right)^{m+n} (e + fx)^{-3+n} \left( \frac{-be - bfx}{-be + af} \right)^{3-n} \\
 & \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^{-m-n} \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2+n} \text{Gamma}[2 + m] \\
 & \left( \frac{2 \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\text{Gamma}[3 + m]} + \right. \\
 & \left. m \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \text{Gamma}[3 + m] + \\
 & \frac{f(a + bx) \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{(be - af) \text{Gamma}[3 + m]} + \\
 & \left( (de - cf) (a + bx) \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)(c + dx) \text{Gamma}[4 + m] \text{Gamma}[m + n] \right) - \\
 & \left( f(-de + cf) (a + bx)^2 \text{Gamma}[1 + m + n] \text{Hypergeometric2F1}\left[2, 1 + m + n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)^2 (c + dx) \text{Gamma}[4 + m] \text{Gamma}[m + n] \right) \Bigg) + \\
 & \frac{1}{b(1 + m)} (a + bx)^{1+m} (c + dx)^{-m-n} \left( \frac{-bc - bdx}{-bc + ad} \right)^{m+n} (e + fx)^{-3+n} \left( \frac{-be - bfx}{-be + af} \right)^{3-n} \\
 & \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^{-m-n} \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2+n} \text{Gamma}[2 + m] \\
 & \left( \frac{bf \text{Hypergeometric2F1}\left[1, m + n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{(be - af) \text{Gamma}[3 + m]} + \right. \\
 & \left. \left( 2(m + n) \left( -\frac{d(de - cf)(a + bx)}{(be - af)(c + dx)^2} + \frac{b(de - cf)}{(be - af)(c + dx)} \right) \text{Hypergeometric2F1}\left[ \right. \right. \right. \\
 & \left. \left. \left. 2, 1 + m + n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)} \right] \right) / \left( (3 + m) \text{Gamma}[3 + m] \right) + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( m(m+n) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \text{Hypergeometric2F1} \left[ \right. \right. \\
 & \quad \left. \left. 2, 1+m+n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / ((3+m) \text{Gamma}[3+m]) + \\
 & \left( f(m+n)(a+bx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \text{Hypergeometric2F1} \left[ \right. \right. \\
 & \quad \left. \left. 2, 1+m+n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / ((be-af)(3+m) \text{Gamma}[3+m]) - \\
 & \left( d(de-cf)(a+bx) \text{Gamma}[1+m+n] \text{Hypergeometric2F1} \left[ 2, 1+m+n, 4+m, \right. \right. \\
 & \quad \left. \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / ((be-af)(c+dx)^2 \text{Gamma}[4+m] \text{Gamma}[m+n]) + \\
 & \left( df(-de+cf)(a+bx)^2 \text{Gamma}[1+m+n] \text{Hypergeometric2F1} \left[ 2, 1+m+n, \right. \right. \\
 & \quad \left. \left. 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / ((be-af)^2(c+dx)^2 \text{Gamma}[4+m] \text{Gamma}[m+n]) + \\
 & \left( b(de-cf) \text{Gamma}[1+m+n] \text{Hypergeometric2F1} \left[ 2, 1+m+n, 4+m, \right. \right. \\
 & \quad \left. \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / ((be-af)(c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n]) - \\
 & \left( 2bf(-de+cf)(a+bx) \text{Gamma}[1+m+n] \text{Hypergeometric2F1} \left[ 2, 1+m+n, \right. \right. \\
 & \quad \left. \left. 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / ((be-af)^2(c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n]) + \\
 & \left( 2(de-cf)(1+m+n)(a+bx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \right. \\
 & \quad \left. \text{Gamma}[1+m+n] \text{Hypergeometric2F1} \left[ 3, 2+m+n, 5+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \\
 & \quad ((be-af)(4+m)(c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n]) - \\
 & \left( 2f(-de+cf)(1+m+n)(a+bx)^2 \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \right. \\
 & \quad \left. \text{Gamma}[1+m+n] \text{Hypergeometric2F1} \left[ 3, 2+m+n, 5+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \\
 & \quad ((be-af)^2(4+m)(c+dx) \text{Gamma}[4+m] \text{Gamma}[m+n]) \Big) \Big) \Big)
 \end{aligned}$$

Problem 3139: Attempted integration timed out after 120 seconds.

$$\int (a+bx)^m (c+dx)^{-m-n} (e+fx)^{-4+n} dx$$

Optimal (type 5, 428 leaves, 4 steps):

$$\begin{aligned} & - \frac{f (a+bx)^{1+m} (c+dx)^{1-m-n} (e+fx)^{-3+n}}{(be-af)(de-cf)(3-n)} + \\ & \left( f(adf(2+m) - b(de(4-n) - cf(2-m-n))) (a+bx)^{1+m} (c+dx)^{1-m-n} (e+fx)^{-2+n} \right) / \\ & \left( (be-af)^2 (de-cf)^2 (2-n)(3-n) \right) + \\ & \left( (a^2 d^2 f^2 (2+3m+m^2) - 2abdf(1+m)(de(3-n) - cf(1-m-n)) - \right. \\ & \quad \left. b^2 (2cdef(3-n)(1-m-n) - d^2 e^2 (6-5n+n^2) - c^2 f^2 (2+m^2 - m(3-2n) - 3n+n^2)) \right) \\ & (a+bx)^{1+m} (c+dx)^{-m-n} \left( \frac{(be-af)(c+dx)^{m+n}}{(bc-ad)(e+fx)} (e+fx)^{-1+n} \right. \\ & \quad \left. \text{Hypergeometric2F1}\left[1+m, m+n, 2+m, -\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)}\right] \right) / \\ & \left( (be-af)^3 (de-cf)^2 (1+m)(2-n)(3-n) \right) \end{aligned}$$

Result (type 1, 1 leaves):

???

**Problem 3140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a+bx)^m (c+dx)^n \left( \frac{bcf+adf+adf m+bcfn}{bd(2+m+n)} + fx \right)^{-3-m-n} dx$$

Optimal (type 3, 88 leaves, 1 step):

$$\frac{bd(2+m+n)(a+bx)^{1+m}(c+dx)^{1+n} \left( \frac{f(ad(1+m)+bc(1+n))}{bd(2+m+n)} + fx \right)^{-2-m-n}}{(bc-ad)^2 f(1+m)(1+n)}$$

Result (type 5, 5681 leaves):

$$\begin{aligned} & \left( (a+bx)^{1+2m} (c+dx)^{2n} \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \left( -\frac{-bc-ad-adm-bcn-2bdx-bdmx-bdnx}{(bc-ad)(1+n)} \right)^{3+m+n} \right. \\ & \quad \left. (bc+ad+adm+bcn+2bdx+bdmx+bdnx)^{-3-m-n} \right. \\ & \quad \left. \left( \frac{f(ad(1+m)+bc(1+n)+bd(2+m+n)x)}{bd(2+m+n)} \right)^{-3-m-n} \right. \\ & \quad \left. \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \right. \\ & \quad \left. \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\text{Gamma}[3+m]} \right) + \right. \end{aligned}$$

$$\begin{aligned}
 & \frac{m \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \\
 & \left( d(2+m+n)(a+bx) \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \\
 & \left( (bc-ad)(1+n)\Gamma[3+m] - \right. \\
 & \left. \left( d(1+m)(a+bx)\Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. (b(1+n)(c+dx)\Gamma[4+m]\Gamma[-n]) - \left( d^2(1+m)(2+m+n)(a+bx)^2 \right. \right. \\
 & \left. \left. \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left. (b(bc-ad)(1+n)^2(c+dx)\Gamma[4+m]\Gamma[-n]) \right) \right) / \\
 & \left( b(1+m) \left( \frac{1}{(bc-ad)(1+m)(1+n)} d(-2-m-n)(2+m+n)(a+bx)^{1+m}(c+dx)^n \right. \right. \\
 & \left. \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \left( -\frac{-bc-ad-adm-bcn-2bdx-bdmx-bdnx}{(bc-ad)(1+n)} \right)^{3+m+n} \right. \\
 & \left. (bc+ad+adm+bcn+2bdx+bdmx+bdnx)^{-3-m-n} \right. \\
 & \left. \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-3-m-n} \Gamma[2+m] \right. \\
 & \left. \frac{2 \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \frac{m \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \left( d(2+m+n)(a+bx) \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \\
 & \left( (bc-ad)(1+n)\Gamma[3+m] - \left( d(1+m)(a+bx)\Gamma[1-n] \right. \right. \\
 & \left. \left. \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. (b(1+n)(c+dx)\Gamma[4+m]\Gamma[-n]) - \left( d^2(1+m)(2+m+n)(a+bx)^2 \right. \right. \\
 & \left. \left. \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left. (b(bc-ad)(1+n)^2(c+dx)\Gamma[4+m]\Gamma[-n]) \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{(-bc+ad)(1+m)} d^n (a+bx)^{1+m} (c+dx)^n \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \\
 & \left( -\frac{-bc-ad-adm-bcn-2bdx-bdmx-bdnx}{(bc-ad)(1+n)} \right)^{3+m+n} \\
 & (bc+ad+adm+bcn+2bdx+bdmx+bdnx)^{-3-m-n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^{-1+n} \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \left( d(2+m+n)(a+bx) \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left( (bc-ad)(1+n) \Gamma[3+m] \right) - \left( d(1+m)(a+bx) \Gamma[1-n] \right. \right. \\
 & \left. \left. \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left( b(1+n)(c+dx) \Gamma[4+m] \Gamma[-n] \right) - \left( d^2(1+m)(2+m+n)(a+bx)^2 \right. \right. \\
 & \left. \left. \Gamma[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left( b(bc-ad)(1+n)^2(c+dx) \Gamma[4+m] \Gamma[-n] \right) \right) + \\
 & \frac{1}{b(1+m)} (-3-m-n)(2bd+bdm+bdn)(a+bx)^{1+m} (c+dx)^n \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \\
 & \left( -\frac{-bc-ad-adm-bcn-2bdx-bdmx-bdnx}{(bc-ad)(1+n)} \right)^{3+m+n} \\
 & (bc+ad+adm+bcn+2bdx+bdmx+bdnx)^{-4-m-n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \left( d(2+m+n)(a+bx) \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( (bc - ad) (1+n) \Gamma[3+m] \right) - \left( d (1+m) (a+bx) \Gamma[1-n] \right. \\
 & \quad \left. \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) / \\
 & (b(1+n)(c+dx) \Gamma[4+m] \Gamma[-n]) - \left( d^2 (1+m) (2+m+n) (a+bx)^2 \right. \\
 & \quad \left. \Gamma[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) / \\
 & \left. \left( b(bc - ad) (1+n)^2 (c+dx) \Gamma[4+m] \Gamma[-n] \right) \right) - \\
 & \frac{1}{b(bc - ad) (1+m) (1+n)} (3+m+n) (-2bd - bdm - bdn) (a+bx)^{1+m} (c+dx)^n \\
 & \left( \frac{-bc - bdx}{-bc + ad} \right)^{-n} \left( -\frac{-bc - ad - adm - bcn - 2bdx - bdmx - bdnx}{(bc - ad) (1+n)} \right)^{2+m+n} \\
 & (bc + ad + adm + bcn + 2bdx + bdmx + bdnx)^{-3-m-n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc + ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc - ad) (1+n)} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \quad \left. \frac{m \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \quad \left. \left( d(2+m+n) (a+bx) \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \right) / \\
 & \left( (bc - ad) (1+n) \Gamma[3+m] \right) - \left( d (1+m) (a+bx) \Gamma[1-n] \right. \\
 & \quad \left. \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) / \\
 & (b(1+n)(c+dx) \Gamma[4+m] \Gamma[-n]) - \left( d^2 (1+m) (2+m+n) (a+bx)^2 \right. \\
 & \quad \left. \Gamma[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) / \\
 & \left. \left( b(bc - ad) (1+n)^2 (c+dx) \Gamma[4+m] \Gamma[-n] \right) \right) + \\
 & \frac{1}{(-bc + ad) (1+m)} dn (a+bx)^{1+m} (c+dx)^n \left( \frac{-bc - bdx}{-bc + ad} \right)^{-1-n} \\
 & \left( -\frac{-bc - ad - adm - bcn - 2bdx - bdmx - bdnx}{(bc - ad) (1+n)} \right)^{3+m+n}
 \end{aligned}$$

$$\begin{aligned}
 & (bc + ad + adm + bcn + 2bdx + bdmx + bdnx)^{-3-m-n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \\
 & \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{m \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \left( d(2+m+n)(a+bx) \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \quad \left. \left( (bc-ad)(1+n) \text{Gamma}[3+m] \right) - \left( d(1+m)(a+bx) \text{Gamma}[1-n] \right. \right. \\
 & \quad \left. \left. \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \quad \left. \left( b(1+n)(c+dx) \text{Gamma}[4+m] \text{Gamma}[-n] \right) - \left( d^2(1+m)(2+m+n)(a+bx)^2 \right. \right. \\
 & \quad \left. \left. \text{Gamma}[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \quad \left. \left( b(bc-ad)(1+n)^2(c+dx) \text{Gamma}[4+m] \text{Gamma}[-n] \right) \right) + \\
 & \frac{1}{b(1+m)} dn (a+bx)^{1+m} (c+dx)^{-1+n} \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \\
 & \left( -\frac{-bc-ad-adm-bcn-2bdx-bdmx-bdnx}{(bc-ad)(1+n)} \right)^{3+m+n} \\
 & (bc + ad + adm + bcn + 2bdx + bdmx + bdnx)^{-3-m-n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-2-m-n} \text{Gamma}[2+m] \\
 & \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{m \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \left( d(2+m+n)(a+bx) \text{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \quad \left. \left( (bc-ad)(1+n) \text{Gamma}[3+m] \right) - \left( d(1+m)(a+bx) \text{Gamma}[1-n] \right. \right. \\
 & \quad \left. \left. \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( (b(1+n)(c+dx) \Gamma[4+m] \Gamma[-n]) - \left( d^2(1+m)(2+m+n)(a+bx)^2 \right. \right. \\
 & \left. \left. \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left( (b(bc-ad)(1+n)^2(c+dx) \Gamma[4+m] \Gamma[-n]) \right) + \right. \\
 & (a+bx)^m (c+dx)^n \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \left( -\frac{-bc-ad-adm-bcn-2bdx-bdmx-bdnx}{(bc-ad)(1+n)} \right)^{3+m+n} \\
 & (bc+ad+adm+bcn+2bdx+bdmx+bdnx)^{-3-m-n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \frac{2 \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \frac{m \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \left( d(2+m+n)(a+bx) \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left( (bc-ad)(1+n) \Gamma[3+m] \right) - \left( d(1+m)(a+bx) \Gamma[1-n] \right. \right. \\
 & \left. \left. \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. (b(1+n)(c+dx) \Gamma[4+m] \Gamma[-n]) - \left( d^2(1+m)(2+m+n)(a+bx)^2 \right. \right. \\
 & \left. \left. \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left( (b(bc-ad)(1+n)^2(c+dx) \Gamma[4+m] \Gamma[-n]) \right) + \frac{1}{b(1+m)} \right. \\
 & (a+bx)^{1+m} (c+dx)^n \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \left( -\frac{-bc-ad-adm-bcn-2bdx-bdmx-bdnx}{(bc-ad)(1+n)} \right)^{3+m+n} \\
 & (bc+ad+adm+bcn+2bdx+bdmx+bdnx)^{-3-m-n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 + \frac{d(2+m+n)(a+bx)}{(bc-ad)(1+n)} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \left( bd(2+m+n) \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \right) / \right. \\
 & \left. \left( (bc-ad)(1+n) \Gamma[3+m] \right) - \left( 2n \left( \frac{d^2(1+m)(a+bx)}{b(1+n)(c+dx)^2} - \frac{d(1+m)}{(1+n)(c+dx)} \right) \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right] \Big/ \left((3+m) \Gamma[3+m]\right) - \\
 & \left(m n \left(\frac{d^2(1+m)(a+bx)}{b(1+n)(c+dx)^2} - \frac{d(1+m)}{(1+n)(c+dx)}\right) \text{Hypergeometric2F1}\left[2, 1-n, \right. \right. \\
 & \quad \left. \left. 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \left((3+m) \Gamma[3+m]\right) - \\
 & \left(d n (2+m+n)(a+bx) \left(\frac{d^2(1+m)(a+bx)}{b(1+n)(c+dx)^2} - \frac{d(1+m)}{(1+n)(c+dx)}\right) \text{Hypergeometric2F1}\left[ \right. \right. \\
 & \quad \left. \left. 2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \left((bc-ad)(3+m)(1+n) \Gamma[3+m]\right) + \\
 & \left(d^2(1+m)(a+bx) \Gamma[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, \right. \right. \\
 & \quad \left. \left. -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \left(b(1+n)(c+dx)^2 \Gamma[4+m] \Gamma[-n]\right) + \\
 & \left(d^3(1+m)(2+m+n)(a+bx)^2 \Gamma[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, \right. \right. \\
 & \quad \left. \left. -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \left(b(bc-ad)(1+n)^2(c+dx)^2 \Gamma[4+m] \Gamma[-n]\right) - \\
 & \left(d(1+m) \Gamma[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \\
 & \quad \left((1+n)(c+dx) \Gamma[4+m] \Gamma[-n]\right) - \\
 & \left(2d^2(1+m)(2+m+n)(a+bx) \Gamma[1-n] \text{Hypergeometric2F1}\left[2, 1-n, 4+m, \right. \right. \\
 & \quad \left. \left. -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \left((bc-ad)(1+n)^2(c+dx) \Gamma[4+m] \Gamma[-n]\right) - \\
 & \left(2d(1+m)(1-n)(a+bx) \left(\frac{d^2(1+m)(a+bx)}{b(1+n)(c+dx)^2} - \frac{d(1+m)}{(1+n)(c+dx)}\right) \right. \\
 & \quad \left. \Gamma[1-n] \text{Hypergeometric2F1}\left[3, 2-n, 5+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \\
 & \quad \left(b(4+m)(1+n)(c+dx) \Gamma[4+m] \Gamma[-n]\right) - \\
 & \left(2d^2(1+m)(1-n)(2+m+n)(a+bx)^2 \left(\frac{d^2(1+m)(a+bx)}{b(1+n)(c+dx)^2} - \frac{d(1+m)}{(1+n)(c+dx)}\right) \right. \\
 & \quad \left. \Gamma[1-n] \text{Hypergeometric2F1}\left[3, 2-n, 5+m, -\frac{d(1+m)(a+bx)}{b(1+n)(c+dx)}\right]\right) \Big/ \\
 & \quad \left(b(bc-ad)(4+m)(1+n)^2(c+dx) \Gamma[4+m] \Gamma[-n]\right) \Big) \Big) \Big)
 \end{aligned}$$

**Problem 3141: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (a + b x)^m (c + d x)^{-1 - \frac{d (b e - a f) (1 + m)}{b (d e - c f)}} (e + f x)^{-1 + \frac{(b c - a d) f (1 + m)}{b (d e - c f)}} dx$$

Optimal (type 3, 101 leaves, 1 step):

$$\frac{b (a + b x)^{1+m} (c + d x)^{-\frac{d (b e - a f) (1 + m)}{b (d e - c f)}} (e + f x)^{\frac{(b c - a d) f (1 + m)}{b (d e - c f)}}}{(b c - a d) (b e - a f) (1 + m)}$$

Result (type 6, 1616 leaves):

$$\frac{1}{1 + m} (a + b x)^{1+m} (c + d x)^{-\frac{d (b e - a f) (1 + m)}{b (d e - c f)}} (e + f x)^{\frac{(b c - a d) f (1 + m)}{b (d e - c f)}} \left( \left( \left( f \operatorname{AppellF1} \left[ 1 + m, \frac{d (b e - a f) (1 + m)}{b (d e - c f)}, 1 + \frac{(b c - a d) f (1 + m)}{b (-d e + c f)}, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / \left( (-d e + c f) \left( \frac{1}{1 + m} f \left( 1 + \frac{(b c - a d) f (1 + m)}{b (-d e + c f)} \right) (a + b x) \operatorname{AppellF1} \left[ 1 + m, \frac{d (b e - a f) (1 + m)}{b (d e - c f)}, 1 + \frac{(b c - a d) f (1 + m)}{b (-d e + c f)}, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + \left( f (-b d e - a d f (1 + m) + b c f (2 + m)) (a + b x) \operatorname{AppellF1} \left[ 1 + m, \frac{d (b e - a f) (1 + m)}{b (d e - c f)}, 1 + \frac{(b c - a d) f (1 + m)}{b (-d e + c f)}, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / (b (d e - c f) (1 + m)) + b (e + f x) \operatorname{AppellF1} \left[ 1 + m, \frac{d (b e - a f) (1 + m)}{b (d e - c f)}, 1 + \frac{(b c - a d) f (1 + m)}{b (-d e + c f)}, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + \left( (a + b x) (e + f x) \left( f (b d e + a d f (1 + m) - b c f (2 + m)) \operatorname{AppellF1} \left[ 2 + m, \frac{d (b e - a f) (1 + m)}{b (d e - c f)}, 2 + \frac{(b c - a d) f (1 + m)}{b (-d e + c f)}, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] + \frac{1}{b c - a d} d^2 (b e - a f)^2 (1 + m) \operatorname{AppellF1} \left[ 2 + m, 1 + \frac{d (b e - a f) (1 + m)}{b (d e - c f)}, 1 + \frac{(b c - a d) f (1 + m)}{b (-d e + c f)}, 3 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) / ((b e - a f) (-d e + c f) (2 + m)) \right) \right) + \left( d \operatorname{AppellF1} \left[ 1 + m, 1 + \frac{d (b e - a f) (1 + m)}{b (d e - c f)}, \frac{(b c - a d) f (1 + m)}{b (-d e + c f)}, 2 + m, \frac{d (a + b x)}{-b c + a d}, \frac{f (a + b x)}{-b e + a f} \right] \right) /$$

$$\begin{aligned}
 & \left( (de - cf) \left( \frac{1}{1+m} d \left( 1 + \frac{d (be - af) (1+m)}{b (de - cf)} \right) (a + bx) \operatorname{AppellF1} \left[ 1+m, \right. \right. \right. \\
 & \quad \left. \left. \left. 1 + \frac{d (be - af) (1+m)}{b (de - cf)}, \frac{(bc - ad) f (1+m)}{b (-de + cf)}, 2+m, \frac{d (a + bx)}{-bc + ad}, \frac{f (a + bx)}{-be + af} \right] - \right. \\
 & \quad \left. \left( d (-bcf - adf (1+m) + bde (2+m)) (a + bx) \operatorname{AppellF1} \left[ 1+m, 1 + \frac{d (be - af) (1+m)}{b (de - cf)}, \right. \right. \right. \\
 & \quad \quad \left. \left. \left. \frac{(bc - ad) f (1+m)}{b (-de + cf)}, 2+m, \frac{d (a + bx)}{-bc + ad}, \frac{f (a + bx)}{-be + af} \right] \right) / (b (de - cf) (1+m)) + \right. \\
 & \quad b (c + dx) \operatorname{AppellF1} \left[ 1+m, 1 + \frac{d (be - af) (1+m)}{b (de - cf)}, \frac{(bc - ad) f (1+m)}{b (-de + cf)}, \right. \\
 & \quad \left. 2+m, \frac{d (a + bx)}{-bc + ad}, \frac{f (a + bx)}{-be + af} \right] + \left( (a + bx) (c + dx) \right. \\
 & \quad \left. \left( \frac{1}{be - af} (bc - ad)^2 f^2 (1+m) \operatorname{AppellF1} \left[ 2+m, 1 + \frac{d (be - af) (1+m)}{b (de - cf)}, 1 + \right. \right. \right. \\
 & \quad \quad \left. \left. \left. \frac{(bc - ad) f (1+m)}{b (-de + cf)}, 3+m, \frac{d (a + bx)}{-bc + ad}, \frac{f (a + bx)}{-be + af} \right] - d (-bcf - adf (1+m) + \right. \right. \\
 & \quad \quad \left. \left. bde (2+m)) \operatorname{AppellF1} \left[ 2+m, 2 + \frac{d (be - af) (1+m)}{b (de - cf)}, \frac{(bc - ad) f (1+m)}{b (-de + cf)}, \right. \right. \right. \\
 & \quad \quad \left. \left. \left. 3+m, \frac{d (a + bx)}{-bc + ad}, \frac{f (a + bx)}{-be + af} \right] \right) / ((bc - ad) (de - cf) (2+m)) \right) \right)
 \end{aligned}$$

### Problem 3142: Result more than twice size of optimal antiderivative.

$$\int (a + bx)^m (c + dx)^n (e + fx)^{-m-n} dx$$

Optimal (type 6, 129 leaves, 3 steps):

$$\begin{aligned}
 & \frac{1}{b (1+m)} (a + bx)^{1+m} (c + dx)^n \left( \frac{b (c + dx)}{bc - ad} \right)^{-n} (e + fx)^{-m-n} \\
 & \left( \frac{b (e + fx)}{be - af} \right)^{m+n} \operatorname{AppellF1} \left[ 1+m, -n, m+n, 2+m, -\frac{d (a + bx)}{bc - ad}, -\frac{f (a + bx)}{be - af} \right]
 \end{aligned}$$

Result (type 6, 303 leaves):

$$\left( (bc - ad) (be - af) (2 + m) (a + bx)^{1+m} (c + dx)^n (e + fx)^{-m-n} \operatorname{AppellF1}\left[1 + m, -n, m + n, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] \right) / \left( b(1 + m) \left( (bc - ad) (be - af) (2 + m) \operatorname{AppellF1}\left[1 + m, -n, m + n, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] - (a + bx) \left( d(-be + af) n \operatorname{AppellF1}\left[2 + m, 1 - n, m + n, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] + (bc - ad) f(m + n) \operatorname{AppellF1}\left[2 + m, -n, 1 + m + n, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] \right) \right) \right)$$

**Problem 3143: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^m (c + dx)^n (e + fx)^{-1-m-n} dx$$

Optimal (type 6, 137 leaves, 3 steps):

$$\left( (a + bx)^{1+m} (c + dx)^n \left( \frac{b(c + dx)}{bc - ad} \right)^{-n} (e + fx)^{-m-n} \left( \frac{b(e + fx)}{be - af} \right)^{m+n} \operatorname{AppellF1}\left[1 + m, -n, 1 + m + n, 2 + m, -\frac{d(a + bx)}{bc - ad}, -\frac{f(a + bx)}{be - af}\right] \right) / ((be - af)(1 + m))$$

Result (type 6, 308 leaves):

$$\left( (bc - ad) (be - af) (2 + m) (a + bx)^{1+m} (c + dx)^n (e + fx)^{-1-m-n} \operatorname{AppellF1}\left[1 + m, -n, 1 + m + n, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] \right) / \left( b(1 + m) \left( (bc - ad) (be - af) (2 + m) \operatorname{AppellF1}\left[1 + m, -n, 1 + m + n, 2 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] - (a + bx) \left( d(-be + af) n \operatorname{AppellF1}\left[2 + m, 1 - n, 1 + m + n, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] + (bc - ad) f(1 + m + n) \operatorname{AppellF1}\left[2 + m, -n, 2 + m + n, 3 + m, \frac{d(a + bx)}{-bc + ad}, \frac{f(a + bx)}{-be + af}\right] \right) \right) \right)$$

**Problem 3145: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^m (c + dx)^n (e + fx)^{-3-m-n} dx$$

Optimal (type 5, 227 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{f (a+bx)^{1+m} (c+dx)^{1+n} (e+fx)^{-2-m-n}}{(be-af)(de-cf)(2+m+n)} - \\
 & \left( (adf(1+m) + b(cf(1+n) - de(2+m+n))) (a+bx)^{1+m} (c+dx)^n \left( \frac{(be-af)(c+dx)}{(bc-ad)(e+fx)} \right)^{-n} \right. \\
 & \quad \left. (e+fx)^{-1-m-n} \text{Hypergeometric2F1} \left[ 1+m, -n, 2+m, -\frac{(de-cf)(a+bx)}{(bc-ad)(e+fx)} \right] \right) / \\
 & \left( (be-af)^2 (de-cf)(1+m)(2+m+n) \right)
 \end{aligned}$$

Result(type 5, 5212 leaves):

$$\begin{aligned}
 & \left( (a+bx)^{1+2m} (c+dx)^{2n} \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} (e+fx)^{-6-2m-2n} \left( \frac{-be-bfx}{-be+af} \right)^{3+m+n} \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \right. \\
 & \quad \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-2-m-n} \text{Gamma}[2+m] \left( \frac{2 \text{Hypergeometric2F1} \left[ 1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{m \text{Hypergeometric2F1} \left[ 1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{\text{Gamma}[3+m]} + \right. \\
 & \quad \left. \frac{f(a+bx) \text{Hypergeometric2F1} \left[ 1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{(be-af) \text{Gamma}[3+m]} + \right. \\
 & \quad \left. \left( (de-cf)(a+bx) \text{Gamma}[1-n] \text{Hypergeometric2F1} \left[ 2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \right. \\
 & \quad \left. \left( (be-af)(c+dx) \text{Gamma}[4+m] \text{Gamma}[-n] \right) - \right. \\
 & \quad \left. \left( f(-de+cf)(a+bx)^2 \text{Gamma}[1-n] \text{Hypergeometric2F1} \left[ 2, 1-n, 4+m, \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \left( (be-af)^2 (c+dx) \text{Gamma}[4+m] \text{Gamma}[-n] \right) \right) \Bigg) / \\
 & \left( b(1+m) \left( -\frac{1}{(-be+af)(1+m)} f(-2-m-n) (a+bx)^{1+m} (c+dx)^n \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} \right. \right. \\
 & \quad \left. \left. (e+fx)^{-3-m-n} \left( \frac{-be-bfx}{-be+af} \right)^{3+m+n} \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-3-m-n} \right. \right. \\
 & \quad \left. \left. \text{Gamma}[2+m] \left( \frac{2 \text{Hypergeometric2F1} \left[ 1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{\text{Gamma}[3+m]} + \right. \right. \right. \\
 & \quad \left. \left. \frac{m \text{Hypergeometric2F1} \left[ 1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{\text{Gamma}[3+m]} + \right. \right. \\
 & \quad \left. \left. \frac{f(a+bx) \text{Hypergeometric2F1} \left[ 1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right]}{(be-af) \text{Gamma}[3+m]} + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( (d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
 & \left( f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) \Bigg) - \\
 & \frac{1}{(-b c + a d) (1 + m)} d n (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \\
 & \left( \frac{-b e - b f x}{-b e + a f} \right)^{3+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^{-1+n} \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
 & \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \right. \\
 & \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \\
 & \left. \frac{f (a + b x) \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) \text{Gamma}[3 + m]} \right) + \\
 & \left( (d e - c f) (a + b x) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f) (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
 & \left( f (-d e + c f) (a + b x)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f)^2 (c + d x) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) \Bigg) - \\
 & \frac{1}{(-b e + a f) (1 + m)} f (3 + m + n) (a + b x)^{1+m} (c + d x)^n \left( \frac{-b c - b d x}{-b c + a d} \right)^{-n} (e + f x)^{-3-m-n} \\
 & \left( \frac{-b e - b f x}{-b e + a f} \right)^{2+m+n} \left( 1 - \frac{d (a + b x)}{-b c + a d} \right)^n \left( 1 - \frac{f (a + b x)}{-b e + a f} \right)^{-2-m-n} \\
 & \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \right. \\
 & \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{\text{Gamma}[3 + m]} + \\
 & \left. \frac{f (a + b x) \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right]}{(b e - a f) \text{Gamma}[3 + m]} \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \left( (de - cf) (a + bx) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)(c + dx) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
 & \left( f(-de + cf) (a + bx)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)^2 (c + dx) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) \Bigg) + \\
 & \frac{1}{b(1+m)} f(-3 - m - n) (a + bx)^{1+m} (c + dx)^n \left( \frac{-bc - bdx}{-bc + ad} \right)^{-n} (e + fx)^{-4-m-n} \\
 & \left( \frac{-be - bfx}{-be + af} \right)^{3+m+n} \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^n \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2-m-n} \\
 & \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\text{Gamma}[3 + m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\text{Gamma}[3 + m]} + \right. \\
 & \left. \frac{f(a + bx) \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{(be - af) \text{Gamma}[3 + m]} \right) + \\
 & \left( (de - cf) (a + bx) \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)(c + dx) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) - \\
 & \left( f(-de + cf) (a + bx)^2 \text{Gamma}[1 - n] \text{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right] \right) / \left( (be - af)^2 (c + dx) \text{Gamma}[4 + m] \text{Gamma}[-n] \right) \Bigg) + \\
 & \frac{1}{(-bc + ad)(1+m)} dn (a + bx)^{1+m} (c + dx)^n \left( \frac{-bc - bdx}{-bc + ad} \right)^{-1-n} (e + fx)^{-3-m-n} \\
 & \left( \frac{-be - bfx}{-be + af} \right)^{3+m+n} \left( 1 - \frac{d(a + bx)}{-bc + ad} \right)^n \left( 1 - \frac{f(a + bx)}{-be + af} \right)^{-2-m-n} \\
 & \text{Gamma}[2 + m] \left( \frac{2 \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\text{Gamma}[3 + m]} + \right. \\
 & \left. \frac{m \text{Hypergeometric2F1}\left[1, -n, 3 + m, \frac{(de - cf)(a + bx)}{(be - af)(c + dx)}\right]}{\text{Gamma}[3 + m]} \right) +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{f(a+bx) \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af) \Gamma[3+m]} + \\
 & \left( \frac{(de-cf)(a+bx) \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af)(c+dx)} \right) / \left( (be-af)(c+dx) \Gamma[4+m] \Gamma[-n] \right) - \\
 & \left( \frac{f(-de+cf)(a+bx)^2 \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af)(c+dx)} \right) / \left( (be-af)^2(c+dx) \Gamma[4+m] \Gamma[-n] \right) \Bigg) + \\
 & \frac{1}{b(1+m)} d n (a+bx)^{1+m} (c+dx)^{-1+n} \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} (e+fx)^{-3-m-n} \left( \frac{-be-bfx}{-be+af} \right)^{3+m+n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \frac{2 \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \frac{m \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \frac{f(a+bx) \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af) \Gamma[3+m]} + \right. \\
 & \left( \frac{(de-cf)(a+bx) \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af)(c+dx)} \right) / \left( (be-af)(c+dx) \Gamma[4+m] \Gamma[-n] \right) - \\
 & \left( \frac{f(-de+cf)(a+bx)^2 \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af)(c+dx)} \right) / \left( (be-af)^2(c+dx) \Gamma[4+m] \Gamma[-n] \right) \Bigg) + \\
 & (a+bx)^m (c+dx)^n \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} (e+fx)^{-3-m-n} \left( \frac{-be-bfx}{-be+af} \right)^{3+m+n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \frac{2 \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\Gamma[3+m]} + \right. \\
 & \left. \frac{m \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{\Gamma[3+m]} + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{f (a+bx) \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af) \Gamma[3+m]} + \\
 & \left( (de-cf)(a+bx) \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \left( (be-af)(c+dx) \Gamma[4+m] \Gamma[-n] \right) - \\
 & \left( f(-de+cf)(a+bx)^2 \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \left( (be-af)^2 (c+dx) \Gamma[4+m] \Gamma[-n] \right) + \\
 & \frac{1}{b(1+m)} (a+bx)^{1+m} (c+dx)^n \left( \frac{-bc-bdx}{-bc+ad} \right)^{-n} (e+fx)^{-3-m-n} \left( \frac{-be-bfx}{-be+af} \right)^{3+m+n} \\
 & \left( 1 - \frac{d(a+bx)}{-bc+ad} \right)^n \left( 1 - \frac{f(a+bx)}{-be+af} \right)^{-2-m-n} \Gamma[2+m] \\
 & \left( \frac{bf \operatorname{Hypergeometric2F1}\left[1, -n, 3+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right]}{(be-af) \Gamma[3+m]} - \right. \\
 & \left. \left( 2n \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \right) \right) / \left( (3+m) \Gamma[3+m] \right) - \\
 & \left( mn \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \operatorname{Hypergeometric2F1}\left[ \right. \right. \\
 & \left. \left. 2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \left( (3+m) \Gamma[3+m] \right) - \\
 & \left( fn(a+bx) \left( -\frac{d(de-cf)(a+bx)}{(be-af)(c+dx)^2} + \frac{b(de-cf)}{(be-af)(c+dx)} \right) \operatorname{Hypergeometric2F1}\left[ \right. \right. \\
 & \left. \left. 2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)} \right] \right) / \left( (be-af)(3+m) \Gamma[3+m] \right) - \\
 & \left( d(de-cf)(a+bx) \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \left( (be-af)(c+dx)^2 \Gamma[4+m] \Gamma[-n] \right) + \\
 & \left( df(-de+cf)(a+bx)^2 \Gamma[1-n] \operatorname{Hypergeometric2F1}\left[2, 1-n, 4+m, \frac{(de-cf)(a+bx)}{(be-af)(c+dx)}\right] \right) / \left( (be-af)^2 (c+dx)^2 \Gamma[4+m] \Gamma[-n] \right) +
 \end{aligned}$$

$$\left( b (d e - c f) \Gamma[1 - n] \operatorname{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) /$$

$$\left( (b e - a f) (c + d x) \Gamma[4 + m] \Gamma[-n] \right) -$$

$$\left( 2 b f (-d e + c f) (a + b x) \Gamma[1 - n] \operatorname{Hypergeometric2F1}\left[2, 1 - n, 4 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] \right) / \left( (b e - a f)^2 (c + d x) \Gamma[4 + m] \Gamma[-n] \right) +$$

$$\left( 2 (d e - c f) (1 - n) (a + b x) \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \right)$$

$$\Gamma[1 - n] \operatorname{Hypergeometric2F1}\left[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] /$$

$$\left( (b e - a f) (4 + m) (c + d x) \Gamma[4 + m] \Gamma[-n] \right) -$$

$$\left( 2 f (-d e + c f) (1 - n) (a + b x)^2 \left( -\frac{d (d e - c f) (a + b x)}{(b e - a f) (c + d x)^2} + \frac{b (d e - c f)}{(b e - a f) (c + d x)} \right) \right)$$

$$\Gamma[1 - n] \operatorname{Hypergeometric2F1}\left[3, 2 - n, 5 + m, \frac{(d e - c f) (a + b x)}{(b e - a f) (c + d x)}\right] /$$

$$\left( (b e - a f)^2 (4 + m) (c + d x) \Gamma[4 + m] \Gamma[-n] \right) \Bigg) \Bigg) \Bigg)$$

**Problem 3146: Attempted integration timed out after 120 seconds.**

$$\int (a + b x)^m (c + d x)^n (e + f x)^{-4-m-n} dx$$

Optimal (type 5, 402 leaves, 4 steps):

$$-\frac{f (a + b x)^{1+m} (c + d x)^{1+n} (e + f x)^{-3-m-n}}{(b e - a f) (d e - c f) (3 + m + n)} +$$

$$\frac{(f (a d f (2 + m) + b (c f (2 + n) - d e (4 + m + n))) (a + b x)^{1+m} (c + d x)^{1+n} (e + f x)^{-2-m-n})}{((b e - a f)^2 (d e - c f)^2 (2 + m + n) (3 + m + n))} +$$

$$\frac{(a^2 d^2 f^2 (2 + 3 m + m^2) + 2 a b d f (1 + m) (c f (1 + n) - d e (3 + m + n)) - b^2 (2 c d e f (1 + n) (3 + m + n) - c^2 f^2 (2 + 3 n + n^2) - d^2 e^2 (6 + m^2 + 5 n + n^2 + m (5 + 2 n))))}{(a + b x)^{1+m} (c + d x)^n \left( \frac{(b e - a f) (c + d x)}{(b c - a d) (e + f x)} \right)^{-n} (e + f x)^{-1-m-n}}$$

$$\operatorname{Hypergeometric2F1}\left[1 + m, -n, 2 + m, -\frac{(d e - c f) (a + b x)}{(b c - a d) (e + f x)}\right] /$$

$$((b e - a f)^3 (d e - c f)^2 (1 + m) (2 + m + n) (3 + m + n))$$

Result (type 1, 1 leaves):

???

### Problem 3147: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^m (c+dx)^n (e+fx)^p dx$$

Optimal (type 6, 123 leaves, 3 steps):

$$\frac{1}{b(1+m)} (a+bx)^{1+m} (c+dx)^n \left( \frac{b(c+dx)}{bc-ad} \right)^{-n} (e+fx)^p \left( \frac{b(e+fx)}{be-af} \right)^{-p} \text{AppellF1} \left[ 1+m, -n, -p, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right]$$

Result (type 6, 296 leaves):

$$\left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^n (e+fx)^p \text{AppellF1} \left[ 1+m, -n, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(1+m) \left( (bc-ad)(be-af)(2+m) \text{AppellF1} \left[ 1+m, -n, -p, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - (a+bx) \left( d(-be+af)^n \text{AppellF1} \left[ 2+m, 1-n, -p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + (-bc+ad) f^p \text{AppellF1} \left[ 2+m, -n, 1-p, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right)$$

### Problem 3148: Result unnecessarily involves higher level functions.

$$\int (a+bx)^m (c+dx)^n (e+fx)^2 dx$$

Optimal (type 5, 259 leaves, 4 steps):

$$\left( f(bde(4+m+n) - f(bc(2+m) + ad(2+n))) (a+bx)^{1+m} (c+dx)^{1+n} \right) / \left( b^2 d^2 (2+m+n)(3+m+n) + \frac{f(a+bx)^{1+m} (c+dx)^{1+n} (e+fx)}{bd(3+m+n)} \right) + \left( (f(bc(1+m) + ad(1+n))(bde(4+m+n) - f(bc(2+m) + ad(2+n))) + bd(2+m+n)(af(cf+de(1+n)) + be(cf(1+m) - de(3+m+n))) (a+bx)^{1+m} (c+dx)^{1+n} \text{Hypergeometric2F1} \left[ 1, 2+m+n, 2+n, \frac{b(c+dx)}{bc-ad} \right] \right) / (b^2 d^2 (bc-ad)(1+n)(2+m+n)(3+m+n))$$

Result (type 6, 330 leaves):

$$\frac{1}{3} (a+bx)^m (c+dx)^n \left( \left( 9acfx^2 \operatorname{AppellF1}\left[2, -m, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \right. \\ \left( 3ac \operatorname{AppellF1}\left[2, -m, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + bcmx \operatorname{AppellF1}\left[3, 1-m, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + \right. \\ \left. \left. adnx \operatorname{AppellF1}\left[3, -m, 1-n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) + \right. \\ \left( 4acfx^3 \operatorname{AppellF1}\left[3, -m, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \\ \left( 4ac \operatorname{AppellF1}\left[3, -m, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + bcmx \operatorname{AppellF1}\left[4, 1-m, -n, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] + \right. \\ \left. \left. adnx \operatorname{AppellF1}\left[4, -m, 1-n, 5, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) + \frac{1}{d(1+n)} \\ \left. 3e^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \operatorname{Hypergeometric2F1}\left[-m, 1+n, 2+n, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 3149: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^m (c+dx)^n (e+fx) dx$$

Optimal (type 5, 131 leaves, 3 steps):

$$\frac{f(a+bx)^{1+m} (c+dx)^{1+n}}{bd(2+m+n)} - \left( (bde(2+m+n) - f(bc(1+m) + ad(1+n))) (a+bx)^{1+m} (c+dx)^{1+n} \right. \\ \left. \operatorname{Hypergeometric2F1}\left[1, 2+m+n, 2+n, \frac{b(c+dx)}{bc-ad}\right] \right) / (bd(bc-ad)(1+n)(2+m+n))$$

Result (type 6, 202 leaves):

$$(a+bx)^m (c+dx)^n \left( \left( 3acfx^2 \operatorname{AppellF1}\left[2, -m, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \right. \\ \left( 6ac \operatorname{AppellF1}\left[2, -m, -n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + 2bcmx \operatorname{AppellF1}\left[3, 1-m, -n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] + \right. \\ \left. \left. 2adnx \operatorname{AppellF1}\left[3, -m, 1-n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) + \frac{1}{d(1+n)} \right. \\ \left. e \left( \frac{d(a+bx)}{-bc+ad} \right)^{-m} (c+dx) \operatorname{Hypergeometric2F1}\left[-m, 1+n, 2+n, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 3151: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^n}{e+fx} dx$$

Optimal (type 6, 100 leaves, 2 steps):

$$\left( (a+bx)^{1+m} (c+dx)^n \left( \frac{b(c+dx)}{bc-ad} \right)^{-n} \text{AppellF1} \left[ 1+m, -n, 1, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right] \right) / ((be-af)(1+m))$$

Result (type 6, 298 leaves):

$$\begin{aligned} & - \left( \left( (bc-ad)(be-af)^2(2+m)(a+bx)^{1+m}(c+dx)^n \right. \right. \\ & \quad \left. \left. \text{AppellF1} \left[ 1+m, -n, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(-be+af)(1+m)(e+fx) \right. \right. \\ & \quad \left( (bc-ad)(be-af)(2+m) \text{AppellF1} \left[ 1+m, -n, 1, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \\ & \quad \left. (a+bx) \left( d(-be+af)n \text{AppellF1} \left[ 2+m, 1-n, 1, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \\ & \quad \left. \left. \left. \left. (bc-ad) f \text{AppellF1} \left[ 2+m, -n, 2, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right] \right) \right) \right) \end{aligned}$$

**Problem 3152: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^n}{(e+fx)^2} dx$$

Optimal (type 6, 101 leaves, 2 steps):

$$\left( b(a+bx)^{1+m} (c+dx)^n \left( \frac{b(c+dx)}{bc-ad} \right)^{-n} \text{AppellF1} \left[ 1+m, -n, 2, 2+m, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right] \right) / ((be-af)^2(1+m))$$

Result (type 6, 286 leaves):

$$\begin{aligned} & \left( (bc-ad)(be-af)(2+m)(a+bx)^{1+m}(c+dx)^n \right. \\ & \quad \left. \text{AppellF1} \left[ 1+m, -n, 2, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \left( b(1+m)(e+fx)^2 \right. \\ & \quad \left( (bc-ad)(be-af)(2+m) \text{AppellF1} \left[ 1+m, -n, 2, 2+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \\ & \quad \left. (a+bx) \left( d(-be+af)n \text{AppellF1} \left[ 2+m, 1-n, 2, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \\ & \quad \left. \left. \left. \left. 2(bc-ad) f \text{AppellF1} \left[ 2+m, -n, 3, 3+m, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right] \right) \right) \right) \end{aligned}$$

**Problem 3153: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^m (c+dx)^n}{(e+fx)^3} dx$$



$$\begin{aligned}
 & \frac{1}{35 d^2 (a + b x)^{2/3}} \\
 & 6 b \sqrt{c + d x} \left( \frac{7 d (a + b x)}{f} + \left( (c + d x) \left( -26 (b c - a d) (3 b d e + 2 b c f - 5 a d f) \operatorname{AppellF1} \left[ \frac{7}{6}, \frac{2}{3}, 1, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] \left( b (-3 d e + 3 c f) \operatorname{AppellF1} \left[ \frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{-d e + c f}{f (c + d x)} \right] + 2 (b c - a d) f \operatorname{AppellF1} \left[ \frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] \right) \right) - \\
 & \quad 7 b (c + d x) \operatorname{AppellF1} \left[ \frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] \left( 13 f (5 a^2 d^2 f + a \right. \\
 & \quad \left. b d (-3 d e + 42 c f + 49 d f x) - b^2 (12 c^2 f + 35 d^2 e x + 2 c d (16 e + 7 f x)) \right) \\
 & \quad \operatorname{AppellF1} \left[ \frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] + 14 (5 b d e + 2 b c f - 7 a d f) \\
 & \quad \left( 3 b (d e - c f) \operatorname{AppellF1} \left[ \frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] + 2 \right. \\
 & \quad \left. (-b c + a d) f \operatorname{AppellF1} \left[ \frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] \right) \Big) \Big) \Big) \Big) / \\
 & \left( d (e + f x) \left( 7 b f (c + d x) \operatorname{AppellF1} \left[ \frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] + \right. \right. \\
 & \quad b (-6 d e + 6 c f) \operatorname{AppellF1} \left[ \frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] + \\
 & \quad \left. 4 (b c - a d) f \operatorname{AppellF1} \left[ \frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] \right) \\
 & \quad \left( 13 b f (c + d x) \operatorname{AppellF1} \left[ \frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] + \right. \\
 & \quad b (-6 d e + 6 c f) \operatorname{AppellF1} \left[ \frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] + \\
 & \quad \left. 4 (b c - a d) f \operatorname{AppellF1} \left[ \frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)} \right] \right) \Big) \Big) \Big) \Big) /
 \end{aligned}$$

**Problem 3159: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^{2/5} (e + f x)^{3/5}}{\sqrt{a + b x}} dx$$

Optimal (type 6, 123 leaves, 3 steps):

$$\begin{aligned}
 & \left( 2 \sqrt{a + b x} (c + d x)^{2/5} (e + f x)^{3/5} \operatorname{AppellF1} \left[ \frac{1}{2}, -\frac{2}{5}, -\frac{3}{5}, \frac{3}{2}, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right] \right) / \\
 & \left( b \left( \frac{b (c + d x)}{b c - a d} \right)^{2/5} \left( \frac{b (e + f x)}{b e - a f} \right)^{3/5} \right)
 \end{aligned}$$

Result (type 6, 661 leaves):

$$\frac{1}{45 b^3 (c + d x)^{3/5} (e + f x)^{2/5}} 2 \sqrt{a + b x} \left( 15 b^2 (c + d x) (e + f x) - 2 (a + b x) \left( \left( 9 (25 a^2 d^2 f^2 - 10 a b d f (3 d e + 2 c f) + b^2 (3 d^2 e^2 + 24 c d e f - 2 c^2 f^2)) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{5}, \frac{2}{5}, \frac{3}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) / \right. \right. \\ \left. \left( 15 d f (a + b x) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{3}{5}, \frac{2}{5}, \frac{3}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right. \right. \\ \left. (-4 b d e + 4 a d f) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{5}, \frac{7}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right. \\ \left. \left. 6 (-b c + a d) f \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{8}{5}, \frac{2}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) \right) + \\ \frac{1}{(a + b x)^2} (3 b d e + 2 b c f - 5 a d f) \left( -\frac{3 b^2 (c + d x) (e + f x)}{d f} + \right. \\ \left. \left( 25 (b c - a d) (b e - a f) (a + b x) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{5}, \frac{2}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) / \right. \\ \left. \left( 25 d f (a + b x) \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{5}, \frac{2}{5}, \frac{5}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right. \right. \\ \left. (-4 b d e + 4 a d f) \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{3}{5}, \frac{7}{5}, \frac{7}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] + \right. \\ \left. \left. 6 (-b c + a d) f \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{8}{5}, \frac{2}{5}, \frac{7}{2}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)} \right] \right) \right) \right) \right)$$

Problem 3160: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{a + b x} (e + f x)^n}{\sqrt{c + d x}} dx$$

Optimal (type 6, 123 leaves, 3 steps):

$$\frac{1}{3 b \sqrt{c + d x}} 2 (a + b x)^{3/2} \sqrt{\frac{b (c + d x)}{b c - a d}} (e + f x)^n \left( \frac{b (e + f x)}{b e - a f} \right)^{-n} \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, -\frac{d (a + b x)}{b c - a d}, -\frac{f (a + b x)}{b e - a f} \right]$$

Result (type 6, 289 leaves):

$$\left( 10 (bc - ad) (be - af) (a + bx)^{3/2} (e + fx)^n \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) /$$

$$\left( 3b\sqrt{c+dx} \left( 5 (bc - ad) (be - af) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right.$$

$$(a + bx) \left( 2 (-bc + ad) f n \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 1 - n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right.$$

$$\left. \left. \left. d (be - af) \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, -n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right)$$

**Problem 3161: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{c+dx} (e+fx)^n}{\sqrt{a+bx}} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$\frac{1}{b \sqrt{\frac{b(c+dx)}{bc-ad}}} 2 \sqrt{a+bx} \sqrt{c+dx} (e+fx)^n$$

$$\left( \frac{b(e+fx)}{be-af} \right)^{-n} \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]$$

Result (type 6, 287 leaves):

$$\left( 6 (bc - ad) (be - af) \sqrt{a+bx} \sqrt{c+dx} \right.$$

$$\left. (e + fx)^n \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) /$$

$$\left( b \left( 3 (bc - ad) (be - af) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right.$$

$$(a + bx) \left( 2 (-bc + ad) f n \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1 - n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right.$$

$$\left. \left. \left. d (-be + af) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right)$$

**Problem 3162: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx)^n}{\sqrt{a+bx} (c+dx)^{3/2}} dx$$

Optimal (type 6, 128 leaves, 3 steps):

$$\left( 2 \sqrt{a+bx} \sqrt{\frac{b(c+dx)}{bc-ad}} (e+fx)^n \left( \frac{b(e+fx)}{be-af} \right)^{-n} \right. \\ \left. \text{AppellF1}\left[\frac{1}{2}, \frac{3}{2}, -n, \frac{3}{2}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right] \right) / ((bc-ad) \sqrt{c+dx})$$

Result (type 6, 816 leaves):

$$\frac{1}{3(c+dx)^{3/2}} 2(b e - a f) \sqrt{a+bx} (e+fx)^n \\ \left( \left( 9 b (c+dx)^2 \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \right. \\ \left( (bc-ad) \left( 3 (bc-ad) (be-af) \text{AppellF1}\left[\frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right. \\ (a+bx) \left( 2 (-bc+ad) f n \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, 1-n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right. \\ \left. \left. d (-be+af) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right) - \\ \left( 5 d (a+bx) (c+dx) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \\ \left( (bc-ad) \left( 5 (bc-ad) (be-af) \text{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right. \\ (a+bx) \left( 2 (-bc+ad) f n \text{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, 1-n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right. \\ \left. \left. d (be-af) \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, -n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right) - \\ \left( 5 d (a+bx) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) / \\ \left( b \left( 5 (bc-ad) (be-af) \text{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] - \right. \right. \\ (a+bx) \left( 2 (-bc+ad) f n \text{AppellF1}\left[\frac{5}{2}, \frac{3}{2}, 1-n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] + \right. \\ \left. \left. 3 d (be-af) \text{AppellF1}\left[\frac{5}{2}, \frac{5}{2}, -n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af}\right] \right) \right) \right) \right)$$

**Problem 3163: Result more than twice size of optimal antiderivative.**

$$\int \frac{(e+fx)^n}{(a+bx)^{3/2} \sqrt{c+dx}} dx$$

Optimal (type 6, 121 leaves, 3 steps):

$$- \left( \left( 2 \sqrt{\frac{b(c+dx)}{bc-ad}} (e+fx)^n \left( \frac{b(e+fx)}{be-af} \right)^{-n} \right. \right. \\ \left. \left. \text{AppellF1} \left[ -\frac{1}{2}, \frac{1}{2}, -n, \frac{1}{2}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right] \right) / \left( b \sqrt{a+bx} \sqrt{c+dx} \right) \right)$$

Result (type 6, 825 leaves):

$$\frac{1}{3(bc-ad)\sqrt{a+bx}\sqrt{c+dx}} 2(b e - a f) (e + f x)^n \\ \left( \left( 3(bc-ad)^2(c+dx) \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -n, \frac{1}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \right. \\ \left( (-bc+ad) \left( (bc-ad)(be-af) \text{AppellF1} \left[ -\frac{1}{2}, -\frac{1}{2}, -n, \frac{1}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \right. \\ \left. \left. (a+bx) \left( 2(-bc+ad) f^n \text{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, 1-n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \right. \\ \left. \left. \left. d(-be+af) \text{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right) - \\ \left( 9d(a+bx)(c+dx) \text{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \\ \left( 3(bc-ad)(be-af) \text{AppellF1} \left[ \frac{1}{2}, -\frac{1}{2}, -n, \frac{3}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \\ \left. (a+bx) \left( 2(-bc+ad) f^n \text{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, 1-n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \\ \left. \left. d(-be+af) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) + \\ \left( 5d^2(a+bx)^2 \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) / \\ \left( b \left( 5(bc-ad)(be-af) \text{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, -n, \frac{5}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] - \right. \right. \\ \left. \left. (a+bx) \left( 2(-bc+ad) f^n \text{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, 1-n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] + \right. \right. \right. \\ \left. \left. \left. d(be-af) \text{AppellF1} \left[ \frac{5}{2}, \frac{3}{2}, -n, \frac{7}{2}, \frac{d(a+bx)}{-bc+ad}, \frac{f(a+bx)}{-be+af} \right] \right) \right) \right) \right)$$

**Problem 3164: Result more than twice size of optimal antiderivative.**

$$\int \frac{\sqrt{a+bx} (c+dx)^{1/3}}{e+fx} dx$$

Optimal (type 6, 100 leaves, 2 steps):

$$\frac{2 (a + b x)^{3/2} (c + d x)^{1/3} \text{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, 1, \frac{5}{2}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]}{3 (b e - a f) \left(\frac{b(c+dx)}{bc-ad}\right)^{1/3}}$$

Result (type 6, 901 leaves):

$$\frac{1}{35 (c + d x)^{2/3}} 6 \sqrt{a + b x} \left( \frac{7 (c + d x)}{f} - \left( d (a + b x) \left( 78 (b c - a d) (b e - a f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{-b c + a d}{d (a + b x)}\right], \frac{-b e + a f}{f (a + b x)} \right) \left( 3 d (b e - a f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] + 2 (b c - a d) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] \right) - 7 (a + b x) \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] \right) \left( 13 d f (3 b^2 c e - 3 a d f (6 a + 7 b x) + b (a (32 d e - 17 c f) + 7 b (5 d e - 2 c f) x)) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] - 14 (5 b d e - 2 b c f - 3 a d f) \left( 3 d (b e - a f) \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] + 2 (b c - a d) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] \right) \right) / \left( b^2 (e + f x) \left( 7 d f (a + b x) \text{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] + (-6 b d e + 6 a d f) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] + 4 (-b c + a d) f \text{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] \right) \left( 13 d f (a + b x) \text{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] + (-6 b d e + 6 a d f) \text{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] + 4 (-b c + a d) f \text{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{-b c + a d}{d (a + b x)}, \frac{-b e + a f}{f (a + b x)}\right] \right) \right)$$

**Problem 3165: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{1/3} \sqrt{c + d x}}{e + f x} dx$$

Optimal (type 6, 100 leaves, 2 steps):

$$\frac{3 (a + b x)^{4/3} \sqrt{c + d x} \operatorname{AppellF1}\left[\frac{4}{3}, -\frac{1}{2}, 1, \frac{7}{3}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right]}{4 (b e - a f) \sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Result (type 6, 895 leaves):

$$\frac{1}{35 (a + b x)^{2/3}} 6 \sqrt{c + d x} \left( \frac{7 (a + b x)}{f} + \left( b (c + d x) \left( -78 (b c - a d) (d e - c f) \operatorname{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + \frac{-d e + c f}{f (c + d x)} \right) \left( b (-3 d e + 3 c f) \operatorname{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 2 (b c - a d) f \operatorname{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) - 7 (c + d x) \operatorname{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right. \\ \left. \left( 13 b f (a d (-3 d e + 17 c f + 14 d f x) + b (-32 c d e + 18 c^2 f - 35 d^2 e x + 21 c d f x)) \operatorname{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 14 (5 b d e - 3 b c f - 2 a d f) \right. \right. \\ \left. \left. \left( 3 b (d e - c f) \operatorname{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 2 (-b c + a d) f \operatorname{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right) \right) / \\ \left( d^2 (e + f x) \left( 7 b f (c + d x) \operatorname{AppellF1}\left[\frac{1}{6}, \frac{2}{3}, 1, \frac{7}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-6 d e + 6 c f) \operatorname{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 2, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 4 (b c - a d) f \operatorname{AppellF1}\left[\frac{7}{6}, \frac{5}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right. \\ \left. \left( 13 b f (c + d x) \operatorname{AppellF1}\left[\frac{7}{6}, \frac{2}{3}, 1, \frac{13}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + b (-6 d e + 6 c f) \operatorname{AppellF1}\left[\frac{13}{6}, \frac{2}{3}, 2, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] + 4 (b c - a d) f \operatorname{AppellF1}\left[\frac{13}{6}, \frac{5}{3}, 1, \frac{19}{6}, \frac{b c - a d}{b c + b d x}, \frac{-d e + c f}{f (c + d x)}\right] \right) \right)$$

Problem 3166: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a + b x} (c + d x)^{1/3} (e + f x)^{1/4} dx$$

Optimal (type 6, 125 leaves, 3 steps):

$$\left( 2 (a+bx)^{3/2} (c+dx)^{1/3} (e+fx)^{1/4} \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{3}, -\frac{1}{4}, \frac{5}{2}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af}\right] \right) / \left( 3b \left( \frac{b(c+dx)}{bc-ad} \right)^{1/3} \left( \frac{b(e+fx)}{be-af} \right)^{1/4} \right)$$

Result (type 6, 1077 leaves):

$$\begin{aligned}
 & \left( \frac{12(3bde + 4bcf + 6adf)}{325bdf} + \frac{12x}{25} \right) \sqrt{a+bx} (c+dx)^{1/3} (e+fx)^{1/4} - \\
 & \frac{1}{82225b^3df} \left( c + \frac{(a+bx)(d-\frac{ad}{a+bx})}{b} \right)^{2/3} \left( e + \frac{(a+bx)(f-\frac{af}{a+bx})}{b} \right)^{3/4} \\
 & 72(a+bx)^{3/2} \left( \left( 1058(-21a^3d^3f^3 + 9a^2bd^2f^2(3de + 4cf) - \right. \right. \\
 & \quad \left. \left. ab^2df(20d^2e^2 + 14cdef + 29c^2f^2) + b^3(5d^3e^3 + 5cd^2e^2f + 2c^2def^2 + 9c^3f^3) \right) \right. \\
 & \quad \left. \text{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right) / \\
 & \left( (a+bx) \left( -23df \text{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + \frac{1}{a+bx} \right. \right. \\
 & \quad \left( 9d(be-af) \text{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{7}{4}, \frac{35}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + \right. \\
 & \quad \left. \left. \left. \left. 8(bc-ad) f \text{AppellF1} \left[ \frac{23}{12}, \frac{5}{3}, \frac{3}{4}, \frac{35}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right] \right) \right) \right) + \\
 & \left( 11(7a^2d^2f^2 - 2abd f(3de + 4cf) + b^2(5d^2e^2 - 4cdef + 6c^2f^2)) \right. \\
 & \quad \left. \left( 35df \left( \frac{bc \left( \frac{17be}{a+bx} + f \left( 23 - \frac{17a}{a+bx} \right) \right)}{a+bx} + d \left( f \left( 23 + \frac{17a^2}{(a+bx)^2} - \frac{46a}{a+bx} \right) + \frac{be \left( 23 - \frac{17a}{a+bx} \right)}{a+bx} \right) \right) \right) \right. \\
 & \quad \left. \text{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{3}{4}, \frac{35}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] - \right. \\
 & \quad \left. \frac{1}{a+bx} 23 \left( d + \frac{bc}{a+bx} - \frac{ad}{a+bx} \right) \left( f + \frac{be}{a+bx} - \frac{af}{a+bx} \right) \right. \\
 & \quad \left( 9d(be-af) \text{AppellF1} \left[ \frac{35}{12}, \frac{2}{3}, \frac{7}{4}, \frac{47}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + \right. \\
 & \quad \left. \left. \left. \left. 8(bc-ad) f \text{AppellF1} \left[ \frac{35}{12}, \frac{5}{3}, \frac{3}{4}, \frac{47}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right] \right) \right) \right) / \\
 & \left( df \left( 35df \text{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{3}{4}, \frac{35}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + \frac{1}{a+bx} \right. \right. \\
 & \quad \left( (-9bde + 9adf) \text{AppellF1} \left[ \frac{35}{12}, \frac{2}{3}, \frac{7}{4}, \frac{47}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] + \right. \\
 & \quad \left. \left. \left. \left. 8(-bc+ad) f \text{AppellF1} \left[ \frac{35}{12}, \frac{5}{3}, \frac{3}{4}, \frac{47}{12}, \frac{-bc+ad}{d(a+bx)}, \frac{-be+af}{f(a+bx)} \right] \right] \right) \right) \right) \right)
 \end{aligned}$$

### Problem 3167: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^{1/3} \sqrt{c+dx} (e+fx)^{1/4} dx$$

Optimal (type 6, 125 leaves, 3 steps):

$$\left( 3 (a+bx)^{4/3} \sqrt{c+dx} (e+fx)^{1/4} \operatorname{AppellF1} \left[ \frac{4}{3}, -\frac{1}{2}, -\frac{1}{4}, \frac{7}{3}, -\frac{d(a+bx)}{bc-ad}, -\frac{f(a+bx)}{be-af} \right] \right) /$$

$$\left( 4b \sqrt{\frac{b(c+dx)}{bc-ad}} \left( \frac{b(e+fx)}{be-af} \right)^{1/4} \right)$$

Result (type 6, 1078 leaves):

$$\begin{aligned}
 & \frac{1}{3575 (a+bx)^{2/3} (e+fx)^{3/4}} \\
 & \sqrt{c+dx} \left( \frac{132 (a+bx) (e+fx) (4adf+b(3de+6cf+13dfx))}{bdf} - \left( 72 (c+dx) \right. \right. \\
 & \quad \left. \left. - 23 (bc-ad) (de-cf) (3bde-7bcf+4adf) \operatorname{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{bc-ad}{bc+bdx}, \right. \right. \right. \\
 & \quad \left. \left. \frac{-de+cf}{f(c+dx)} \right] \left( 9b(de-cf) \operatorname{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{7}{4}, \frac{23}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] + \right. \right. \\
 & \quad \left. \left. 8(-bc+ad) f \operatorname{AppellF1} \left[ \frac{11}{12}, \frac{5}{3}, \frac{3}{4}, \frac{23}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] \right) \right) - \\
 & 11 (c+dx) \operatorname{AppellF1} \left[ -\frac{1}{12}, \frac{2}{3}, \frac{3}{4}, \frac{11}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] \\
 & \left( 23bf(-2a^2d^2f(-2de+35cf+33dfx)) - b^2(7c^2f^2(12c+11dx) - 2cdef \right. \\
 & \quad \left. (38c+33dx) + d^2e^2(58c+55dx)) + abd(99c^2f^2+d^2e(3e+44fx) + \right. \\
 & \quad \left. 2cdf(15e+44fx)) \right) \operatorname{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] + \\
 & 11(6a^2d^2f^2 - 4abdf(de+2cf) + b^2(5d^2e^2 - 6cdef + 7c^2f^2)) \\
 & \left( 9b(de-cf) \operatorname{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{7}{4}, \frac{35}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] + 8 \right. \\
 & \quad \left. (-bc+ad) f \operatorname{AppellF1} \left[ \frac{23}{12}, \frac{5}{3}, \frac{3}{4}, \frac{35}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] \right) \Big) \Big) \Big) \Big) \Big) / \\
 & \left( d^3 \left( 11bf(c+dx) \operatorname{AppellF1} \left[ -\frac{1}{12}, \frac{2}{3}, \frac{3}{4}, \frac{11}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] + \right. \right. \\
 & \quad b(-9de+9cf) \operatorname{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{7}{4}, \frac{23}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] + \\
 & \quad \left. 8(bc-ad) f \operatorname{AppellF1} \left[ \frac{11}{12}, \frac{5}{3}, \frac{3}{4}, \frac{23}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] \right) \Big) \\
 & \left( 23bf(c+dx) \operatorname{AppellF1} \left[ \frac{11}{12}, \frac{2}{3}, \frac{3}{4}, \frac{23}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] + \right. \\
 & \quad b(-9de+9cf) \operatorname{AppellF1} \left[ \frac{23}{12}, \frac{2}{3}, \frac{7}{4}, \frac{35}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] + \\
 & \quad \left. 8(bc-ad) f \operatorname{AppellF1} \left[ \frac{23}{12}, \frac{5}{3}, \frac{3}{4}, \frac{35}{12}, \frac{bc-ad}{bc+bdx}, \frac{-de+cf}{f(c+dx)} \right] \right) \Big) \Big) \Big) \Big) \Big)
 \end{aligned}$$

**Problem 3168: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^4 (A+Bx) (d+ex)^m dx$$

Optimal (type 3, 234 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(bd - ae)^4 (Bd - Ae) (d + ex)^{1+m}}{e^6 (1+m)} + \frac{(bd - ae)^3 (5bBd - 4Abe - aBe) (d + ex)^{2+m}}{e^6 (2+m)} \\
 & - \frac{2b (bd - ae)^2 (5bBd - 3Abe - 2aBe) (d + ex)^{3+m}}{e^6 (3+m)} + \\
 & - \frac{2b^2 (bd - ae) (5bBd - 2Abe - 3aBe) (d + ex)^{4+m}}{e^6 (4+m)} - \\
 & - \frac{b^3 (5bBd - Abe - 4aBe) (d + ex)^{5+m}}{e^6 (5+m)} + \frac{b^4 B (d + ex)^{6+m}}{e^6 (6+m)}
 \end{aligned}$$

Result (type 3, 635 leaves):

$$\begin{aligned}
 & \frac{1}{e^6 (1+m) (2+m) (3+m) (4+m) (5+m) (6+m)} \\
 & (d + ex)^{1+m} (a^4 e^4 (360 + 342m + 119m^2 + 18m^3 + m^4) (-Bd + Ae (2+m) + Be (1+m)x) + \\
 & 4a^3 b e^3 (120 + 74m + 15m^2 + m^3) \\
 & (Ae (3+m) (-d + e (1+m)x) + B (2d^2 - 2de (1+m)x + e^2 (2 + 3m + m^2) x^2) + \\
 & 6a^2 b^2 e^2 (30 + 11m + m^2) (Ae (4+m) (2d^2 - 2de (1+m)x + e^2 (2 + 3m + m^2) x^2) + \\
 & B (-6d^3 + 6d^2 e (1+m)x - 3de^2 (2 + 3m + m^2) x^2 + e^3 (6 + 11m + 6m^2 + m^3) x^3)) + 4ab^3 e \\
 & (6+m) (Ae (5+m) (-6d^3 + 6d^2 e (1+m)x - 3de^2 (2 + 3m + m^2) x^2 + e^3 (6 + 11m + 6m^2 + m^3) x^3) + \\
 & B (24d^4 - 24d^3 e (1+m)x + 12d^2 e^2 (2 + 3m + m^2) x^2 - \\
 & 4de^3 (6 + 11m + 6m^2 + m^3) x^3 + e^4 (24 + 50m + 35m^2 + 10m^3 + m^4) x^4)) - \\
 & b^4 (-Ae (6+m) (24d^4 - 24d^3 e (1+m)x + 12d^2 e^2 (2 + 3m + m^2) x^2 - \\
 & 4de^3 (6 + 11m + 6m^2 + m^3) x^3 + e^4 (24 + 50m + 35m^2 + 10m^3 + m^4) x^4) + \\
 & B (120d^5 - 120d^4 e (1+m)x + 60d^3 e^2 (2 + 3m + m^2) x^2 - 20d^2 e^3 (6 + 11m + 6m^2 + m^3) x^3 + \\
 & 5de^4 (24 + 50m + 35m^2 + 10m^3 + m^4) x^4 - e^5 (120 + 274m + 225m^2 + 85m^3 + 15m^4 + m^5) x^5))
 \end{aligned}$$

**Problem 3169: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^3 (A + Bx) (d + ex)^m dx$$

Optimal (type 3, 186 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(bd - ae)^3 (Bd - Ae) (d + ex)^{1+m}}{e^5 (1+m)} - \frac{(bd - ae)^2 (4bBd - 3Abe - aBe) (d + ex)^{2+m}}{e^5 (2+m)} + \\
 & - \frac{3b (bd - ae) (2bBd - Abe - aBe) (d + ex)^{3+m}}{e^5 (3+m)} - \\
 & - \frac{b^2 (4bBd - Abe - 3aBe) (d + ex)^{4+m}}{e^5 (4+m)} + \frac{b^3 B (d + ex)^{5+m}}{e^5 (5+m)}
 \end{aligned}$$

Result (type 3, 391 leaves):

$$\frac{1}{e^5 (1+m) (2+m) (3+m) (4+m) (5+m)} \\
 (d+ex)^{1+m} (a^3 e^3 (60+47m+12m^2+m^3) (-Bd+Ae(2+m)+Be(1+m)x) + 3a^2 b e^2 (20+9m+m^2) \\
 (Ae(3+m)(-d+e(1+m)x) + B(2d^2-2de(1+m)x+e^2(2+3m+m^2)x^2)) + \\
 3a b^2 e (5+m) (Ae(4+m)(2d^2-2de(1+m)x+e^2(2+3m+m^2)x^2) + \\
 B(-6d^3+6d^2e(1+m)x-3de^2(2+3m+m^2)x^2+e^3(6+11m+6m^2+m^3)x^3)) + \\
 b^3 (Ae(5+m)(-6d^3+6d^2e(1+m)x-3de^2(2+3m+m^2)x^2+e^3(6+11m+6m^2+m^3)x^3) + \\
 B(24d^4-24d^3e(1+m)x+12d^2e^2(2+3m+m^2)x^2- \\
 4de^3(6+11m+6m^2+m^3)x^3+e^4(24+50m+35m^2+10m^3+m^4)x^4))$$

**Problem 3174: Unable to integrate problem.**

$$\int \frac{(A+Bx)(d+ex)^m}{(a+bx)^2} dx$$

Optimal (type 5, 112 leaves, 2 steps):

$$-\frac{(Ab-aB)(d+ex)^{1+m}}{b(bd-ae)(a+bx)} + \\
 \left( (aBe(1+m) - b(Bd+Aem)) (d+ex)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{b(d+ex)}{bd-ae}\right] \right) / \\
 (b(bd-ae)^2(1+m))$$

Result (type 8, 22 leaves):

$$\int \frac{(A+Bx)(d+ex)^m}{(a+bx)^2} dx$$

**Problem 3181: Result more than twice size of optimal antiderivative.**

$$\int \frac{(2+3x)^m (3+5x)^3}{1-2x} dx$$

Optimal (type 5, 90 leaves, 3 steps):

$$-\frac{5135(2+3x)^{1+m}}{216(1+m)} - \frac{725(2+3x)^{2+m}}{108(2+m)} - \frac{125(2+3x)^{3+m}}{54(3+m)} + \\
 \frac{1331(2+3x)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, \frac{2}{7}(2+3x)\right]}{56(1+m)}$$

Result (type 5, 240 leaves):

$$\frac{1}{432} (2+3x)^m \left( -\frac{32670(2+3x)}{1+m} + \frac{2475(40+36x-36x^2-7^{2+m}(4+6x)^{-m}-6m(-2+x+6x^2))}{2+3m+m^2} + \right. \\ \left. (250(4+6x)^{-m}(7^{3+m}-316(4+6x)^m-162x(4+6x)^m+324x^2(4+6x)^m-216x^3(4+6x)^m-9m^2(1-2x)^2(2+3x)(4+6x)^m-3m(4+6x)^m(46-59x-120x^2+108x^3)) \right) / \\ \left( (1+m)(2+m)(3+m) - \frac{35937\left(\frac{4+6x}{-3+6x}\right)^{-m} \text{Hypergeometric2F1}\left[-m, -m, 1-m, \frac{7}{3-6x}\right]}{m} \right)$$

**Problem 3187: Unable to integrate problem.**

$$\int \frac{(a+bx)^m}{(e+fx)^2} dx$$

Optimal (type 5, 52 leaves, 1 step):

$$\frac{b(a+bx)^{1+m} \text{Hypergeometric2F1}\left[2, 1+m, 2+m, -\frac{f(a+bx)}{be-af}\right]}{(be-af)^2(1+m)}$$

Result (type 8, 17 leaves):

$$\int \frac{(a+bx)^m}{(e+fx)^2} dx$$

**Problem 3188: Unable to integrate problem.**

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)^2} dx$$

Optimal (type 5, 187 leaves, 4 steps):

$$-\frac{f(a+bx)^{1+m}}{(be-af)(de-cf)(e+fx)} + \frac{d^2(a+bx)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, -\frac{d(a+bx)}{bc-ad}\right]}{(bc-ad)(de-cf)^2(1+m)} + \\ \left( \frac{f(adf-b(de(1-m)+cfm))(a+bx)^{1+m} \text{Hypergeometric2F1}\left[1, 1+m, 2+m, -\frac{f(a+bx)}{be-af}\right]}{(be-af)^2(de-cf)^2(1+m)} \right) /$$

Result (type 8, 24 leaves):

$$\int \frac{(a+bx)^m}{(c+dx)(e+fx)^2} dx$$

**Problem 3189: Unable to integrate problem.**

$$\int \frac{(a+bx)^m}{(c+dx)^2(e+fx)^2} dx$$

Optimal (type 5, 281 leaves, 5 steps):

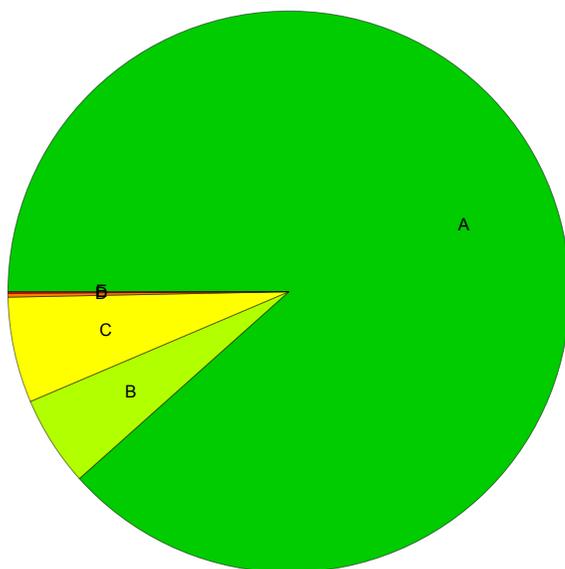
$$\frac{f (b d e + b c f - 2 a d f) (a + b x)^{1+m}}{(b c - a d) (b e - a f) (d e - c f)^2 (e + f x)} + \frac{d (a + b x)^{1+m}}{(b c - a d) (d e - c f) (c + d x) (e + f x)} + \left( \frac{d^2 (2 a d f - b (c f (2 - m) + d e m)) (a + b x)^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 (d e - c f)^3 (1 + m)} - \frac{f^2 (2 a d f - b (d e (2 - m) + c f m)) (a + b x)^{1+m} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, -\frac{f (a + b x)}{b e - a f}\right]}{(b e - a f)^2 (d e - c f)^3 (1 + m)} \right) /$$

Result (type 8, 24 leaves):

$$\int \frac{(a + b x)^m}{(c + d x)^2 (e + f x)^2} dx$$

## Summary of Integration Test Results

3189 integration problems



A - 2819 optimal antiderivatives

B - 166 more than twice size of optimal antiderivatives

C - 194 unnecessarily complex antiderivatives

D - 7 unable to integrate problems

E - 3 integration timeouts